Midterm

1. Consider a transiting planet, of mass M_p, in an orbit with semi-major axis a and period P. If the planet is itself orbited by a moon of mass m_s and orbital radius a_s (around the planet), find an expression for the magnitude of variations δt in the onset of transits caused by the presence of the moon. For a typical hot Jupiter, what sort of timing precision would be needed to detect the presence of moons similar to the Galilean satellites via their effect on transit times?

2. Suppose that the equation of state of gas in the protoplanetary disk can be written in polytropic form, $P = K\rho^\gamma$. For $\gamma \neq 1$, derive the vertical density profile assuming hydrostatic equilibrium (and $z \ll r$). How does the solution compare to the isothermal ($\gamma = 1$) case we worked out previously?

3. Consider a model in which Neptune formed in place, by accreting from an annulus of material situated between 25 AU and 35 AU. First, use a Minimum Mass Solar Nebula-type argument to calculate the surface density Σ_p in solid material that would have had to be present in this region of the disk. Second, assume that as Neptune grows the gravitational focusing term remains constant,

$$F_g = \left[1 + \frac{{v_{\rm esc}}^2}{\sigma^2}\right].$$

   Determine the minimum value of F_g that would allow Neptune to form within 10 Myr.

4. Suppose that a planetesimal of radius R is internally heated by the decay of ^{26}Al, with a volumetric heating rate ϵ (units erg cm^{-3} s^{-1}). The specific heat of the material making up the planetesimal is C (units erg g^{-1} K^{-1}) and the melting temperature is T_m. The thermal diffusivity of the material is D (units cm^2 s^{-1}). By considering the heat equation $\partial T / \partial t = D \nabla^2 T$, derive an approximate expression for the size of the smallest body whose interior will be melted.

5. A stable planetary system orbits a star of mass M_*. Describe how you would expect the orbits of the planets to change, if the star loses mass (a) almost instantaneously (e.g. it blows off a shell of gas at much greater than the escape velocity) and (b) extremely slowly (e.g. via a weak spherically symmetric stellar wind). If the stability of the planetary system is set by the planetary separation in units of the Hill radius, how would the stability change in case (b)?