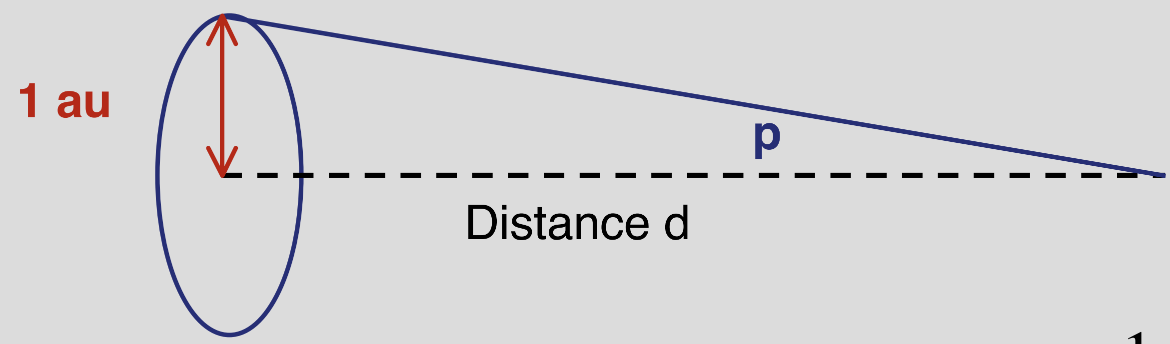
The Initial Mass Function of stars

Motion of the Earth around the Sun leads to an apparent shift of nearby stars relative to more distant objects - this is **trignometric parallax**.



Measure the parallax angle p, then: $\tan p = \frac{1 \text{ au}}{d}$

approximating tan p = p as angle p is very small $\longrightarrow$ $p = \frac{1 \text{ au}}{d}$

In this equation, p is in radians and d is in astronomical units (au).

Definition: the parsec is the distance at which a star has a parallax of one arcsecond.

$$1 \text{ arcsec} = 1'' = \frac{1}{3600} \text{degree} = \frac{1}{3600} \times \frac{2\pi}{360} \text{radians}$$

Thus: $$1 \text{ pc} = \frac{1 \text{ au}}{\frac{1}{3600} \times \frac{2\pi}{360}} = 2.06 \times 10^{5} \text{ au} = 3.09 \times 10^{18} \text{ cm}$$

Hipparcos satellite measured positions of ~10^5 bright stars to an accuracy of ~0.001 arcsec, and thereby determined distances to a large sample of stars with ~100 parsecs.

Luminosity functions

Suppose we measure the distance and apparent magnitude m of all stars within some limiting distance d_{max} (a `volume limited sample' - easier in theory than in practice!).

Convert from apparent magnitude to absolute magnitude M using the known distance d to each star and the definition:

$$M = m - 5\log_{10}\left(\frac{d}{10\ \mathrm{pc}}\right)$$

absolute magnitude in some waveband, e.g. in the visual M_V

Finally count the number of stars with M between (M-0.5) and (M+0.5), and divide by the volume $\frac{4}{3}\pi d_{\mathrm{max}}^3$ surveyed. Gives the **luminosity function**.

More formally:

$$\Phi(M)\Delta M$$ = number density (stars per pc^3) of stars with absolute magnitude M between M and $M+\Delta M$

↑

luminosity function

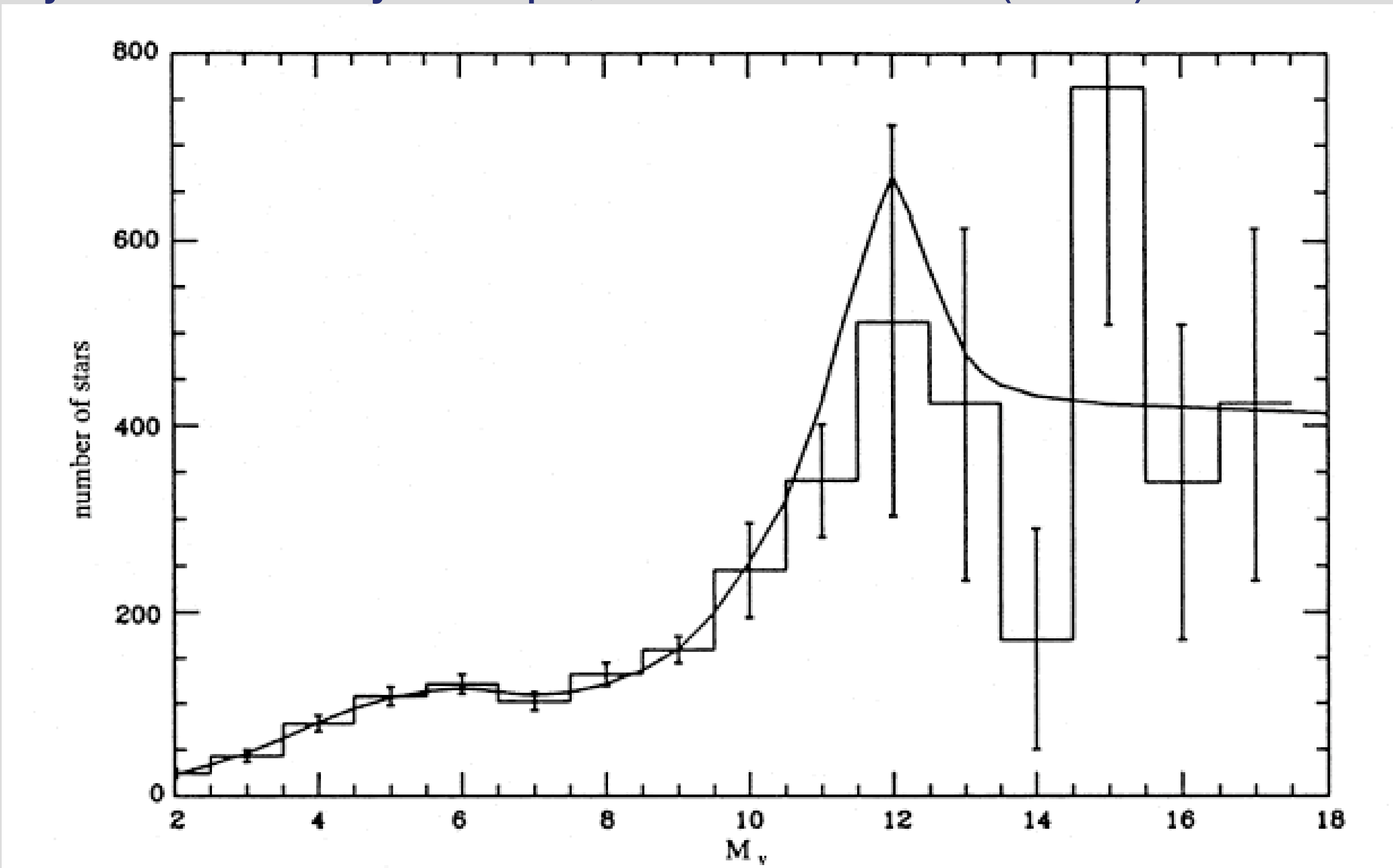
Identical concept applies to galaxies (though typically measure numbers of galaxies per Mpc^3 rather than per pc^3).

Can be hard to measure $\Phi(M)$:

- for very low mass stars (M large), which are dim unless very close to the Sun
- for massive stars (M small), which are rare

Luminosity function is the basic observable for studying a population of stars.

Local luminosity function (stars with d < 20 pc) for the Milky Way measured by Kroupa, Tout & Gilmore (1993):



bright stars

faint stars

Initial Mass Function (IMF)

Starting from the observed luminosity function, possible to derive an estimate for the Initial Mass Function (IMF). To define the IMF, imagine that we form a large number of stars. Then:

$$\xi(M)\Delta M \quad = \quad$$ the **number** of stars that have been born with initial masses between M and M+ΔM (careful not to confuse mass and absolute magnitude here)

this is the Initial Mass Function or IMF

The IMF is a more fundamental theoretical quantity which is obviously related to the star formation process. Note that the IMF only gives the distribution of stellar masses immediately after stars have formed - it is **not** the mass distribution in, say, the Galactic disk today.