STABILIZING AND MEASURING OPTICAL FREQUENCIES

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Many factors are coming together to make this an exciting time in the development of optical frequency standards. These include improved strategies for interrogation of the resonant quantum reference sample, improved accuracy and control of the modulation process, demonstration of an external modulation-removal strategy, and a rich field of new possibilities for frequency measurement, particularly the arrival of broad optical comb generation techniques. Frequency-domain methods based on broad femtosec laser combs are very promising, as made clear by the work of Udem and Hänsch. We easily see beats at 805 nm with optical comb lines whose center is at 840 nm, a 16 THz interval. Adjustment of the laser compensation broadens the comb to >140 nm width at – 3 dB ! Stabilization of both optical center frequency and pulse rate has been implemented. Interesting combs are also generated near 1060 nm with Kourogi's modulator-in-a-cavity approach, augmented by an intracavity LiNbO₃ crystal which provides OPO gain and frequency connection to the stable pump. So far, about 15 000 coherent comb lines are generated, limited by uncompensated dispersion in the OPO cavity. Useful data regarding S/N necessary for accurate counting and a powerful adaptive Phase-Tracking Filter/Oscillator are discussed.

1 Introduction

Optical frequencies are interesting for precision measurement since, in comparison with microwave domain, the broadening processes are similar in magnitude, while the carrier frequency may be 50 000-fold higher in frequency. This huge frequency ratio enhances the relative sharpness of suitable resonances in the optical domain. It also inserts a massive impediment to the universal use of frequency-based techniques in the optical domain . Of course, many interesting experiments can be cast into differential form, for example wondering if right-handed and left-handed molecules will have precisely the same resonance frequencies in view of the Weak Neutral Currents which resolve this handedness degeneracy[1]. Another category of questions concerns residual effects of our acceleration in the frame of the Cosmic Black-Body Relic Radiation. With modern fast-ion storage-ring technology, augmented by laser cooling, a serious new experimental approach could address this question at a new level of precision. A third fascinating measurement will be that of the speed of light in vacuum in the presence of a powerful

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transverse magnetic field : robust QED theory leads to a prediction of 2×10^{-22} polarization-dependent differential index of refraction in the 6.2 Tesla field of some long SSC dipole magnets which have recently become available, and are presently being installed at FermiLab[2].

Another family of interesting experiments depends upon the actual value of the optical frequency we observe. Of course the units in which we express such a result are man-made and adopted by consensus. More broadly, then, we are looking at experiments in which the quantum frequencies of two different transitions are compared. For example a clock based on a rotation/vibration transition vs one using a transition between electronic energy levels, or vs one using hyperfine levels. Using the current (for 32 years!) definition of frequency, as 9 192 631 770 vibrations of the Cs ground state hyperfine levels $|3,0\rangle$ to $|4,0\rangle$, we wish for the technical means accurately and simply to compare our optical-domain frequencies of interest with this microwave domain standard. In this report we note the present status of our frequency stabilization and accuracy-of-locking issues using transitions near 532 nm in I_2 and 1064 nm in HCCD. Because of the increasing facility in optical measurement - the main subject of this report - we draw attention to simple measurement errors and the techniques known to overcome them. Specifically we discuss technical issues in accurately counting optical beat frequencies, and the necessary S/N. We then discuss the Phase-Tracking Oscillator/Filter, NASA's favorite trick for relaying data over vast interplanetary distances without significant error. All of these tools were part of our Nd:YAG frequency measurement system, and we provide a brief status report on this measurement. Our report concludes with an indication of a new paradigm of optical frequency measurement, based on broad spectral combs associated with well-stabilized femtosecond or OPO-based sources.

2 Line Center Locking: Issues and Prospects

In precision measurement - including the best national frequency standards - one has some resonance which can be observed with high S/N ratio by scanning an appropriate tunable source. A suitable processing of the resonance leads to a "discriminator" curve which forms the basis for locking that oscillator source to the quantum reference. Now the problems (and lots of the fun!) begin: "the Devil is in the details." It is inevitable that our system will be locked away from the resonance center. Perhaps to resolve a degenerate spectrum we used a small Zeeman shift, inducing a small shift which we must calibrate. In some systems there may be non-negligible atomic velocities which lead to second-order Doppler shifts. Additionally, velocity-dependent effects would include recoil-induced substructure, wavefront curvature and velocity selectivity effects, among others. With increasing accuracy, even atom interferometry and gravitational effects may play a role.

To these physical effects must be added subtle defects of the interrogation source and techniques. Source noise variations at the modulation frequency become mixed with the signal amplitude. (Sampled systems with deadtime are particularly dangerous.) Once the noise is mixed in, nothing can reduce it, save averaging. For accuracy, more damaging are even harmonics of the modulation frequency, as these produce a modulation wave which dwells unequal times around the center of the resonance, and thus surely will shift the apparent center.

Of course the long-term performance will not show a stability better than that of the Residual Amplitude Modulation (RAM) produced by the EOM. A number of methods have been tried to actively stabilize this to zero: one of the easiest effective methods is to thermally control the modulator crystal to yield zero RAM.

Turning now to the resonances themselves, an old problem has reappeared in the form of a beam size dependent power shift for I_2 systems, which may arise from the recoil splitting, 5.6 kHz for the 532 nm lines: the originally-equal populations in lower and excited states, produced by the saturating beam, have different decay rates. The ground state is perturbed only by weak velocity-changing collisions, while the upper state also undergoes normal spontaneous emission decay, mainly to levels other than the original level. At higher powers the molecules oscillate between ground and excited state, so somewhat similar populations will be in the upper and lower state recoil-split peaks[3]. But at low power, the fluoresence keeps the excited state mainly empty, even though optical pumping is deeply depleting ground-state molecules of the chosen velocity. One possible solution to this dilemma is being tried. The CH₄ case was attractive: the long lifetime led to pure transit broadening, the same in each level.

Some real joy awaits us with new laser sources near 1030 nm, such as Yb:YAG microchip lasers. Doubling to the green, these would access the sharp I₂ lines near 515 nm with 50 kHz linewidths[4]. The wide tunability supports tuning higher into the I₂ upper ³B state vibrational manifold where the lifetimes continue to increase: at 501.7 nm reported I₂ lifetimes imply an 8 kHz-wide optical resonance! The 1037 nm oscillator will be attractive for acetylene as well, since $3v_3$ bands in this region are ~130-fold stronger than the ones at 1064 nm used by Ye[5].

3 Frequency Measurement Issues, Tools, and Status

Heterodyne detection and frequency measurement arguably are the optimal modes of obtaining data, and a number of groups use frequency-based measurements to extend the precision and accuracy of the obtained spectra. Inevitably one will wish for more bandwidth in these frequency measurements, leading in turn to weaker signals: it is a general experience that the optical heterodyne waveform can be marginally weak for us to count reliably. At least to count directly. In this section we estimate the minimum optical power for a successful

measurement, and note the deadly role of excess bandwidth. (Later we show how a simple tracking oscillator can be built which will function as an *adaptive* narrow-band filter.) Data are presented showing actual counting-rate errors which increase rapidly as the input S/N is decreased. We will see that a Signal to Integrated Noise-within-the-measurement-bandwidth ratio of ~10 dB can be sufficient for accurate counting.

3.1 Minimum heterodyne Power for cycle counting

It is useful to address the question of the weakest measurable heterodyne signal, E_s . We suppose the reference laser field is a clean optical sinewave E_0 , with $E_0 = A_0 \cos(\Omega_0 t)$. Similarly, the heterodyne signal field can be taken as $E_s = A_s \cos(\Omega_s t)$, where we will treat the laser phase/frequency noise by considering Ω_s to be represented by some distribution of values. Noise fields will be additive inputs, they will yield zero time average. The heterodyne

term can be found from the photocurrent, $i = c \frac{(E_0 + E_s)^2}{4\pi} \left(\frac{A}{h\nu}\right) e \eta$, where A is the effective

area of the beam reaching the detector and the factor $\eta < 1$ is the charge collection efficiency. The dc photocurrent i_0 is derived from the E_0^2 component, while the signal bearing cross-term is $i_s = c \frac{2E_0E_s}{4\pi} \left(\frac{A}{h\nu}\right) e\eta$, and leads to a measurable rf power of $P_s = i_s^2 R \propto \left(\sqrt{E_0E_s}\right)^2 R \Rightarrow \left(\sqrt{i_0i_s}\right)^2 R$. The shot noise current associated with the Poissonian photon arrival rate distribution is $i_n = \sqrt{2ei_0B}$, leading to a noise power of $P_n = \left(\sqrt{2ei_0B}\right)^2 R$, where B is our measurement Resolution Bandwidth. The beat power S/N ratio is directly

$$S/N = \frac{P_s}{P_n} = \frac{\left(\sqrt{2ei_0 B}\right)^2 R}{\left(\sqrt{2ei_0 B}\right)^2 R} = \frac{i_s}{2eB}$$
, where it can be seen that the S/N for heterodyne detection

of the weak signal is independent of the level of the local oscillator, assuming it is shot noise limited. In this equation, we also see that the minimum detectable heterodyne signal has the same S/N as if it were directly detected, which would lead to the same fluctuation-based formula. It is impressive that the two detection methods appear to have different physical origins for the noise, but lead to the same sensitivity limit. Optics is interesting!

Concerning the allowed levels for the LO laser, ultimately all real signals will have excess noise of a technical origin: this limits the maximum useful LO power. The smallest LO power to be used is fixed such that its shot noise current must mask the thermal noise current in the measurement resistor R, which will likely be the input impedance of the rf amplifier used to increase the scale of the photobeat signal. This thermal noise current i_{th} is given by

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 $i_{th} = \sqrt{\frac{4kTB}{R}}$. This noise power tends to obscure our signal power unless the LO laser power

is sufficient. We need the shot noise to dominate, that is $i_n > i_{th}$, so $\sqrt{2ei_0B} > \sqrt{\frac{4kTB}{R}}$,

leading to $i_0 R > \frac{2kT}{e}$. This states that the LO light power must produce a current sufficient to make a voltage over the sensing resistor R of at least 1/20 Volts (at room temperature). Considering that the input amplifier will also contribute some noise, one comes to the usual rule of thumb that the LO photocurrent must drop at least 0.1 v over the rf input resistor, which is typically 50 ohms. So the photocurrent must be at least 2 mA. With silicon current responsivity of ~0.8 mA/mW at 800 nm, perhaps 3-5 mW would be a good optical power for the LO.

Now we can calculate the minimum power needed for the heterodyne rf optical frequency measurement. Requiring the S/N to be 10 dB for sub-Hertz cycle counting leads to $i_s / 2eB = 10 dB$. (See below) If we directly count the beat via a 50 MHz lowpass filter, and include the 10 dB needed S/N margin, we come to the usual rule of thumb that the detected photobeat needs to be > 50 pA for a successful beat measurement. Using a Phase-Tracking Filter (PTF), 100 kHz is a reasonable bandwidth B and gives $i_s = 10^{-13} A$. (See below). If the laser sources have a significant and rapid frequency jitter, a wider bandwidth may be needed for the PTF to ensure that much less than 1 rad of phase error would be accumulated within the tracking response time of the system. In such a case the narrow bandwidth advantage of the PTF is partially lost.

3.2 Measurement of the Cycle-slipping Threshold

Trustworthy design of a complex frequency measurement system requires knowledge of the S/N ratio required for a stated counting error rate. Fig 3 shows the spectrum used in these tests. A -14 dBm monochromatic 50 MHz signal from a synthesizer modeled our projected optical frequency beat. This level is 17 dB above the HP 5334B counter threshold. To this carrier was added with variable attenuation the amplified noise of a 50 ohm termination. The composite wave was filtered by a tunable filter centered on 50 MHz. The counter was interfaced to a computer and 100 successive 1 s measurements were performed. With excessive noise, we always observed a frequency decrease which was exceedingly sensitive to the exact noise level. The cycle-slipping data are indicated in Fig 3b. (If the noise spectrum was seriously shifted with respect to the carrier, the error took up the corresponding sign.)



Figure 3. RF carrier with added filtered spectrum of noise. 0 dB on graph is carrier level which was -14 dBm, 17 dB above counter threshold. a) Upper trace is single sweep, Lower is with 100 video averages. b) Frequency error as a function of added noise level. When spectrally-integrated noise (single sweep basis) is 9 dB below carrier, measured frequency is low by 10 Hz.

3.3 Frequency-tracking filter/oscillator

It is natural that one will push to the limits of frequency and low signal levels. The minimum optical power which can be counted depends on many circumstances. Of course it is better to have the laser source emit a spectrally-narrow signal which has little frequency jitter. In this case we can use an appropriate narrow-band filter to remove most of the noise, with the noise power just scaling down with the bandwidth. In this narrow band the signal appears without attenuation, while the remaining narrow range of noise Fourier components mainly fiddle the apparent phase and amplitude of the desired signal. But for an adequate S/N, the counter's discriminator always sees the "right" number of zero-crossings as just shown. But a narrow filter is useful only if it is centered on our signal. What are the constraints?

For the beat frequency to be determined accurately, some maximum "fast" linewidth is tolerable, say 10 kHz. Ordinarily this interesting narrow line will have considerable frequency jitter, perhaps even many MHz, but occurring at slow rates. In this case the adaptive Phase-Tracking Filter (PTF) is beneficial, operating as a narrow filter which dynamically tracks the varying input frequency. As shown in Fig 1, the PTF consists of a voltage-tunable oscillator, a phase-measuring system, and an appropriate feedback circuit. The phase is measured with a Doubly-Balanced Mixer (DBM) which multiplies the noisy input wave by a locally-generated sinewave, providing at quadrature phase a slope ~5-10 mV/degree for the signal, while the noise tends to average to zero. A Proportional-Integral (PI) controller corrects the frequency of the Voltage-Controlled Oscillator(VCO) to match the input frequency. Adding the intrinsic integration from controlling-frequency-while-measuring-phase makes a second-order loop. The parameters of this loop define the effective tracking bandwidth for this PTF.

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Figure 1. Adaptive Frequency Filter, also called a Phase Tracking Filter. Voltage-controlled oscillator is kept in phaselock with the noisy input wave via feedback. Servo loop parameters (unity gain frequency, or servo attack time, and damping) are chosen with Gain and Proportional-Integral (PI) crossover frequency settings. Unmarked 3-port devices are -3 dB power splitters. DBM=Doubly-Balanced Mixer. DC-10 is a -10 dB Directional Coupler.

To track the signal accurately enough to preserve the phaselock, a bandwidth of a few units of the instantaneous linewidth will be needed, say 50 kHz in this example. (If the freqency is randomly slewing rapidly, a larger control bandwidth will be needed.) The output of the system is taken as a sample of the VCO's output waveform - which has no amplitude change as it changes frequency - so the amplitude rejection of the PTF is nearly infinite.

To prove that cycle slips are not occurring, one can count with redundant systems set up with different bandwidths. For a simpler diagnostic, one can appropriately filter the phase-detector's output and pass it to a discriminator and counter system, with the window discriminator thresholds set at perhaps $\pm 60\%$ of the full analog phase discriminator output range. If a phase transient reaches this level we should be aware of it even if a real cycle slip did not occur, since it is clear the operation is marginal.

3.4 External Dither Suppression

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Fig 3. External Dither Suppression. DDS provides ~1000-fold linewidth reduction! See text. As may be inferred from the above discussion, large and rapid phase excursions represent a challenge to the Tracking Filter's phase-lock. But just such a case is presented by a laser system stabilized using the conventional "Third-harmonic" locking idea based on laser dither. For example, the HeNe laser at 633 nm stabilized via an intracavity I₂ cell uses a 6 MHz pp. dither typically at 5 - 8 kHz. This represents a ± 500 rad deviation, certainly a challenge to track. Since this deviation is deterministic, it seems a waste to take a chance on cycle slip. This FM may be externally canceled using a

doubly-passed AOM which is fed by an antiphase frequency-modulated rf waveform. Evidently this could be provided by a VCO, but the linearity may be inadequate and so generate a harmonic structure in the net optical phase. Additionally, the long-term stability of VCO's frequency will likely be unsatisfactory. A new opportunity is provided by a Direct Digital Synthesizer, to which the desired FM waveform can be supplied externally. Internally, this input is periodically digitized and scaled to represent the needed frequency modulation. Recently

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this externally-modulated DDS function has become.commercially available. In Fig. 3 we show the reduction of the HeNe beat width from the original 6 MHz pp. to about 100 kHz with the VCO analog technique, and then digitally to about 8 kHz, near the random noise Gaussian width of the lasers employed in this experiment. The extra sub-peaks are alias components as the digitizing rate is too slow (200 kHz) to accurately represent the dither waveform.

3.5 Status Report on Optical Frequency Standards based on the Nd:YAG laser

As for standards based on Nd:YAG and its doubled output, laser locking to HCCD at 1064 nm provides a stability of 2×10^{-13} / sqrt(tau), while locking to one of the molecular iodine transitions with the 532 nm output provides 5×10^{-14} /sqrt(tau). This near-equivalent performance is remarkable in view of the ~ 5×10^{5} absorption ratio between the two systems, and demonstrates the significant sensitivity gain provided by cavity enhancement and our dual-modulation locking scheme[5]. Importantly, both these systems reach below 3×10^{-15} at 1000 s and are still improving with tau[6].

A useful frequency measurement approach has used the Rb two-photon standard at 778 nm and the HeNe I₂ standard at 633 nm to determine the 532 nm frequency[6]. This was recently improved considerably by using the 3392 nm HeNe/CH₄ reference laser with the 778 nm reference and sum frequency generation in PPLN to generate a reliable proof for the 633 nm system. Good data was obtained at JILA by an international collaboration in September 1998, but the troubling offsets (~ 5 kHz) are not yet adequately understood.

4 Extending the Optical Heterodyne Frequency Range

4.1 Variations/Extensions of Kourogi's Comb generator

A convenient implementation of Kourogi's comb generator[7] can be based on using high finesse Fabry-Perot resonators for the cavity "mirrors." At the output we can enjoy the resonant transmission filter which passes only our chosen sideband[8]. Alignment stability of the F=600 cavity has been problematic, but is now solved via a PZT tube with sectored electrodes which offers a sensitive means for fine tuning the alignment, as well as the length. The resonant transmission is about 50%. On the input side, a mirror spacing of ~25 mm is convenient as that leads to wavefronts that match commercially-available "zero-optical-power" substrates. This allows the input power recycling mirror to be added after the comb generator is aligned and functioning. The reflection dips of this short cavity can be used to lock it to the input wavelength, and reveal within them the fine structure due to the multiple

frequencies within the main cavity. As the present reflectivity is not ideal, the transmission of this input frequency "Power Recycling" cavity is only about 30%. The net "transmission" on our utilized 63^{rd} sideband (of the 10 GHz rf drive) is in the range 4-10%.

Another enhancement of the comb generator can be the provision of intracavity gain with an OPA/OPO. Using LiNbO₃ for the modulator and also as the OPA/OPO, a phase-coherent bandwidth of 20 nm at 1064 nm has been obtained using \sim 300 MHz modulator drive, representing something like 10 000 sidebands on either side of the carrier. This will be challenging relative to the rf source purity. Issues of dispersion compensation remain to be addressed. Present results are noted elsewhere in these Proceedings.

4.2 The femtosecond Road to Coherent Spectral Bandwidth: Combs of 20 % Bandwidth and More!

The Garching MPQ group, under the guidance of Professor T. W. Hänsch, has recently used the fs comb of a commercial fs laser to make a seminal measurement of optical frequency intervals[9], and thereby of the optical frequency itself. This last extension makes use of the Optical Frequency Interval BiSector idea[10] which is now in widespread use. It is an anomaly of this epoch of extremely rapid progress that our group, entering the field second, can offer any interesting measurements to show alongside the remarkable Garching work. It is also a wonderful indication of the generous sharing of knowledge by those colleagues and friends there, and we enthusiastically thank them! For the historical record it should be noted that Venia Chebotayev also had independently made suggestions[11] along these lines, but at such an early stage, before the fs laser epoch, that little notice was taken of these ideas.

But now it is routinely possible to have pulsewidths in the 10-12 fs domain[12]. These pulses contain fewer than 6 cycles of the "carrier" wave and thus show fractional bandwidth approaching 20 %. The phase modulation associated with the Kerr intensity dependent index of the Ti:Sapphire crystal at the peak of the pulse amounts to $\sim 2\pi$ radians, and in this way the fast-rising edges of the pulse lead to a large associated spectral broadening. Amazing as it seems, this process is very reproducible from pulse to pulse, so that one finds a high degree of coherence of this "white" light from one pulse to the next. Actually the cavity length leads to a pulse repeat frequency ~100 MHz, so the optical spectrum is found to be a vast comb of narrow spectral lines with a common 100 MHz separation. Shorter cavities have been used recently to push this Pulse Repetition Frequency (prf) up to the 3.4 GHz domain[13].

Thinking about the fs comb stability, the optical frequencies certainly are increased a little when the cavity is shortened by a wavelength. But they are not exact harmonics of the prf, being offset by the fact that Group Velocity \neq Phase Velocity in the laser cavity. The spectrum is spatially dispersed at one end of the cavity, offering a nice opportunity for the prf servo to "tweak" the GV of the cavity by tipping the associated mirror just a little, for example so that the blue spectral components are returned a little earlier and the reds a little later. In such a way the GV and prf can be adjusted with relatively little consequence on the

central wavelength. Thus with suitable electronic orthogonalization, the two physical features (central wavelength and comb spacing) can be independently stabilized. We are stabilizing the optical frequency with a reference cavity at present, and controlling the prf using a low noise rf synthesizer. Recent progress in microwave reference oscillators[14] has arrived at a good time for this work, as the multiplied output of even the best quartz reference has far too much phase noise. Effectively we are multiplying this source up to about 1/5 the optical frequency, so that "carrier collapse" would already occur before reaching this range[15]. As a result, one finds there is an optimum gain for the prf servo, enough to mainly remove the low frequency drift component of the prf without excessively broadening the distant comb lines.

While the Garching group is far ahead of us in making physical measurements, for the record in Fig. 4 we can show a nice beat across a \sim 35 nm interval (15 THz). Importantly, the comb components continue to be narrow, even with this broader spectrum. Our S/N estimate is that comb components down to 1 % of the central power in Fig. 4 could be measured: this would be more than 100 THz! Recently "two-cycle" pulses have been reported[16,17] with \sim 50 % bandwidth, while fiber-based broadening[18] has exceeded a 4:1 bandwidth:carrier ratio. If good luck continues and these remarkable combs are really phase coherent, the future of optical frequency metrology looks bright indeed!



Figure 4. Left, linear display of fs comb spectrum. At -10 dB below the maximum, the spectral width is >180 nm (76 THz). Right figure shows optical beat with 805 nm auxiliary stable laser. The estimated threshold for phase-tracking is -20 dB (1% in linear scale): good beats were seen to 750 nm, the tunable laser's limit, a 43 THz beat.

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