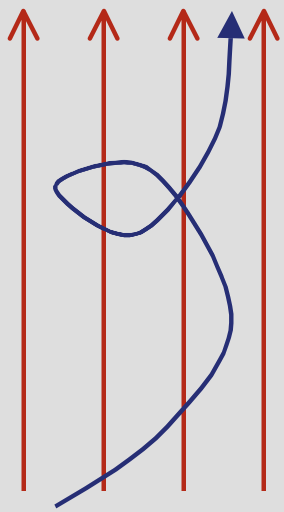
Cyclotron and synchrotron radiation



Electron moving perpendicular to a magnetic field feels a Lorentz force.

➡ Acceleration of the electron.

➡ Radiation (Larmor's formula).

Define the Lorentz factor: $$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

Non-relativistic electrons: ($\gamma \sim 1$) - **cyclotron radiation**

Relativistic electrons: ($\gamma >> 1$) - **synchrotron radiation**

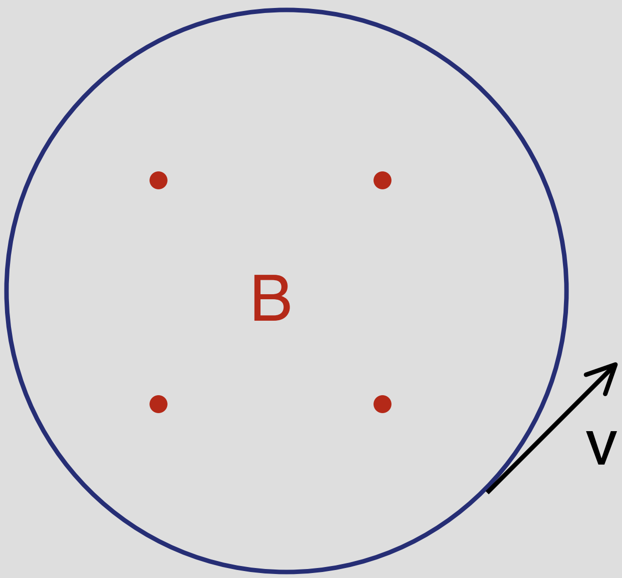
Same physical origin but very different spectra - makes sense to consider separate phenomena.

Start with the non-relativistic case:

Particle of charge q moving at velocity **v** in a magnetic field **B** feels a force:

$$\mathbf{F} = \frac{q}{c}\mathbf{v} \times \mathbf{B}$$

Let v be the component of velocity perpendicular to the field lines (component *parallel* to the field remains constant). Force is constant and normal to direction of motion.



Circular motion: acceleration -

$$a = \frac{qvB}{mc}$$

…for particle mass m.

Let **angular velocity** of the rotation be ω_B. Condition for circular motion:

$$m\frac{v^2}{r} = \frac{qvB}{c}$$

$$m\omega_B v = \frac{qvB}{c}$$

$$\omega_B = \frac{qB}{mc}$$

Use c.g.s. units when applying this formula, i.e.

- electron charge = 4.80 x 10^{-10} esu
- B in Gauss
- m in g
- c in cm/s

Power given by Larmor's formula:

$$P = \frac{2q^2}{3c^3}\left|\mathbf{a}\right|^2 = \frac{2q^2}{3c^3}\left(\frac{qvB}{mc}\right)^2 = \frac{2q^4\beta^2B^2}{3c^3m^2}$$

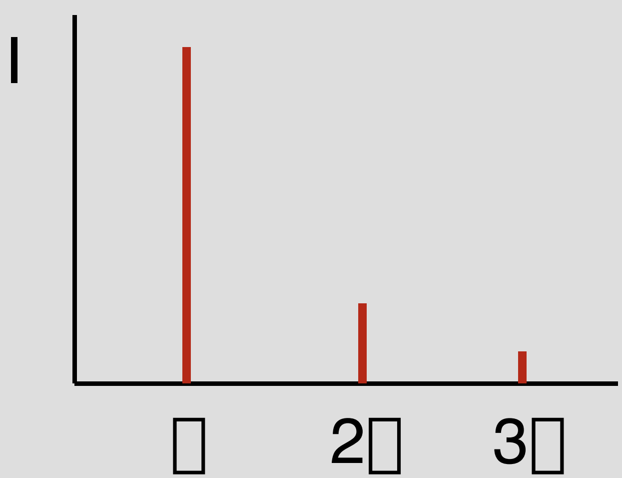
where β=v/c

$$P = \frac{2q^4 \gamma^2 B^2}{3c^3 m^2}$$

Magnetic energy density is $B^2 / 8\pi$ (c.g.s.) - energy loss is proportional to the energy density.

Energy loss is largest for low mass particles, electrons radiate much more than protons (c.f. highest energy particle accelerators are proton / antiproton not electron / positron).

Cyclotron spectrum: ideally a single line at: $\nu = \frac{\omega}{2\pi} = \frac{qB}{2\pi mc}$



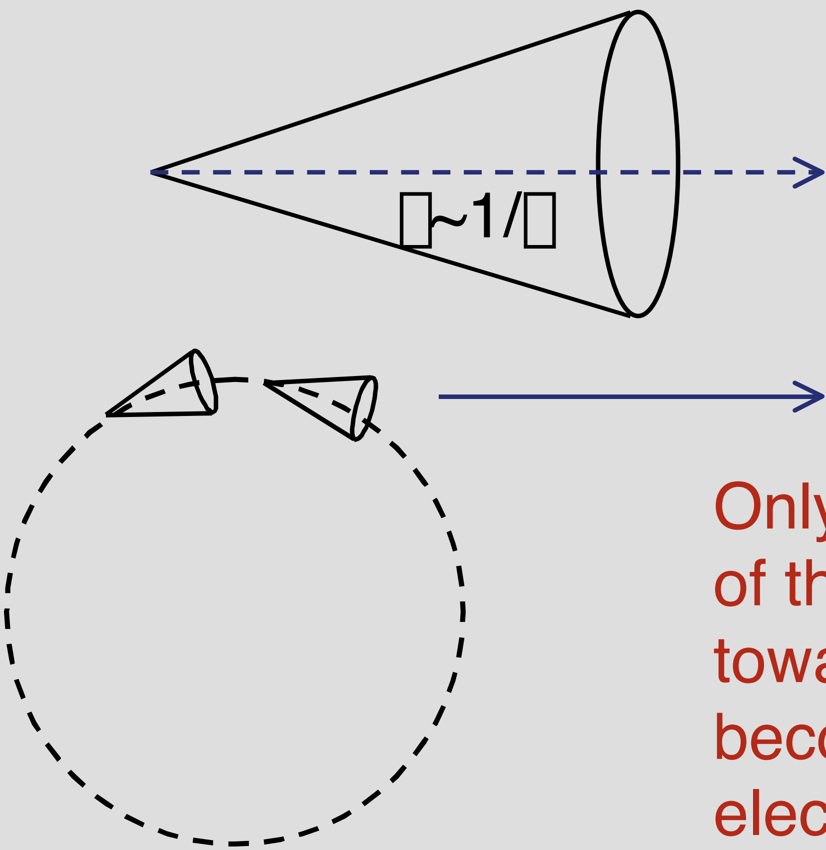
Actually get some emission at the harmonics too - 2ν, 3ν etc.

Very nice way to measure the magnetic field.

Synchrotron radiation

If the electrons are moving at close to the speed of light, two effects alter the nature of the radiation.

1) Radiation is **beamed**:

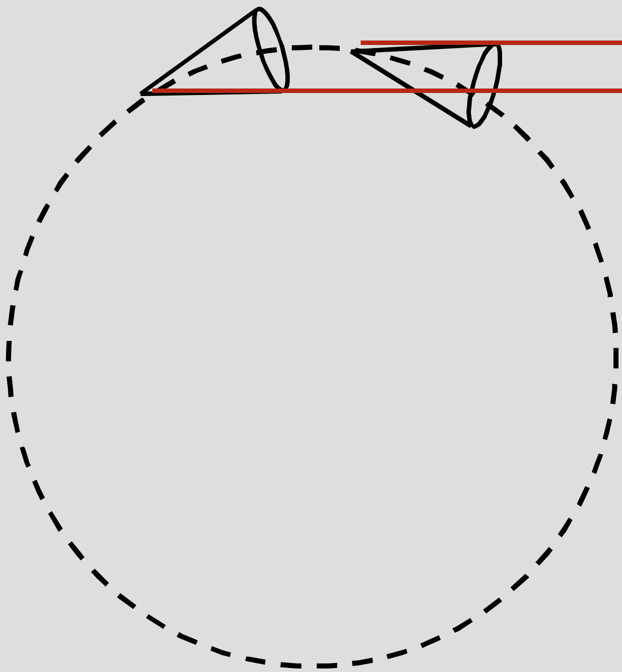


To observer

Particle moving with Lorentz factor γ toward observer emits radiation into cone of opening angle:

$$\theta \approx \gamma^{-1}$$

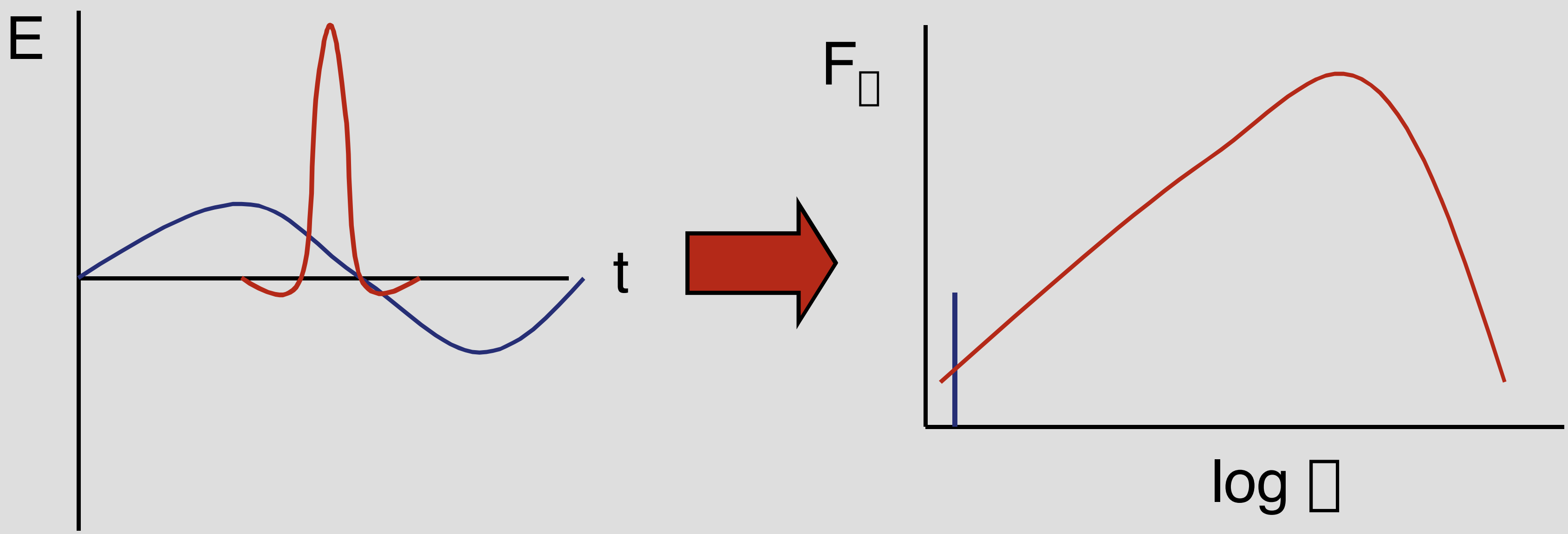
Only see radiation from a small portion of the orbit when the cone is pointed toward us - pulse of radiation which becomes shorter for more energetic electrons.



2) For source moving at v ~ c, photon emitted at end of pulse almost `catches up' with photon from start of pulse.

Pulse is further shortened.

Difference between **cyclotron** and synchrotron radiation.



Useful formulae for synchrotron radiation

For a **single particle**, spectrum extends up to a peak frequency roughly given by:

$$\nu \sim \gamma^2 \nu_c \sim \frac{\gamma^2 qB}{2\pi mc}$$

↑

cyclotron frequency

Can produce very high frequency radiation, with a continuous spectrum (no lines).

Normally, the electrons which produce synchrotron radiation have a (wide) range of energies. If number of particles with energy between E and E+dE can be written as:

$$N(E)dE = CE^{-p}dE$$

i.e. as a power-law in energy, then it turns out that the spectrum of the resulting synchrotron radiation is *also* a power-law, but with a different index:

$$P(\nu) \propto \nu^{-s} \propto \nu^{-(p-1)/2}$$

Measure the spectral index of the radiation (s), this then gives an indication of the distribution of particle energies (p)!

$$s = \frac{p-1}{2}$$