Newton and "Dark Stars"

Units and Scientific Notation

Standard units of mass, length and time

Kilograms (kg) Meters (m) Seconds (s)

Astronomical masses and distances are often *very* large, e.g. the mass of the Sun is:

2,000,000,000,000,000,000,000,000,000,000 kg

Units and Scientific Notation

Frequently use the mass of the Sun as a convenient unit of mass:

Sun itself has mass 1 M_{Sun}
Most massive stars are about 100 M_{Sun}
Milky Way center black hole 4.5 million M_{Sun}

For distance use "light year" – the *distance* light travels in one year, or "parsec":

1 parsec (pc) = 3.26 light years

Units and Scientific Notation

Recall "three squared" is written $3^2 = 9$, "three cubed" is $3^3 = 27$ etc

Write large or small numbers as powers of 10:

$10 = 10^1$
$100=10x10=10^2$
$1{,}000=10^3$
$1{,}000{,}000=10^6$
$2{,}000{,}000=2x10^6$

number of zeroes following the first digit

Units and Scientific Notation

Recall "three squared" is written $3^2 = 9$, "three cubed" is $3^3 = 27$ etc

Write large or small numbers as powers of 10:

$0.1=1/10= 10^{-1}$

$0.01=10^{-2}$

$0.001=10^{-3}$ ← number of zeroes preceding the first non-zero digit

$0.002=2\text{x}10^{-3}$

Units and Scientific Notation

$1\ M_{Sun} = 2 \text{ x } 10^{30}$ kg

1 parsec = $3 \text{ x } 10^{16}$ m

Before Einstein

Nicolaus Copernicus (1473-1543)
planets orbit the Sun, not the Earth

Johannes Kepler (1571-1630)
planetary orbits are not circles, but ellipses, described by simple laws

Galileo Galilei (1564-1642)
discovered the moons of Jupiter…
orbital motion is universal

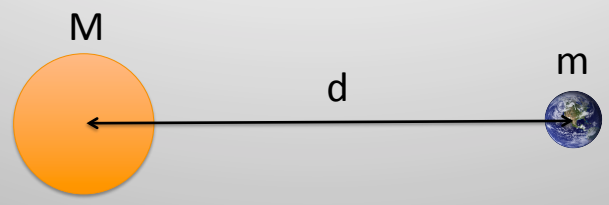
Isaac Newton

Forces cause *acceleration* $F = ma$

Gravity is a force:
same force on Earth extends to the Moon
explains the Moon's orbit about the Earth
gravity also explains planetary orbits

Gravitational forces:
proportional to mass: 2 x m = 2 x force
decrease with distance squared: 2 x d = ¼ force

Isaac Newton



$$F = \frac{GMm}{d^2}$$

G is Newton's gravitational constant: measures how strong gravity is

Newton's theory of gravity is unbelievably successful

Explains the motions of the planets *almost* perfectly, used to model spacecraft trajectories, collisions between galaxies…

Features of Newtonian gravity

"Mass" is used to refer to two (in principle) distinct concepts

- The strength of the gravitational forces a body creates
- How resistant it is to being accelerated by a given force (its "inertia")

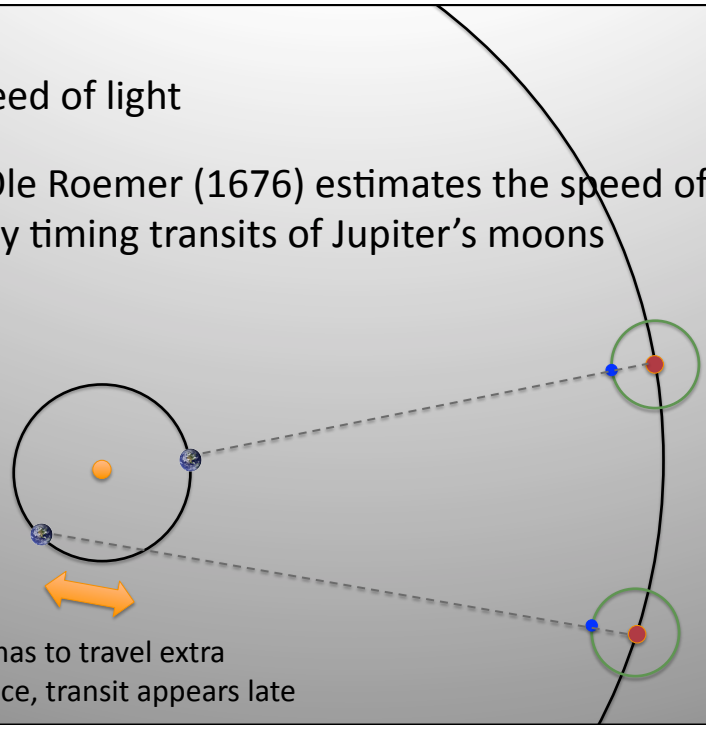
Features of Newtonian gravity

"Mass" is used to refer to two (in principle) distinct concepts

A gold bar and a lead bar that have an inertial mass of 1 kg produce the same gravitational forces too

Speed of light

Ole Roemer (1676) estimates the speed of light by timing transits of Jupiter's moons



Light has to travel extra distance, transit appears late

Speed of light

Modern value:

- ~182,000 miles / s
- ~300,000 km / s
- 299,792,458 m / s

(Roemer was about 20% off the right value)

In Newtonian gravity, the speed of light, denoted as c, has no special role – force between Earth and Sun, say, is communicated instantaneously

Escape velocity

Forms of energy before Einstein included:

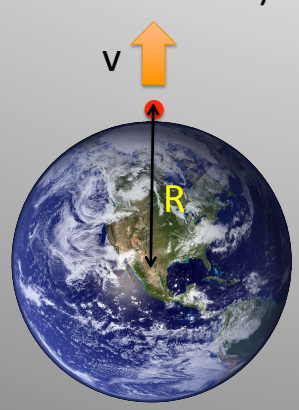
- kinetic energy (associated with motion)
- thermal energy
- potential energy (with position)

Conserved: sum remains the same

Escape velocity is smallest v that gives enough energy for a body to escape to very large distance ("infinity") without falling back under gravity

Escape velocity

Launch a projectile, mass m, with velocity v, from surface of Earth (distance R from center of Earth)



Kinetic energy = ½ m v^2

Gravitational potential energy

$$= -\frac{GM_{Earth}m}{R}$$

(negative: if I fall toward Earth kinetic energy increases, potential energy must become more negative to conserve energy)

Escape velocity

If Kinetic + Potential energy = 0 (or more), escape

$$\frac{1}{2}mv^2 - \frac{GM_{Earth}m}{R} = 0$$

$$\frac{1}{2}mv^2 = \frac{GM_{Earth}m}{R}$$

$$\frac{1}{2}v^2 = \frac{GM_{Earth}}{R}$$

$$v^2 = \frac{2GM_{Earth}}{R}$$

$$v = \sqrt{\frac{2GM_{Earth}}{R}}$$

For the Earth, the escape velocity is 25,000 mph or 11,200 m/s

Dark Stars

Imagine light was just like a projectile launched with some speed c (the speed of light!) from a star of mass M. How compact would the star be to "trap" the light?

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$ set equal to c

Dark Stars

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

set equal to c

$$c = \sqrt{\frac{2GM}{R}}$$

$$c^2 = \frac{2GM}{R}$$

$$R = \frac{2GM}{c^2}$$

a star smaller than this would be a "dark star", even light cannot escape

Dark Stars

42 *Mr.* MICHELL *on the Means of discovering the*

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

John Michell (1784)

Dark Stars

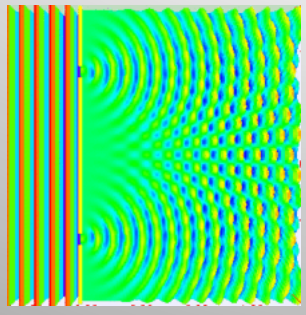
How big is this? For an object the mass of the Sun, the critical radius is:

R = 3 km

$$R = \frac{2GM}{c^2}$$

…very small because c is so large

Dark Stars



This argument depends on seeing light as particles that respond to gravity just like regular projectiles

Experiments at the start of the 19th century convinced physicists that light was instead a *wave*

Dark Stars

Michell's crude concept of a black hole (and, as it turns out, *correct* estimate of what we now call the Schwarzschild radius), was forgotten…