Nuclear processes in stars

Mass of nuclei with several protons and / or neutrons does not exactly equal mass of the constituents - slightly smaller because of the **binding energy** of the nucleus.

Since binding energy differs for different nuclei, can release or absorb energy when nuclei either fuse or fission.

Example: $4 \times {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He}$

4 protons, each of mass 1.0081 atomic mass units: 4.0324 amu

mass of helium nucleus: 4.0039 amu

Mass difference: 0.0285 amu = 4.7 x 10^{-26} g

$$\Delta E = \Delta M c^2 = 4.3 \times 10^{-5}\ \mathrm{erg} = 27\ \mathrm{MeV}$$

General calculation: define the binding energy of a nucleus as the energy required to break it up into constituent protons and neutrons.

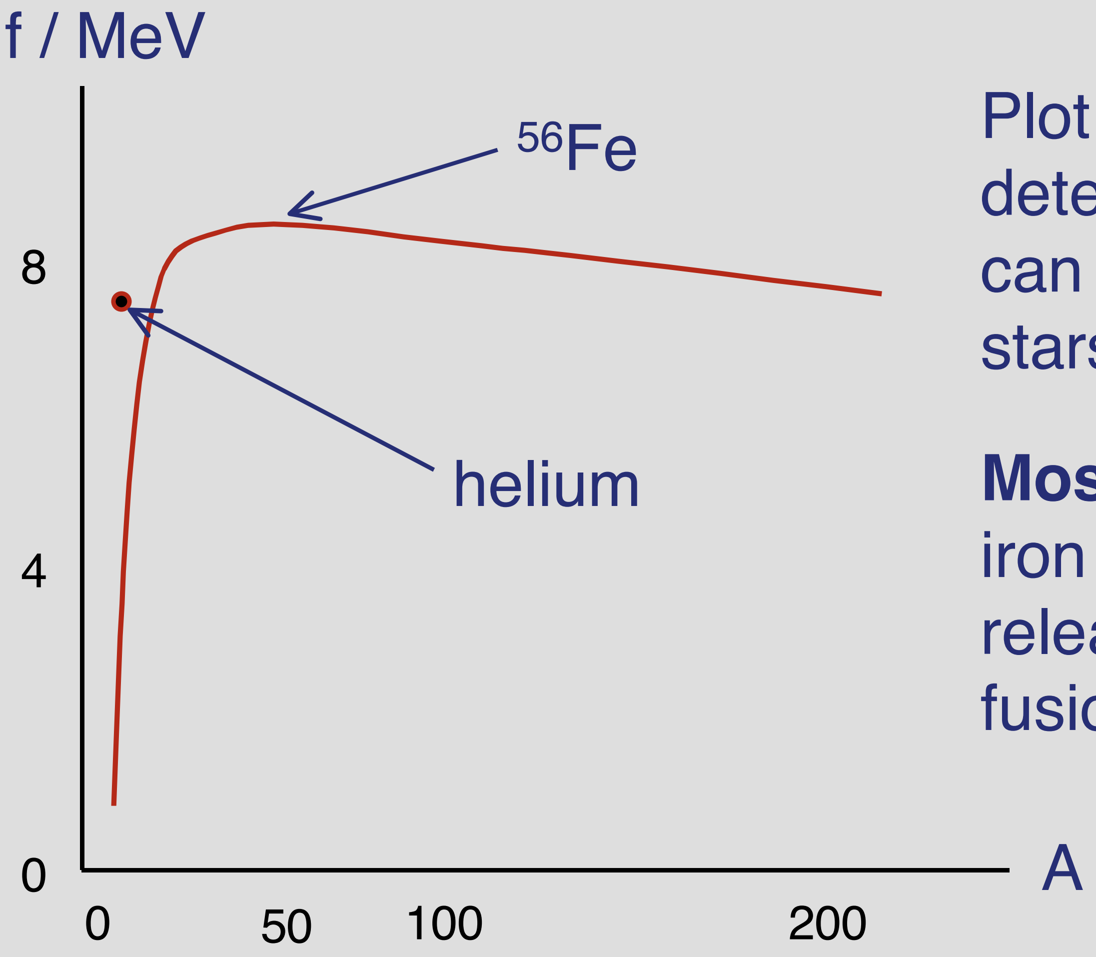
Suppose nucleus has:

- Proton number Z
- Atomic mass number A (number of protons + neutrons)
- Mass M_{nuc}

Binding energy: $E_B = \left[\left(A - Z \right) m_n + Z m_p - M_{nuc} \right] c^2$

Most useful quantity for considering which nuclear reactions yield energy is the binding energy *per nucleon* - defined as:

$$f = \frac{E_B}{A}$$



Plot of f vs A largely determines which elements can be formed in different stars/

Most bound nucleus is iron 56: A < 56 fusion releases energy, A > 56 fusion requires energy.

Yield for fusion of hydrogen to ^{56}Fe: ~8.5 MeV per nucleon
Most of this is already obtained in forming helium (6.6 MeV)

Drawn curve as smooth - actually fluctuates for small A -
He is more tightly bound than `expected'.

Energetics of fusion reactions

Nuclei are positively charged - repel each other.
If charges on the nuclei are Z_1e and Z_2e, then at distance d the electrostatic energy is:

$$E = \frac{Z_1 Z_2 e^2}{d}$$

If the nuclei approach sufficiently closely, short range nuclear forces (attractive) dominate and allow fusion to take place.

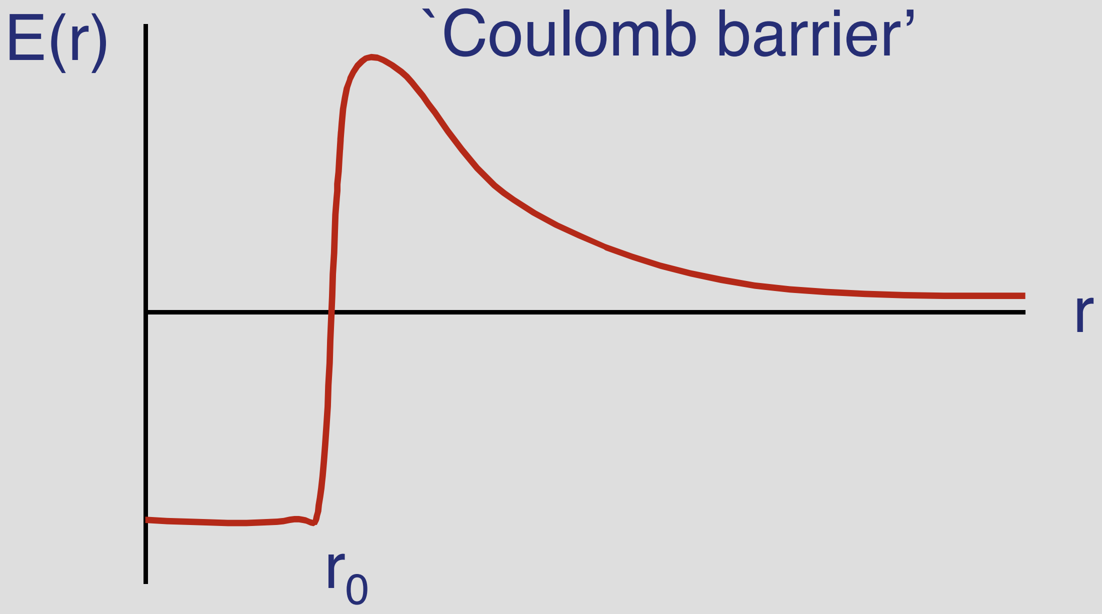
Nuclear material has roughly constant density, so `close enough' means within a distance:

$$r_0 \approx 1.44 \times 10^{-13} A^{1/3} \text{ cm}$$

↑

atomic mass number

Schematically:



At $r = r_0$, height of the Coulomb barrier is:

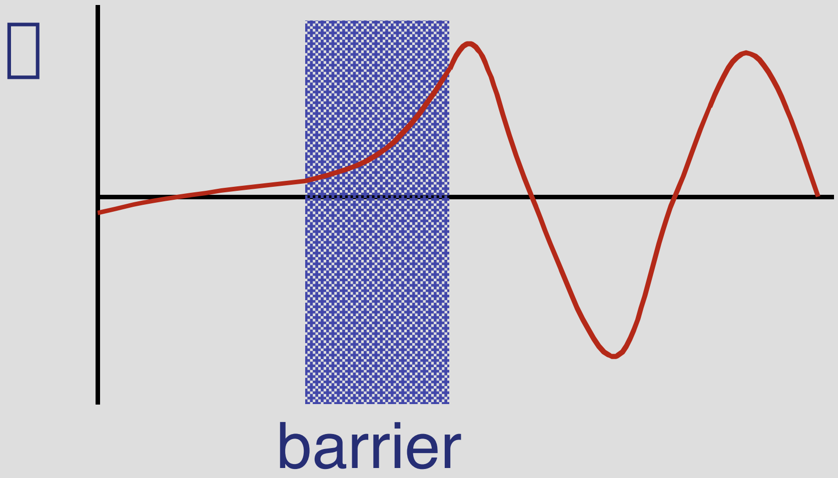
$$E = \frac{Z_1 Z_2 e^2}{r_0} \sim Z_1 Z_2 \text{ MeV}$$

i.e. of the order of 1 MeV for two protons…

For Solar core conditions $T = 1.5 \times 10^7$ K
Thermal energy of particles = kT = 1300 eV = 10^{-3} MeV

Classically, there are *zero* particles in a thermal distribution with enough energy to surmount the Coulomb barrier and fuse.

Quantum mechanically, lower energy particles have a very small but non-zero probability of tunnelling through the barrier:



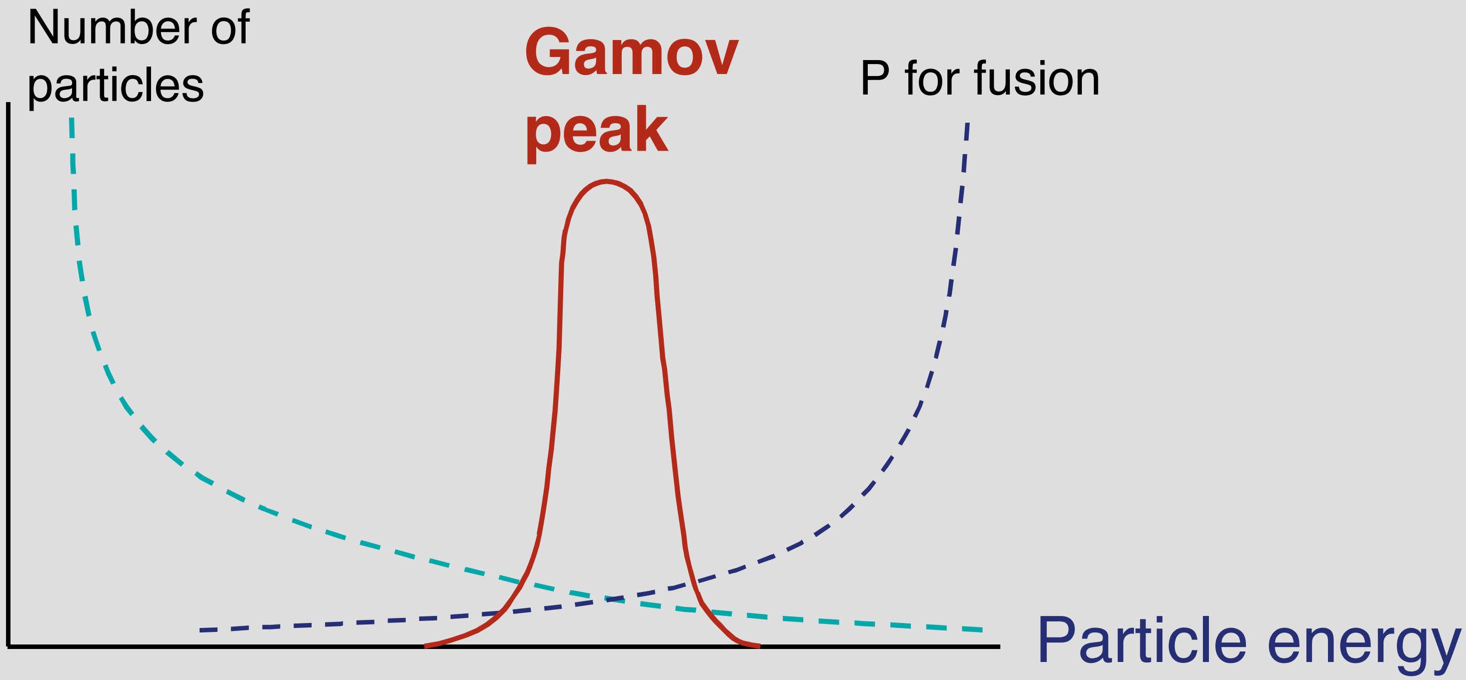
Probability of finding particle ~ $|\psi|^2$ - if barrier is not too wide then non-zero wavefunction allows some probability of tunnelling…

Probability of tunnelling depends upon the energy of the particles, their mass, and the charge:

$$P \propto E^{-1/2} e^{-2\pi\eta} \qquad \eta = \left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}}$$

- P increases rapidly with E
- P decreases with Z_1Z_2 - lightest nuclei can fuse more easily than heavy ones
- Higher energies / temperatures needed to fuse heavier nuclei, so different nuclei burn in well-separated phases during stellar evolution.

Competition: most energetic nuclei most likely to fuse, but very few of them in a thermal distribution of particle speeds:



Narrow range of energies around the Gamov peak where significant numbers of particles in the plasma are able to fuse. Energy is >> typical thermal energy, so fusion is slow.

Nuclear reactions in the Sun

Almost all reactions involve collisions of only two nuclei. So making helium from four protons involves a sequence of steps. In the Sun, this sequence is called the **proton-proton chain**:

Step 1 $^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \square_{e}$

deuteron - one proton + one neutron

positron

electron neutrino

This is the critical reaction in the proton-proton chain. It is slow because forming a deuteron from two protons requires transforming a proton into a neutron - this involves the weak nuclear force so it is slow…

Beyond this point, several possibilities. Simplest:

<u>Step 2</u> **$^{2}\mathrm{H} + {}^{1}\mathrm{H}$ -> $^{3}\mathrm{He}$ + □**

<u>Step 3</u> **$^{3}\mathrm{He} + {}^{3}\mathrm{He}$ -> $^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H}$**

Results of this chain of reactions:

- Form one ^{4}He nucleus from 4 protons
- Inject energy into the gas via energetic particles: one positron, one photon, two protons
- Produce one electron neutrino, which will escape the star without being absorbed.

Energy yield is ~10^{-5} erg per proton, so ~4 x 10^{38} reactions per second needed to yield L_{sun}. About 0.65 billion tonnes of hydrogen fusing per second.