Motion under gravity

Motions of the stars and gas in the disk of a spiral galaxy are approximately circular (v_R and v_z << v_ϕ).

Define the circular velocity at radius r in the galaxy as V(r). Acceleration of the star moving in a circular orbit must be provided by a net inward gravitational acceleration:

$$\frac{V^2(r)}{r} = -a_r(r)$$

To calculate $a_r(r)$, must in principle sum up gravitational force from bulge, disk and halo.

For spherically symmetric mass distributions:

- Gravitational force at radius r due to matter interior to that radius is the same as if all the mass were at the center.
- Gravitational force due to matter outside is zero.

Thus, if the mass enclosed within radius r is M(r), gravitational acceleration is:

$$a_r = -\frac{GM(r)}{r^2}$$

(minus sign reflecting that force is directed inward)

Bulge and halo components of the Galaxy are at least approximately spherically symmetric - assume for now that those dominate the potential.

Self-gravity due to the disk itself is not spherically symmetric…

If you are familiar with vector calculus, *Sparke & Gallagher 3.1* derives Poisson's equation needed to calculate force from an arbitrary mass distribution.

Note: no simple form for the force from disks with realistic surface density profiles…

Rotation curves of simple systems

1. Point mass M:

$$V(r) = \sqrt{\frac{GM}{r}}$$

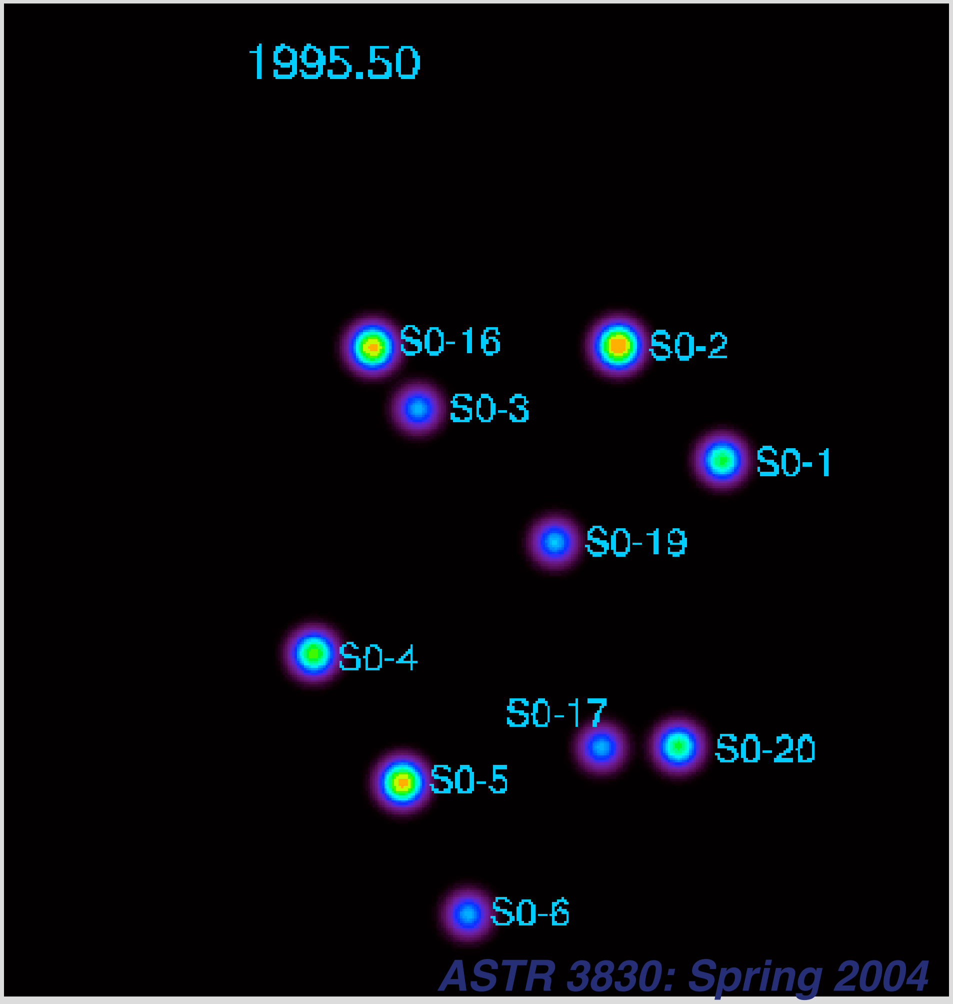
Applications:

- Close to the central black hole (r < 0.1 pc)
- `Sufficiently far out' that r encloses all the Galaxy's mass

eg image of the Galactic center

Note: non-circular orbits and presence of massive stars

Movie: Andrea Ghez's group



2. Uniform sphere:

If the density ρ is constant, then:

$$M(r) = \frac{4}{3}\pi r^3 \rho$$

$$V(r) = \sqrt{\frac{4\pi G\rho}{3}}\, r$$

Rotation curve rises linearly with radius, period of the orbit $2\pi r$ / $V(r)$ is a constant independent of radius.

Roughly appropriate for central regions of spiral galaxies.

3. Power law density profile:

If the density falls off as a power law:

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\alpha}$$

…with $\alpha < 3$ a constant, then:

$$V(r) = \sqrt{\frac{4 \pi G \rho_0 r_0^{\alpha}}{3 - \alpha}}\, r^{1-\alpha/2}$$

For many galaxies, circular speed curves are approximately flat (V(r) = constant). Suggests that mass density in these galaxies may be proportional to r^{-2}.

4. Simple model for a galaxy with a core:

Spherical density distribution:

$$4\pi G\rho(r) = \frac{V_H^2}{r^2 + a_H^2}$$

- Density tends to constant at small r
- Density tends to r^{-2} at large r

Corresponding circular velocity curve is:

$$V(r) = V_H \sqrt{1 - \frac{a_H}{r} \arctan\left(\frac{r}{a_H}\right)}$$

Resulting rotation curve