# Problem Set #5: Solutions

(1) (i) We have 1000 galaxies in a sphere of radius $R = 2\,\text{Mpc}$.

Number density $n = \dfrac{N}{\frac{4}{3}\pi R^3}$ $\quad N = 10^3$, $R = 2\,\text{Mpc} = 6.2 \times 10^{24}\,\text{cm}$

$$\Rightarrow \quad n = 29.8\ \text{Mpc}^{-3} = 1.0 \times 10^{-72}\ \text{cm}^{-3}$$

(ii) Collision time is given as:

$$t_c = \frac{1}{n \underbrace{\Sigma}_{\text{cross-section}} \underbrace{v}_{\text{velocity}}}$$

If a "collision" means any part of the galaxies over-lapping:

$$\Sigma = \pi \left( \underbrace{2}_{\text{depends on def}^{n}\text{ of collision.}} r_{gal} \right)^2$$

$$t_c = \frac{1}{4\pi r_{gal}^2\, n\, v}$$

$$= 8.2 \times 10^{17}\ \text{s}$$

$$= \underline{\underline{2.6 \times 10^{10}\ \text{yr}}}$$

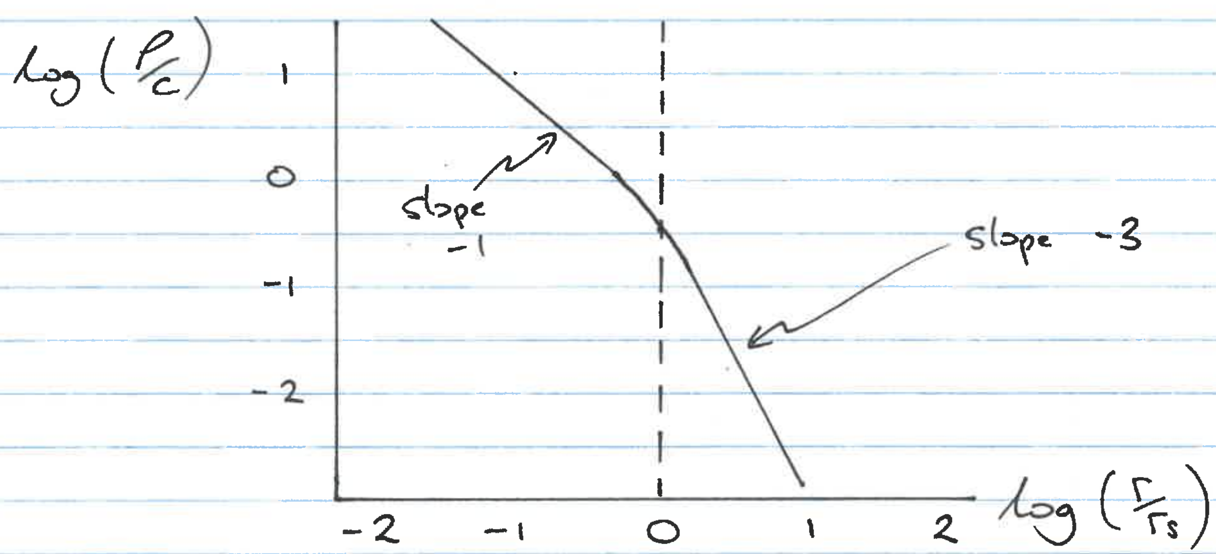
(2) The NFW profile is given by:

$$\rho = \frac{C}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

(i) For $r << r_s$, $\rho \simeq C / \left(\frac{r}{r_s}\right) \propto r^{-1}$

For $r >> r_s$ $\rho \simeq C / \left(\frac{r}{r_s}\right)^3 \propto r^{-3}$.

... so we have a broken power-law.




... graph is obviously schematic only!

(ii) This is just an exercise in integration.

$$M(r) = \int_0^r 4\pi r^2 \rho \, dr$$

$$= 4\pi C \int_0^r \frac{r^2 \, dr}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

$$x \equiv \frac{r}{r_s} \qquad dr = r_s \, dx$$

$$= 4\pi C \int \frac{r_s^3 \, x^2 \, dx}{x\,(1+x)^2}$$

$$t = 1+x \qquad dt = dx.$$

$$= 4\pi C r_s^3 \int \frac{(t-1)}{t^2} \, dt$$

$$= 4\pi C r_s^3 \int t^{-1} - t^{-2} \, dt.$$

$$= 4\pi C r_s^3 \left[ \ln t + t^{-1} \right]$$

$$= 4\pi C r_s^3 \left[ \ln\left(1+\frac{r}{r_s}\right) + \left(1+\frac{r}{r_s}\right)^{-1} \right]_0^r$$

$$= \underline{\underline{4\pi C r_s^3 \left[ \ln\left(1+\frac{r}{r_s}\right) + \left(1+\frac{r}{r_s}\right)^{-1} - 1 \right]}}$$

(3) Definitions:

Luminosity distance $d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2}$

Apparent magnitude $m = -2.5 \log f + \text{const.}$

For a source with $M = -19.6$ mag.

$$f \propto d_L^{-2}$$

$$\Rightarrow \quad m = 5 \log d_L + \text{const}'.$$

@ 10 pc $\quad m = M = -19.6.$

$$-19.6 = 5 \log\left(\frac{d_L}{10\,\text{pc}}\right) + \text{const}'$$

i.e. $$m = 5 \log\left(\frac{d_L}{10\,\text{pc}}\right) - 19.6.$$

$$= 5 \log\left(\frac{d_L}{\text{Mpc}}\right) - 19.6 + 5 \log 10^5.$$

$$= 5 \log\left(\frac{d_L}{\text{Mpc}}\right) + 5.4$$

(b) Model universes have:

$$\Omega_M = 0.26 \quad , \quad \Omega_\Lambda = 0.74 \qquad (A)$$
$$\Omega_M = 0.30 \quad , \quad \Omega_\Lambda = 0.70 \qquad (B)$$

Using the calculator, for $H_0 = 70 \text{ km s}^{-1}$ the luminosity distances and magnitudes are:

MODEL A:

$z = 0.5 \qquad d_L = 2873.2 \text{ Mpc} \qquad m = 22.692$
$z = 1.0 \qquad d_L = 6771.6 \text{ Mpc} \qquad m = 24.553$

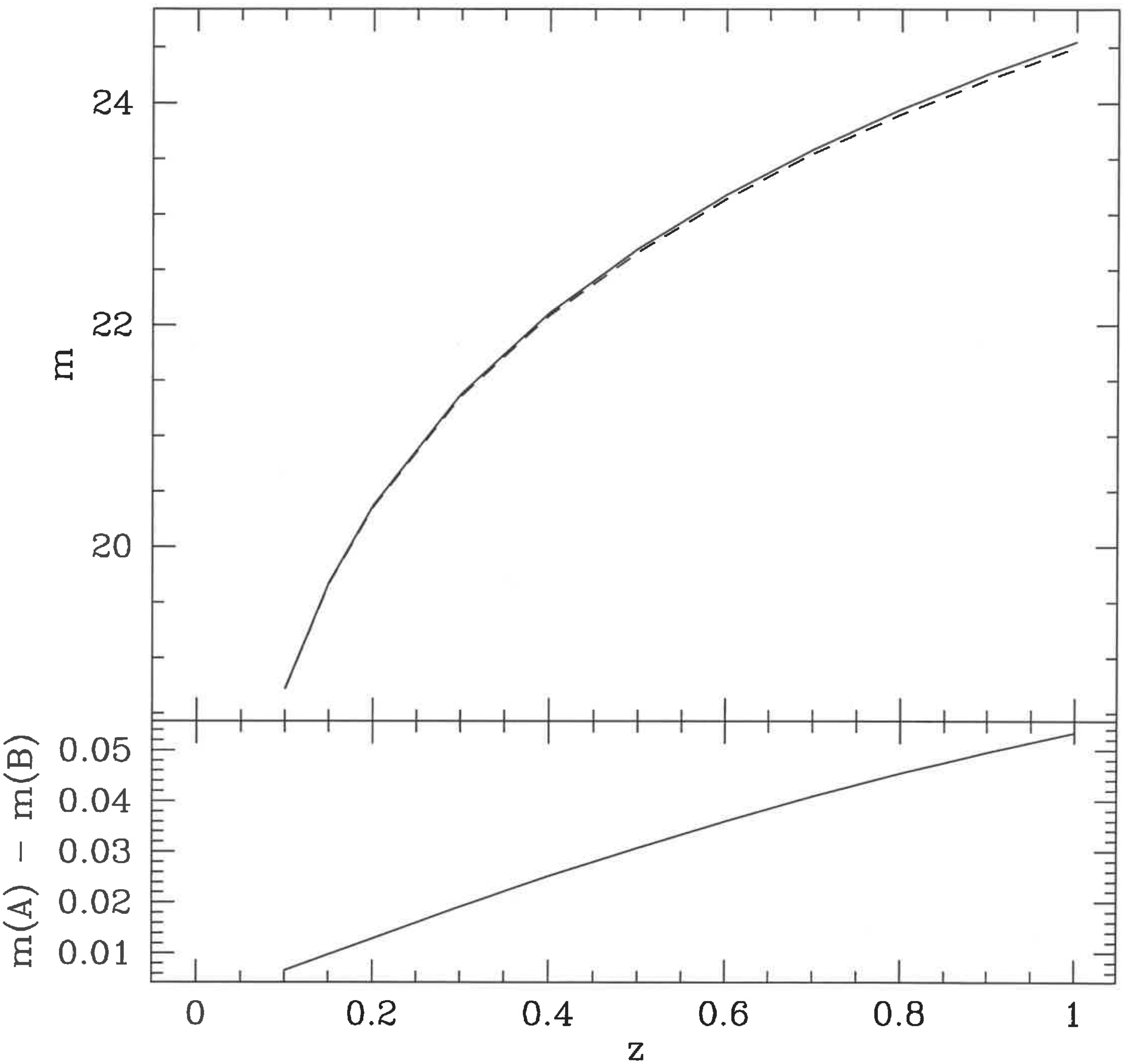
$$m(z=1) - m(z=0.5) = 1.861 \text{ mag.}$$

MODEL B:

$z = 0.5 \qquad d_L = 2832.8 \text{ Mpc} \qquad m = 22.661$
$z = 1.0 \qquad d_L = 6607.1 \text{ Mpc} \qquad m = 24.500$

$$m(z=1) - m(z=0.5) = 1.839 \text{ mag.}$$

$\Rightarrow$ The SN fade more between $z=0.5$ and $z=1.0$ in Model A, by about 0.02 magnitudes.



24
22
20
m
0.05
0.04
0.03
0.02
0.01
m(A) - m(B)
0
0.2
0.4
0.6
0.8
1
z