Problem Set 3 (due Tuesday March 3rd)

(1) A gravitational version of "Gauss' Law" can be written as,

$$\oint_S \mathbf{g} \cdot \mathrm{d}\mathbf{A} = -4\pi G M_{\mathrm{in}},$$

where $\mathbf{g}$ is the local acceleration of gravity at position $\mathrm{d}\mathbf{A}$ and the integral is over a closed surface that bounds an interior mass M_{in}.

Show that if we pick a spherical surface that is centered on and surrounds a spherically symmetric mass distribution, the above equation can be used to solve for g_r, the radial gravitational acceleration. Verify that this gives the usual answer.

(2) We now apply the above equation in a more interesting situation. Consider a galactic disk whose mid-plane is in the plane $z = 0$, at a distance from the center of the galaxy where the disk thickness is very small compared to the radius. In this case, we can approximate the gravitational force from the disk as being that of an infinite slab, and the gravitational acceleration vector is always vertical (i.e. oriented along $\pm z$).

Derive an expression for the gravitational acceleration felt by a star at a height z above the disk mid-plane, assuming that the star always remains within the disk, and that the disk density is a constant ρ.

Show that the motion of the star in the vertical direction can be described by an equation of the form,

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + kz = 0,$$

and express k in terms of ρ and G.

Solve this equation to find z as a function of time.

If the mass density in the Milky Way disk is 0.1 $M_\odot$ pc^{-3}, estimate the vertical oscillation period of the Sun as it orbits the Galactic center.