Black holes are completely specified by their mass M, angular momentum J, and charge Q (likely ~zero). *No-hair theorem:*

Q=0, J=0: **Schwarzschild black hole**

Spherically symmetric. Solution has two important radii for our purposes:

- An **event horizon** at $R_s = \frac{2GM}{c^2}$ (Schwarzschild radius)

  No matter, radiation, or information can propagate outwards through this radius

  - R_s = 3 km for one Solar mass black hole
  - R_s = 2 AU for a 10^8 Solar mass black hole

- A **last stable circular orbit** at $R_{ms} = \frac{6GM}{c^2}$

Outside R_{ms} test particles can orbit indefinitely in stable circular orbits

Inside R_{ms} orbits are unstable, particles spiral rapidly past the event horizon and into the black hole

Defines the inner edge of the gas disk in AGN

Sets a minimum orbital period. Roughly,

$$t = \frac{2\pi R_{ms}}{v_{ms}} = \frac{2\pi R_{ms}}{\sqrt{c^2/6}} = 12\sqrt{6}\pi \frac{GM}{c^3}$$

Newtonian - so this is not quite correct…

For 10^8 Solar masses, this is about 12 hours
For 10 Solar masses, ~ 5 ms

Q=0, J and M arbitrary: **Kerr black hole**

Axisymmetric solution - hole has a preferred rotation axis

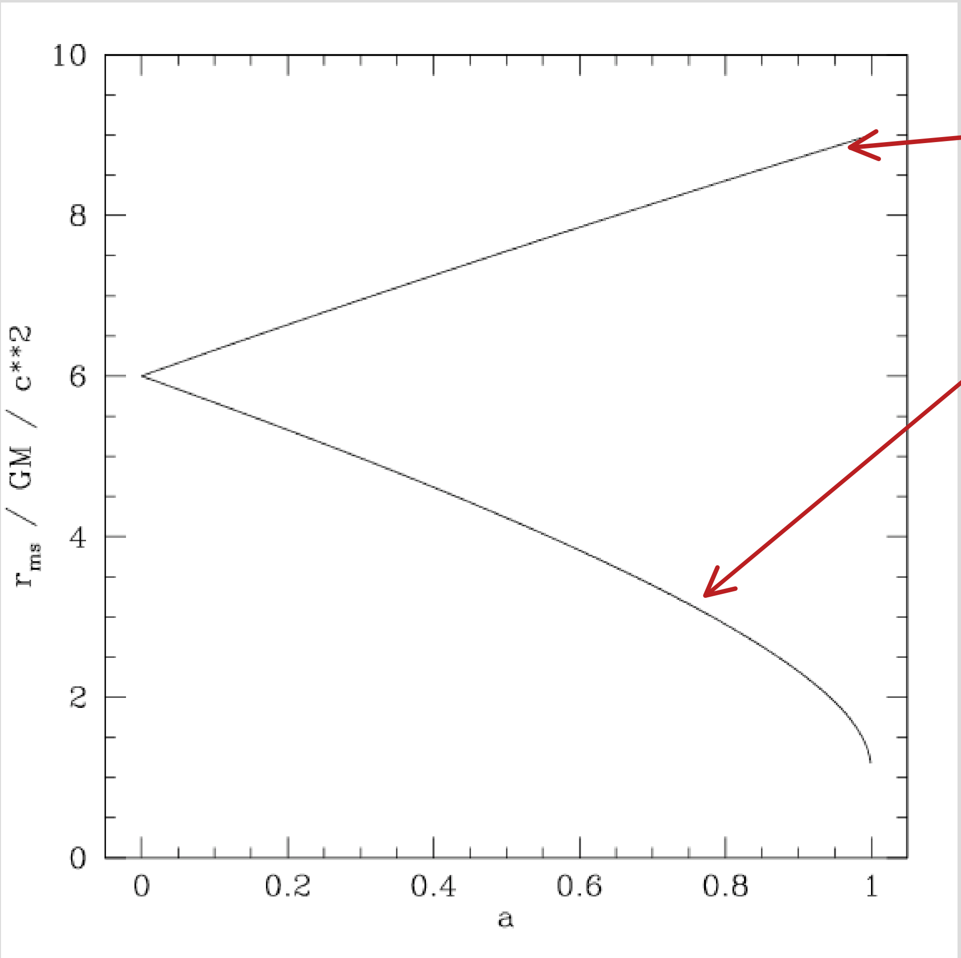
Define the amount of angular momentum via a dimensionless **spin parameter**:

$$a = \frac{cJ}{GM^2}$$

Maximum angular momentum of a Kerr black hole corresponds to a spin parameter a = 1.

Cannot spin a Kerr hole up beyond this limit.

Kerr black holes also have an event horizon, and a last stable circular orbit. For particles orbiting in the equatorial plane of the hole, R_{ms} is *smaller* than the Schwarzschild case if the orbit corotates with the hole, *larger* for counter-rotation



Counter rotating
Corotating

Gas in a disk can spiral deeper into the potential well before reaching R_{ms} around a Kerr black hole

More energy can be extracted