Eddington limit

For an AGN with an observed (bolometric) luminosity L, can estimate the *minimum* mass of the black hole involved

Suppose the gas around the black hole is:
- Spherically symmetric
- Fully ionized hydrogen

At distance r, flux is: $F = \frac{L}{4\pi r^2}$

This is flux of **energy**. Since momentum of a photon of energy E is E / c, **momentum flux** due to radiation is:

$$P_{rad} = \frac{L}{4\pi r^2 c}$$

This is the pressure that would be exerted on a totally absorbing surface at distance r from the source.

Force exerted on the gas depends upon the opacity (ie the fraction of the radiation absorbed per unit mass of gas).

Minimum force is given by the absorption due just to free electrons. This is given by the Thomson cross-section:

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$$

Outward radiation force on a single electron is:

$$F_{rad} = \frac{L\sigma_e}{4\pi r^2 c}$$

Inward force due to gravity of a central point mass M is:

$$F_{grav} = \frac{GM(m_p + m_e)}{r^2} \approx \frac{GMm_p}{r^2}$$

Note: include proton mass in F_{grav} since electrons and protons coupled due to electrostatic forces.

Setting $F_{rad} = F_{grav}$, and solving for L:

$$L = \frac{4\pi G c m_p}{\sigma_e} M$$

$$= 1.26 \times 10^{38} \left(\frac{M}{M_{sun}} \right) \text{erg s}^{-1}$$

This luminosity is known as the **Eddington limit**. It is the maximum luminosity of a source of mass M, which is powered by spherical accretion of gas.

Can invert the argument. If a source with observed luminosity L is radiating at the Eddington limit, the mass would be:

$$M_E = 8 \times 10^5 \left(\frac{L}{10^{44} \text{ erg s}^{-1}} \right) M_{sun}$$

This is a minimum mass - source could actually be radiating at much less than the Eddington limit.

A luminous Seyfert galaxy has $L \sim 10^{44}$ erg s^{-1}. Conclude that the black hole mass in those galaxies must be at least 10^6 Solar masses - i.e. could be comparable to the Milky Way black hole.

For quasars, $L_{QSO} \sim 10^{46}$ erg s^{-1}. Black hole masses in those systems must be at least 10^8 Solar masses.

Consistent with the masses derived from studies of now quiescent galaxy nuclei.

Fuelling Active Galactic Nuclei

How fast must gas be supplied to the black hole to produce typical AGN luminosities (10^{44} - 10^{46} erg s^{-1})?

Define the efficiency of the accretion process ϵ:

$$L = \epsilon \dot{M} c^2$$

Accretion rate: units g s^{-1} or Solar masses per year

A mass Δm of gas at infinity has zero potential energy. Energy available if the gas spirals in to radius r is:

$$\Delta E = \frac{G M_{BH} \Delta m}{r} \longrightarrow L \sim \frac{G M_{BH} \dot{M}}{r}$$

Really an upper limit - not all the potential energy will be radiated as the gas falls in…

Assume that the gas falls in to the last stable orbit at 6 GM / c^2 before being swallowed by the hole. Estimate of the efficiency is then:

$$\eta = \frac{GM_{BH}\dot{M}}{6GM_{BH}/c^2} \times \frac{1}{\dot{M}c^2} \approx 0.17$$

Dubious Newtonian calculation - but gives right order of magnitude. Actual efficiency of disk accretion onto a black hole is estimated to be:

- Schwarzschild black hole: η = 0.06
- Kerr black hole (corotating disk): η = 0.42

Standard estimate is η = 0.1. Using this, mass flow needed to sustain a quasar is:

$$\dot{M} \approx \frac{10^{46} \text{ erg s}^{-1}}{0.1 \times c^2} \approx 10^{26} \text{ g s}^{-1} \approx 2 \text{ Solar masses yr}^{-1}$$

Mass supply needed is fairly modest…