Homework #3: Due in class Tuesday March 7th

Suppose you watch a spaceship pass by at a speed close to the speed of light:

(a) How do clocks on the spaceship appear to run, compared to your own clocks?

Clocks on the spaceship appear to run slow

(b) How would a passenger *on the spaceship* view your time as passing?

The passenger on the spaceship views your time as *also running slowly*. To see this, rememeber that the situation is completely symmetrical – I see you moving past me at (say) 90% of the speed of light, and you see the same thing. So if I reckon your clock is slow, likewise you think my clock is slow.

The supermassive black hole at the Galactic Center has been measured to have a mass of 4 x 10^6 Solar masses (4 million times the mass of our Sun):

(a) What is the Schwarzschild radius of the black hole, in meters?

Use the formula for the Schwarzschild radius:

$$R_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 4 \times 10^6 \times 2 \times 10^{30}}{\left(3 \times 10^8\right)^2} \text{m}$$
$$= 1.2 \times 10^{10} \text{ m}$$

Key here is to put the mass of the black hole in kg. You can also use the textbook formula (which states that the radius is 3 km per Solar mass), and then multiply by 1000 to convert from km to m.

(b) How does the Schwarzschild radius compare to the distance between the Earth and the Sun?

Earth-Sun distance is 1.5 x 10^{11} m, so the Schwarzschild radius is smaller numerically, it is about 0.08 times or about 8% as large).

c) If you were to fall into the black hole, how long would it take to go from the Schwarzschild radius to the central singularity (you will need to *guess* a speed to answer this, so pick something sensible – remember we're dealing with a black hole here – and state your assumption).

Velocities close to the event horizon of a black hole are close to the speed of light, so guess c = 3 x 10^8 ms^{-1} (if you guess half of that or whatever, that's fine too). Time taken is then the distance (calculated) earlier divided by the speed:

$$\text{time} = \frac{1.2 \times 10^{10} \text{m}}{3 \times 10^8 \text{ms}^{-1}} = 40 \text{ s}$$