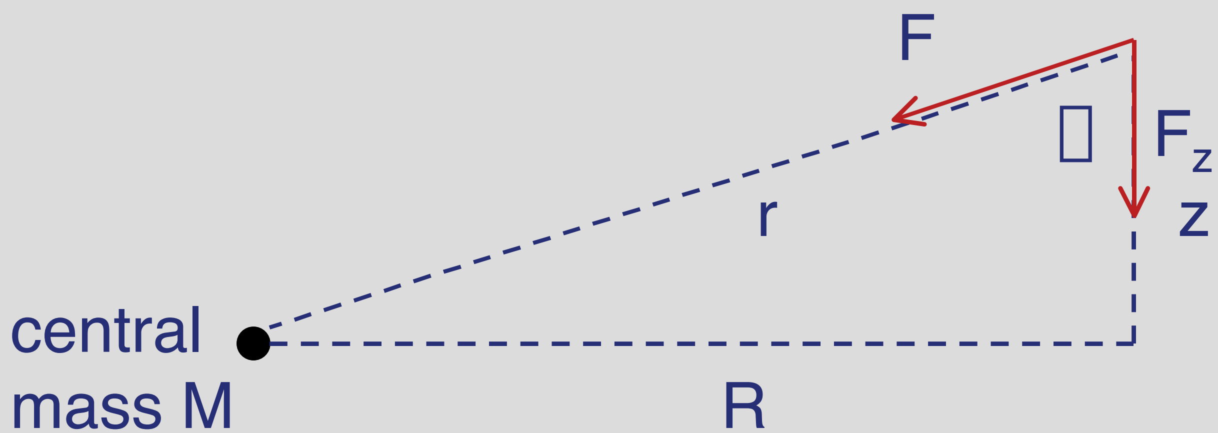
# Accretion Disks

Luminosity of AGN derives from gravitational potential energy of gas spiraling inward through an **accretion disk.** Derive structure of the disk, and characteristic temperatures of the gas.

First consider **vertical structure**:




Gas at radius R (cylindrical) and height z above the disk midplane

Gravitational acceleration in vertical direction:

$$g = \frac{GM}{r^2}\cos\theta = \frac{GM}{r^2}\frac{z}{r} \approx \frac{GM}{R^3}z \qquad (z \ll R)$$

If the gas is supported against gravity by a pressure gradient, force balance in the vertical direction gives:

$$\frac{dP}{dz} = -\rho g$$

Assume the disk is isothermal in the vertical direction with sound speed $c_s$. The pressure is then:

$$P = \rho c_s^2$$

Solve for the vertical structure:

$$c_s^2 \frac{d\rho}{dz} = -\rho \frac{GM}{R^3} z = -\Omega^2 \rho z$$

$\Omega$ is angular velocity in disk

$$\int \frac{d\rho}{\rho} = -\frac{\Omega^2}{c_s^2} \int z\,dz$$

$$\Rightarrow \quad \rho = \rho_{z=0} e^{-\frac{\Omega^2 z^2}{2c_s^2}}$$

$\rho_{z=0}$ is density in disk midplane

Rewrite this equation as: $\rho = \rho_{z=0} e^{-z^2/h^2}$

…where h is the vertical scale height of the disk. Since $\Omega = v_\phi / r$, can write h as:

$$h^2 \equiv \frac{2c_s^2}{\Omega^2} = \frac{2c_s^2 R^2}{v_\phi^2}$$

$$\Rightarrow \frac{h}{R} \sim \frac{c_s}{v_\phi}$$

The thickness of the disk as a fraction of the radius is given by the ratio of the sound speed to the orbital velocity.

A disk for which (h / R) << 1 is described as a geometrically **thin** disk. Structure of thin disks is relatively simple because radial pressure forces can be neglected - i.e. $v_\phi$ for the gas is the same as a particle orbiting at the same radius.

## Angular momentum transport

If the disk is thin, then orbital velocity of the gas is Keplerian:

$$v_\phi = \sqrt{\frac{GM}{R}}$$

Specific angular momentum $v_\phi R$ is: $l = \sqrt{GMR}$

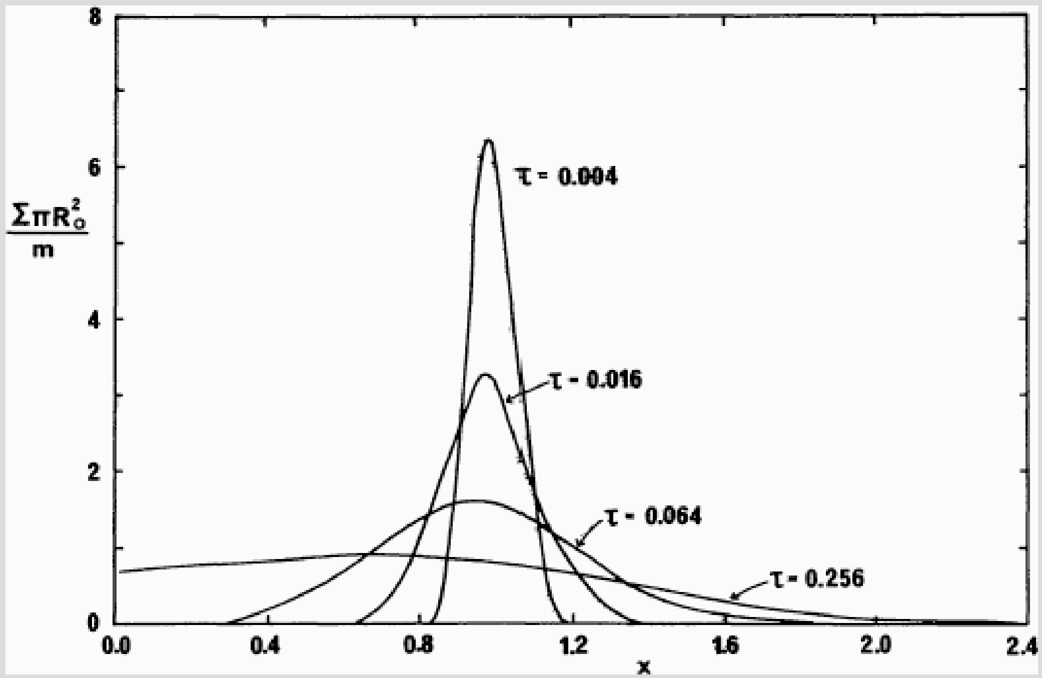
i.e. increasing outwards. Gas at large R has too much angular momentum to be accreted by the black hole.

To flow inwards, gas must lose angular momentum, either:

- By **redistributing** the angular momentum within the disk (gas at small R loses angular momentum to gas further out and flows inward)
- By loss of angular momentum from the entire system. e.g. a wind from the disk could take away angular momentum allowing inflow

Redistribution of angular momentum within a thin disk is a *diffusive* process - a narrow ring of gas spreads out under the action of the disk *viscosity*:

gas surface density

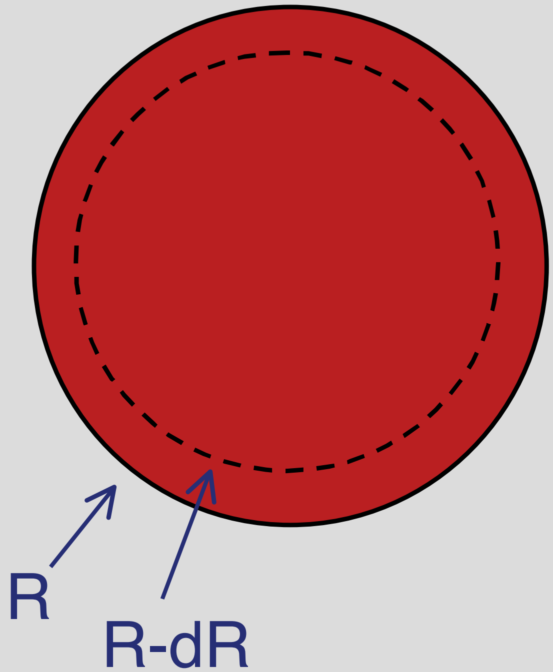



radius R

With increasing time:

- **Mass** all flows inward to small R and is accreted
- **Angular momentum** is carried out to very large R by a vanishingly small fraction of the mass.

# Radiation from thin disk accretion




Consider gas flowing inward through a thin disk. Easy to estimate the radial distribution of temperature.

Potential energy per unit mass at radius R in the disk is:

$$E = -\frac{GM}{R} \quad \Rightarrow \quad \frac{dE}{dR} = \frac{GM}{R^2}$$

Suppose mass dM flows inward distance dR. Change in potential energy is:

$$\Delta E = \frac{GM}{R^2} dM dR$$

Half of this energy goes into increased kinetic energy of the gas. If the other half is radiated, luminosity is:

$$L = \frac{GM\dot{M}}{2R^2} dR$$

Divide by the radiating area, 2 x 2πR x dR to get luminosity per unit area. Equate this to the rate of energy loss via blackbody radiation:

$$\frac{GM\dot{M}}{8\pi R^3} = \sigma T^4$$

σ is Stefan-Boltzmann constant

Gives the radial temperature distribution as:

$$T = \left(\frac{GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4}$$

Correct dependence on mass, accretion rate, and radius, but **wrong** prefactor. Need to account for:

- Radial energy flux through the disk (transport of angular momentum also means transport of energy)
- Boundary conditions at the inner edge of the disk

Correcting for this, radial distribution of temperature is:

$$T(R) = \left[ \frac{3GM\dot{M}}{8\pi\sigma R^3} \left( 1 - \sqrt{\frac{R_{in}}{R}} \right) \right]^{1/4}$$

…where $R_{in}$ is the radius of the disk inner edge. For large radii $R >> R_{in}$, we can simplify the expression to:

$$T(R) \approx \left( \frac{3GM\dot{M}}{8\pi\sigma R_s^3} \right)^{1/4} \left( \frac{R}{R_s} \right)^{-3/4}$$

with $R_s = 2GM / c^2$ the Schwarzschild radius as before.
For a hole accreting at the Eddington limit:

- Accretion rate scales linearly with mass
- Schwarzschild radius also increases linearly with mass

Temperature at fixed number of $R_s$ decreases as $M^{-1/4}$ - disks around **more massive black holes are cooler.**

For a supermassive black hole, rewrite the temperature as:

$$T(R) \approx 6.3 \times 10^5 \left( \frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left( \frac{M}{10^8 M_{sun}} \right)^{-1/4} \left( \frac{R}{R_s} \right)^{-3/4} \text{ K}$$

Accretion rate at the Eddington limiting luminosity (assuming $\eta$=0.1)

A thermal spectrum at temperature T peaks at a frequency:

$$h\nu_{\max} \approx 2.8kT$$

An inner disk temperature of ~$10^5$ K corresponds to strong emission at frequencies of ~$10^{16}$ Hz. Wavelength ~50 nm.

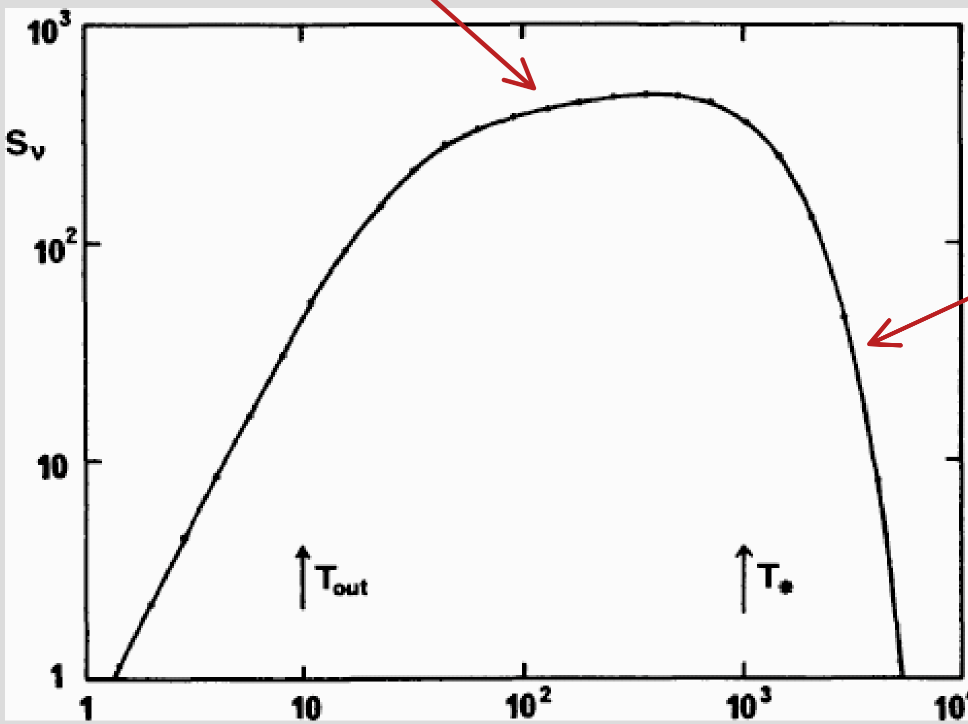
Expect disk emission in AGN accreting at close to the Eddington limit to be strong in the ultraviolet - origin of the broad peak in quasar SEDs in the blue and UV.

Disk has annuli at many different temperatures - spectrum is weighted sum of many blackbody spectra.

flat - $\nu^{1/3}$ - region

exponential cut off at high $\nu$

flux




frequency

Consistent with the broad spectral energy distribution of AGN in the optical and UV regions of the spectrum.