Problem set 2 (due Thursday February 19th)

1 Starting from the full expression for the radial temperature profile of an irradiated disk,

$$\left(\frac{T_{\rm disk}}{T_*}\right)^4 = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{R_*}{r}\right) - \left(\frac{R_*}{r}\right)\sqrt{1-\left(\frac{R_*}{r}\right)^2}\right],$$

Taylor expand the terms on the right hand side to show that for $r/R_* >> 1$, $T_{\rm disk} \propto r^{-3/4}$, as we asserted in class.

2 Consider a disk with mass $M_{\rm disk} \sim \pi r^2 \Sigma$ and thickness h, at radius r from a star of mass M_*. By approximating the self-gravity of the disk as that of an infinite sheet, estimate the minimum Σ such that disk self-gravity dominates the vertical acceleration at $z = h$. Hence, show that,

$$\frac{M_{\rm disk}}{M_*} > \left(\frac{h}{r}\right),$$

is a rough condition for when self-gravity matters for the vertical structure.

3 A disk is vertically isothermal, with a profile,

$$\rho(z) = \rho_0 \exp[-z^2/2h^2],$$

as usual. The mid plane Keplerian velocity is $\Omega_{\rm K} = \sqrt{GM_*/r^3}$. Suppose that the disk has a radial variation of surface density and temperature,

$$\Sigma \propto r^{-\gamma}$$
$$T \propto r^{-\beta}$$

with γ and β constants. In cylindrical co-ordinates, the condition for hydrostatic equilibrium is,

$$r\Omega_{\rm g}^2 = \frac{GM_*}{(r^2+z^2)^{3/2}} r + \frac{1}{\rho}\frac{\partial P}{\partial r},$$

where Ω_g is the gas angular velocity. Find the lowest order expression for the gas angular velocity in the form $\Omega_g = \Omega_{\rm K}[1 - ...]$, where the departure from Keplerian rotation is a function of (h/r), (z/h) and the constants β and γ.