Spin(11,1) String Theory

Andrew J. S. Hamilton

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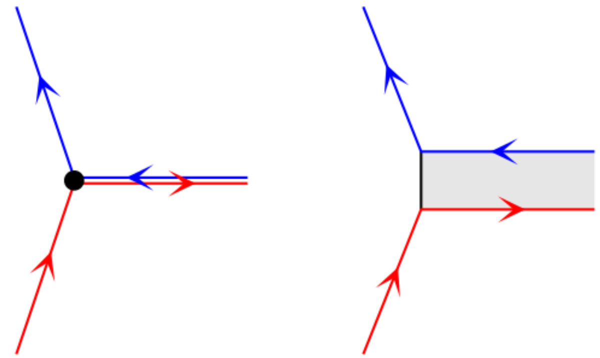
1/15. The strength of string theory is that it is a UV complete theory of quantum gravity

Just 2 kinds of objects have consistent perturbative quantum field theories:

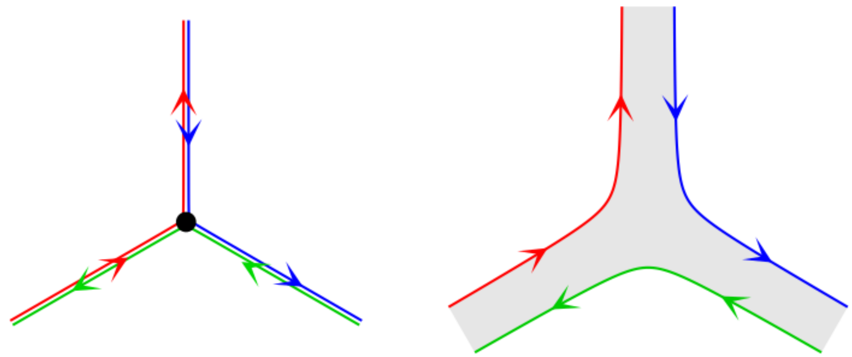
1. Particles (1d worldlines);
2. Strings (2d worldtubes).

Particle qft cannot handle gravity, whose coupling parameter $8\pi G$ of dimension mass^{-2} makes it nonrenormalizable. But string qft automatically includes gravity (spin 2 gravitons) encoded as closed uncharged strings.

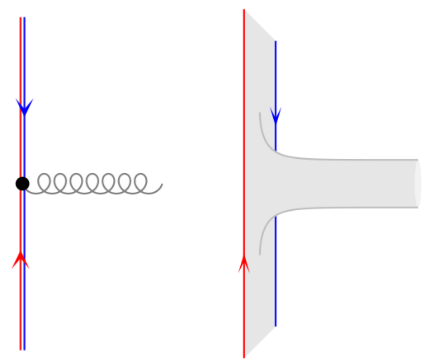
Objects of other dimensions (0d solitions, ≥3d branes) exist, but they do not support perturbative qfts.



Quark-gluon interaction.



Gluon-gluon interaction.

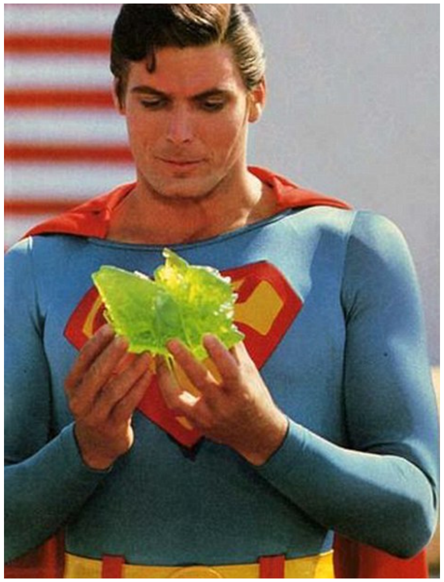


Gluon-graviton interaction.

2/15. The weakness of string theory is supersymmetry

Standard arguments for supersymmetry (super = fermionic):

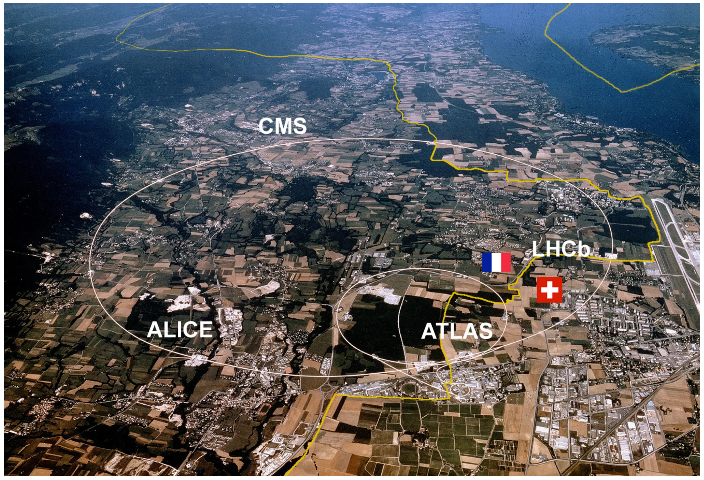
1. Nature has fermions.
2. Evades Coleman-Mandula (1967) no-go theorem.
3. Gets 3 coupling parameters of standard model to meet at grand unification.
4. The ground state of bosonic strings is tachyonic, therefore unstable.
5. Fermion-boson infinities cancel each other in scalar boson masses.
6. Magically cancels gauge anomalies in string theory.



Superman + kryptonite (1978)

Problems with supersymmetry:

1. Predicts symmetries that are not observed.
2. Preempts symmetries that are observed.



[LHC − superpartners (2017)]

Supersymmetry algebra posits that anticommutators of R, L-handed spinor generators Q generate translations P:

$$\{Q_R, Q_L\} = P \ , \quad [P, P] = 0 \ .$$

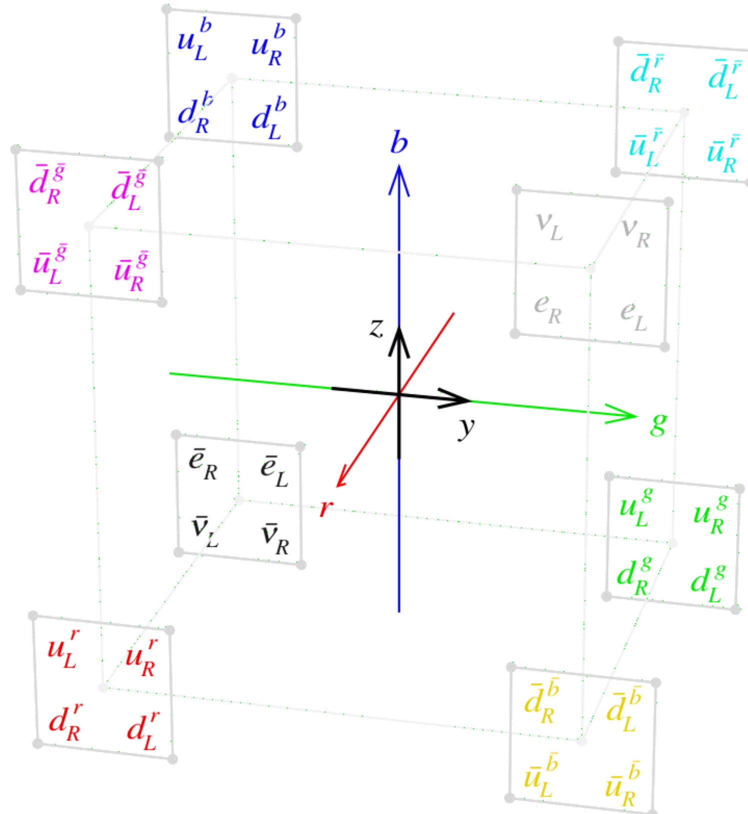
But Brauer-Weyl (1935) theorem shows

$$\big[\{Q_R, Q_L\}, \{Q_R, Q_L\}\big] = \{Q_R, Q_R\} + \{Q_L, Q_L\} \ ,$$

which are the observed symmetries of the standard model.

3/15. Spin(10) spinors

The grandfather of Grand Unified groups is the group Spin(10) of rotations in $N = 10$ ‘internal’ dimensions (Georgi 1975; [Fritzsch & Minkowski 1975]), which elegantly contains the three forces $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_c$ of the Standard Model of Physics.



The electron generation of 32 spinors of the Standard Model in Spin(10).

Spin(10) has $[N/2] = 5$ bits, consisting of 2 weak bits y, z and 3 color bits r, g, b ([Wilczek 1998]).

Spin(10) has $2^{[N/2]} = 32$ basis spinors.

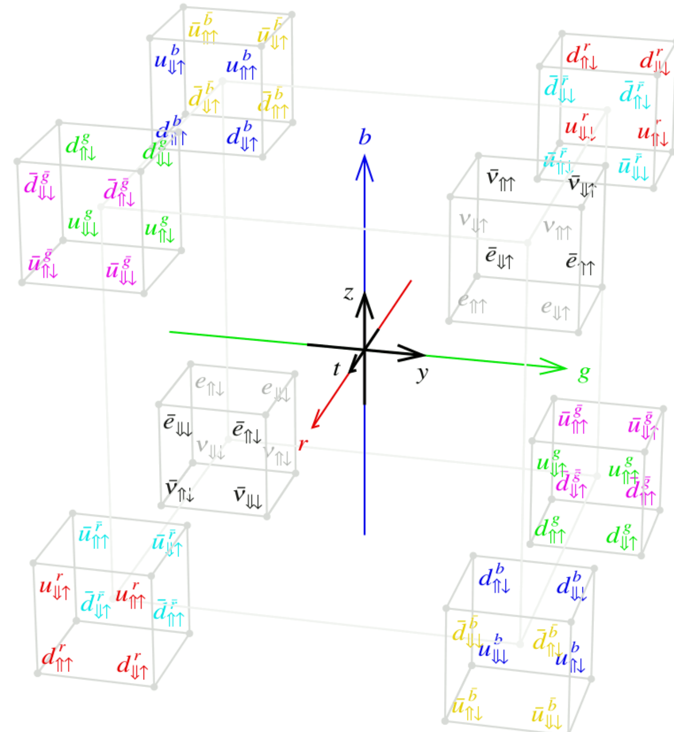
The electron in Spin(10) is:

$$\underset{\bar e_R}{\downarrow\uparrow\downarrow\downarrow\downarrow}\ ,\quad \underset{\bar e_L}{\uparrow\uparrow\downarrow\downarrow\downarrow}\ ,\quad \underset{e_R}{\downarrow\downarrow\uparrow\uparrow\uparrow}\ ,\quad \underset{e_L}{\uparrow\downarrow\uparrow\uparrow\uparrow}\ .$$

4/15. Two reasons unification of forces in Spin(11,1) is “obvious”

1. Each of the $2^5 = 32$ fermions of a generation of Spin(10) is itself a chiral (massless, Weyl) fermion with 2 degrees of freedom transforming under the Lorentz group $\text{Spin}(3,1)$, for a total of $2^6 = 64$ fermions. The smallest spin group that contains 64 fermions and 1 time dimension is the group $\text{Spin}(11,1)$ of rotations in 11+1 spacetime dimensions.

2. Spin(10) chirality (right/left-handed) coincides with Dirac chirality. The reason it is possible to embed Spin(10) and $\text{Spin}(3,1)$ in only 11+1 dimensions (not 13+1 dimensions) is because they redundantly contain the 2 degrees of freedom associated with chirality.



The electron generation of 64 spinors of the Standard Model in $\text{Spin}(11,1)$.

$2^6 = 64$ fermions require one extra bit, the t-bit, or time bit.

The coincidence of Dirac and $\text{Spin}(11,1)$ chiralities is a coincidence of their pseudoscalars:

$$\begin{aligned} \gamma_0\gamma_1\gamma_2\gamma_3 &= \gamma_t^+\gamma_t^-\gamma_y^+\gamma_y^-\gamma_z^+\gamma_z^-\gamma_r^+\gamma_r^-\gamma_g^+\gamma_g^-\gamma_b^+\gamma_b^- \\ = \underset{\text{boost rotation}}{(\gamma_0\gamma_3)\ (\gamma_1\gamma_2)} &= \underset{\text{boost}}{(\gamma_t^+\gamma_t^-\gamma_y^+\gamma_y^-\gamma_z^+\gamma_z^-)}\underset{\text{rotation}}{(\gamma_r^+\gamma_r^-\gamma_g^+\gamma_g^-\gamma_b^+\gamma_b^-)}\,. \end{aligned}$$

5/15. Unification of Dirac and standard-model algebras in the Spin(11,1) Clifford algebra

The relation between the 4 Dirac vectors $\boldsymbol{\gamma}_m$, $m = 0, 1, 2, 3$, and the 12 $\mathrm{Spin}(11,1)$ vectors $\boldsymbol{\gamma}_k^{\pm}$, $k = t, y, z, r, g, b$, is ($\boldsymbol{\gamma}_k^{+}$ and $\boldsymbol{\gamma}_k^{-}$ comprise the real and imaginary parts of a complex vector)

$$\begin{aligned}\boldsymbol{\gamma}_0 &= i\boldsymbol{\gamma}_t^- \ , \\ \boldsymbol{\gamma}_1 &= \boldsymbol{\gamma}_y^-\boldsymbol{\gamma}_z^-\boldsymbol{\gamma}_r^+\boldsymbol{\gamma}_g^+\boldsymbol{\gamma}_b^+ \ , \\ \boldsymbol{\gamma}_2 &= \boldsymbol{\gamma}_y^-\boldsymbol{\gamma}_z^-\boldsymbol{\gamma}_r^-\boldsymbol{\gamma}_g^-\boldsymbol{\gamma}_b^- \ , \\ \boldsymbol{\gamma}_3 &= \boldsymbol{\gamma}_t^+\boldsymbol{\gamma}_y^+\boldsymbol{\gamma}_y^-\boldsymbol{\gamma}_z^+\boldsymbol{\gamma}_z^- \ .\end{aligned}$$

- The inner products and Lie algebra of the above spacetime vectors $\boldsymbol{\gamma}_0$, $\boldsymbol{\gamma}_1$, $\boldsymbol{\gamma}_2$, $\boldsymbol{\gamma}_3$ are precisely those of the Dirac algebra.
- The standard-model algebra is generated by a subset of bivector products of $\boldsymbol{\gamma}_k^{\pm}$ with y, z, r, g, b.
- The Dirac and standard-model algebras are mutually commuting subalgebras of the $\mathrm{Spin}(11,1)$ Clifford algebra, consistent with the Coleman-Mandula (1967) no-go theorem.
- There is a non-obvious trick that allows the algebra to work, which is to modify all imaginary vectors $\boldsymbol{\gamma}_k^-$ by multiplying them by a phase factor times the color pseudoscalar I_{rgb},

$$\hat{\boldsymbol{\gamma}}_k^+ = \boldsymbol{\gamma}_k^+ \ , \quad \hat{\boldsymbol{\gamma}}_k^- = \big(\underset{yz}{i} \text{ or } \underset{rgb}{1}\big)\boldsymbol{\gamma}_k^- I_{rgb} \ .$$

6/15. Energy scales of unification

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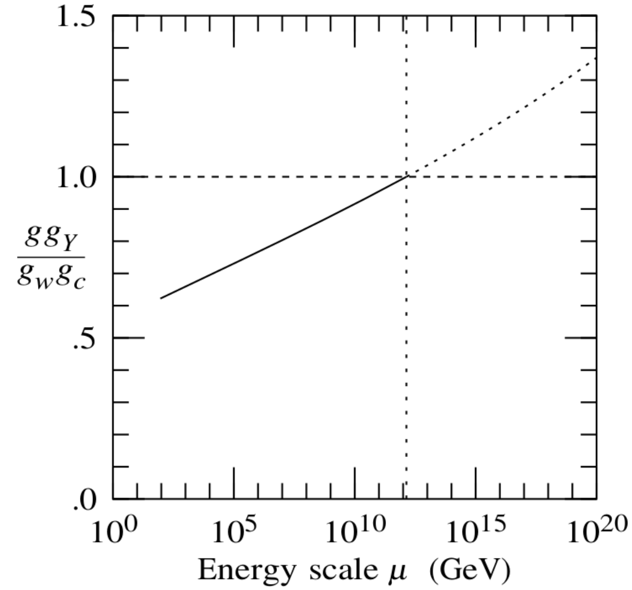
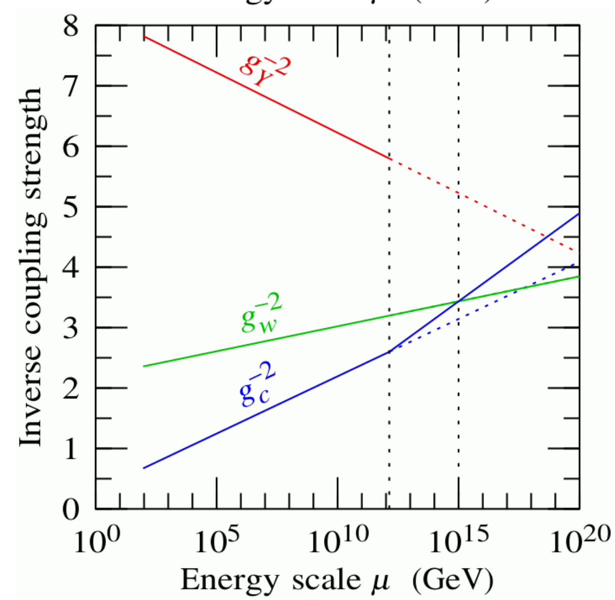
1. The electroweak Higgs field is
$$\langle \boldsymbol{H} \rangle = \langle H \rangle \boldsymbol{\gamma}_t^+ \boldsymbol{\gamma}_y^- I_{rgb} \ ,$$
which carries one unit of y charge, and gives mass to fermions by flipping their y-bit.

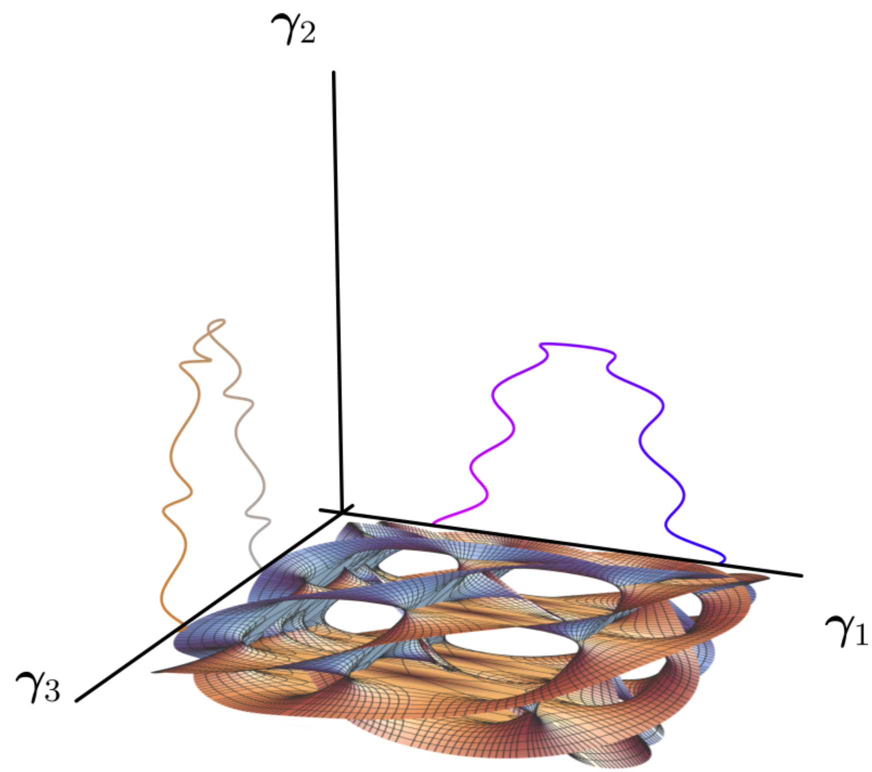
2. $\mathrm{Spin}(11,1)$ predicts an intermediate unification to $\mathrm{Spin}(4)_w \times \mathrm{Spin}(6)_c$ (the Pati-Salam 1974 group) at $\approx 10^{12}$ GeV, where standard-model coupling parameters g_Y, g_w, and g_c satisfy
$$\frac{g g_Y}{g_w g_c} = 1 \ \text{ with } \ g \equiv \sqrt{g_w^2 + g_c^2} \ .$$
At Pati-Salam symmetry breaking, 2 right-handed weak and 6 leptoquark bosons acquire masses in the ratio $m_R / m_C = 1.1$.

3. Grand unification is predicted to occur at $\approx 10^{15}$ GeV, where the $\mathrm{Spin}(4)_w$ weak and $\mathrm{Spin}(6)_c$ color coupling parameters meet,
$$g_w = g_c \ .$$
The grand Higgs field
$$\langle \boldsymbol{T} \rangle = \langle T \rangle \boldsymbol{\gamma}_t^+ \boldsymbol{\gamma}_t^- I_{rgb}$$
breaks t-symmetry, which generates a Majorana mass for the right-handed neutrino.





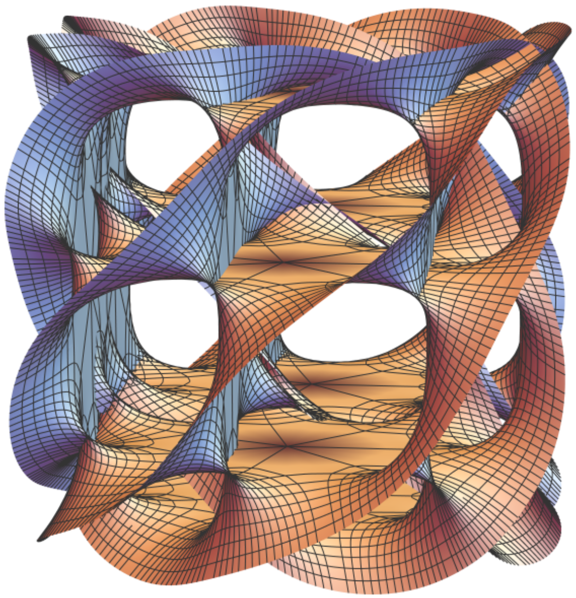
7/15. Spin(11,1) string theory



Not a direct product of internal and spacetime manifolds.

Accomodates 10 internal and 3+1 large spacetime dimensions inside 11+1 dimensions.

The 4 weak vectors $\hat{\boldsymbol{\gamma}}_k^{\pm}$, $k = y, z$, Lorentz transform like Dirac vector $\boldsymbol{\gamma}_3$.
The 6 color vectors $\hat{\boldsymbol{\gamma}}_k^{\pm}$, $k = r, g, b$, Lorentz transform like Dirac vector $\boldsymbol{\gamma}_1$.



The 4 weak and 6 color vectors form a 10d compact complex manifold, a D10-brane, which carries the 2^6 spinors of $\mathrm{Spin}(11,1)$. The internal complex structure is generated by the color pseudoscalar I_{rgb}.

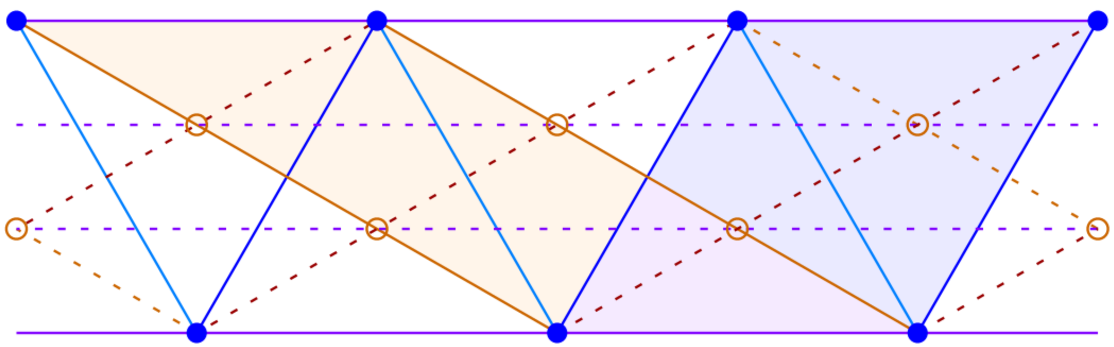
The weak and color gauge fields of the standard model are carried by bosonic open strings whose ends attach to D4 weak and D6 color subbranes of the fermionic D10-brane.

After symmetry breaking to the standard model (which is a subgroup of $\mathrm{SU}(5)$), the compact internal fermionic D10-brane becomes a 10d Calabi-Yau five-fold. A Calabi-Yau n-fold is a complex manifold with $\mathrm{SU}(n)$ holonomy. A Calabi-Yau n-fold has the key property that it carries zero internal energy-momentum.

8/15. Compactification from 26 to 12 dimensions on 14 dimensional SU(8)×SU(8) torus

Gross et al. (1985) came up with heterotic string theory by combining left-moving bosonic strings in 26d with right-moving fermionic strings in 10d by compactifying 16 of the 26 dimensions on a self-dual torus of $\mathrm{E}_8 \times \mathrm{E}_8$ or $\mathrm{SO}(32)$. Spin(11,1) bosonic string theory achieves 12d by compactifying 26d on a 14d maximal torus of the subgroup $\mathrm{SU}(8) \times \mathrm{SU}(8)$ of the group $G^2(10) = \mathrm{U}(16)$ generated by multivectors of grade 2 (mod 4) in 10d.

A standard $\mathrm{SU}(n)$ torus is not self-dual (self-dual means determinant of Cartan matrix is 1), but the $\mathrm{SU}(n)$ torus with one of its roots replaced by a spinor root is self-dual, so that winding numbers and momenta on the torus are dual, as required by modular invariance ($\tau \leftrightarrow \sigma$ symmetry) of string theory. Again, this is similar to heterotic theory, where substituting a root of $\mathrm{SO}(32)$ with a spinor root makes its torus self-dual.



Standard and spinor $\mathrm{SU}(3)$ lattices.
● = standard (integral) vertices;
○ = spinor (1/2 integral) vertices.
The 3-fold cover of the spinor lattice is self-dual.

The (periodically identified) vertices are joined by $\frac{1}{2}(n-1)n = 3$ distinct lattice lines. Fundamental modes of strings along each line in each direction represent $(n-1)n = 6$ massless eigenstates of $\mathrm{SU}(n)$ gauge bosons. These combine with the $n-1 = 2$ Kaluza-Kline modes of the torus to form the $n^2-1 = 8$ gauge bosons of $\mathrm{SU}(n)$.

9/15. Eigenstates of open strings on nonflat complex manifolds

A spatial gauge transformation of the 3+1 large spacetime dimensions rotates the spacetime frame to t-bit up. The 26 dimensions X^κ in which the bosonic strings propagate comprise

$$26 = \underset{\text{spacetime}}{1{+}1} + \underset{\text{fermionic D10-brane}}{10} + \underset{\text{bosonic 14-torus}}{14} \ .$$

Gauge transform the string metric to be conformally flat. Wick rotate the 2 string coordinates $\sigma^\alpha \equiv \{\tau, \sigma\}$ to the conformally Euclidean complex plane (L is a fundamental scale, the string scale),

$$z \equiv e^{(\tau+i\sigma)/L} \ , \quad \tilde{z} \equiv e^{(\tau-i\sigma)/L} \ .$$

In curved spacetime, the Nambu-Goto string action implies the string wave equation

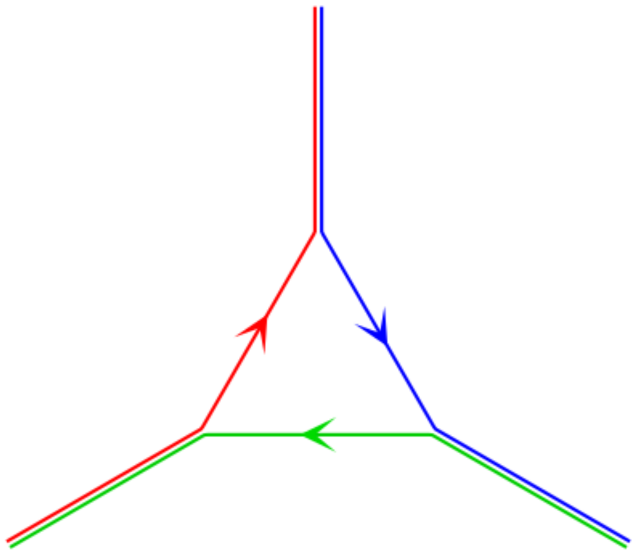
$$\partial_z \partial_{\tilde{z}} X^\kappa + \Gamma^\kappa_{\mu\nu} \partial_z X^\mu \partial_{\tilde{z}} X^\nu = 0 \ .$$

In Minkowski spacetime all coordinate connections $\Gamma^\kappa_{\mu\nu}$ vanish, and the general solution of the wave equation is a sum $X^\kappa(z) + \tilde{X}^\kappa(\tilde{z})$ of right- and left-moving modes. But in a curved complex manifold the wave equation also has open-string solutions in which all modes are right-moving, functions only of z. Expand right-moving modes as $X^\kappa(z)$ as

$$X^\kappa(z) = x^\kappa + iL\left((k^\kappa - l^\kappa) \ln z + \sum_{m \neq 0} \frac{\alpha^\kappa_m}{m} z^m \right) \ ,$$

where k^κ and l^κ are dual momentum and winding modes on the 14-torus, and α^κ_m are creation ($m < 0$) and destruction ($m > 0$) operators for a ladder of Kaluza-Klein modes for each coordinate κ. The z can be treated as a Fourier mode $z = e^{i\theta}$, with conjugate $1/z = e^{-i\theta}$, and quantization of open-string eigenmodes carries through in a curved complex manifold the same as in Minkowski space.

10/15. Gauge and gravitational anomalies vanish



In quantum field theory in $2n$ spacetime dimensions, gauge anomalies potentially arise from chiral loops with $n+1$ vertices attached to bosonic lines. Gauge anomalies destroy classical gauge invariance, and must be avoided. Gauge anomalies are called gravitational if the bosonic lines are gravitons.

In standard supersymmetric string theory in 10d, gauge anomalies vanish only for gauge groups of dimension 496, specifically $\mathrm{E}_8 \times \mathrm{E}_8$ or $\mathrm{SO}(32)$ (Alvarez-Gaumé+ 1984, 2022). Supersymmetry demands "Majorana" spinors of just one handedness, and requires a magical cancellation of anomalies between fermion and multivector boson fields. This mathematical miracle (Green & Schwarz 1984) precipitated the first superstring revolution.

In Spin(11,1) string theory on the other hand, vanishing of gauge and gravitational anomalies is almost trivial. Fermions are chiral, not Majorana, occupying spinor representations of

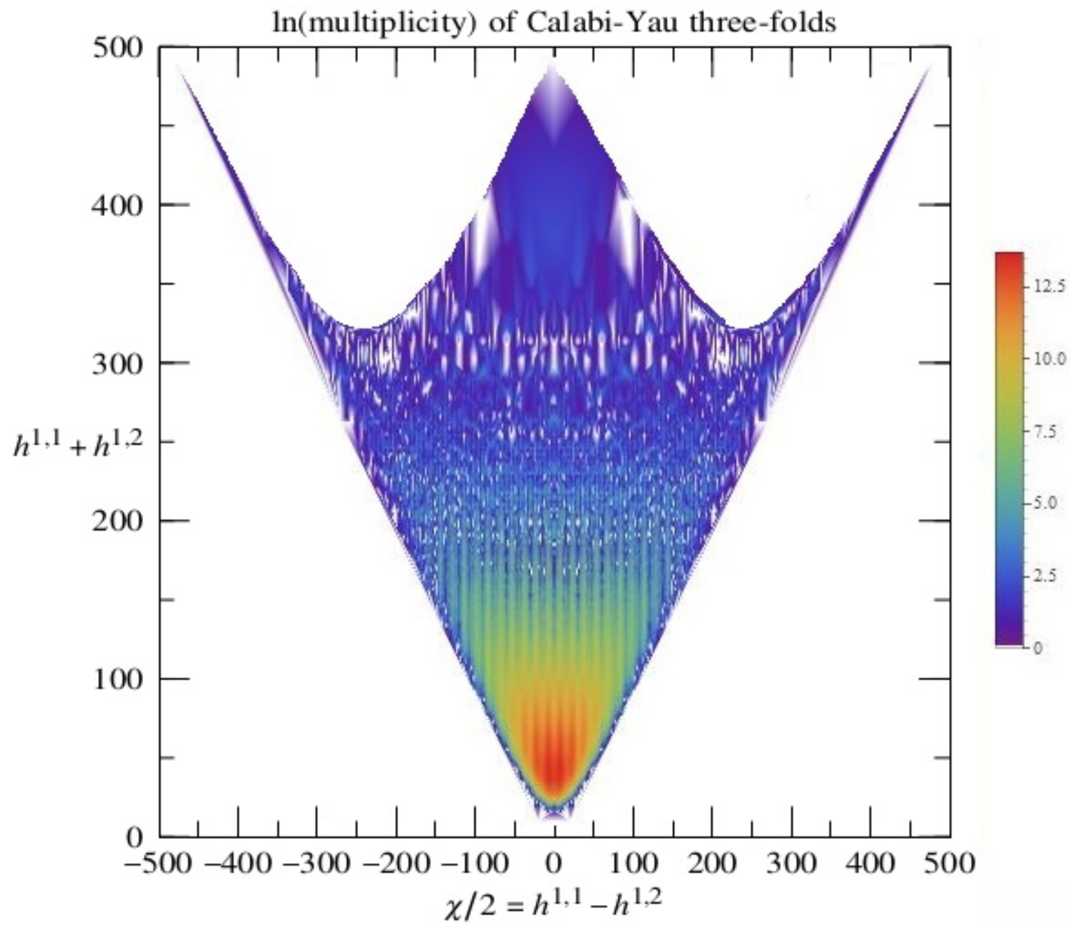
$$\mathrm{SU}(8)_R \times \mathrm{SU}(8)_L \ ,$$

and their antifermion partners occupy a matching conjugate representation. A sufficient condition for the fermionic contribution to anomalies to vanish is that fermions of opposite chirality come in matching representations of the gauge group, which is true here. The same applies to bosons.

11/15. Fermion generations

There are 3 generations of fermion (e, μ, τ), but only 1 generation of bosons.

In string theories with fermions living on a compact internal manifold whose tangent bundle is, as here, the gauge bundle, the number of generations (number of massless gauge-multiplet solutions of the Dirac equation, also known as the index of the Dirac operator $\boldsymbol{\gamma}^m D_m$) equals $\frac{1}{2}|\chi|$, half the absolute value of the Euler characteristic χ of the compact manifold (Atiyah-Singer index theorem).



He+ (2015).

The first Calabi-Yau three-fold discovered with $\frac{1}{2}|\chi| = 3$ was a core ingredient of the influential proposal by Candelas+ (1985) to compactify 10d $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string theory to 4d on a 6d Calabi-Yau three-fold.

There have been extensive investigations of Calabi-Yau three-folds, and some investigation of Calabi-Yau four-folds, but relatively little on Calabi-Yau five-folds. There is a plethora of Calabi-Yau manifolds, although only 9 known three-folds with $\frac{1}{2}|\chi| = 3$ (Candelas+ 2018).

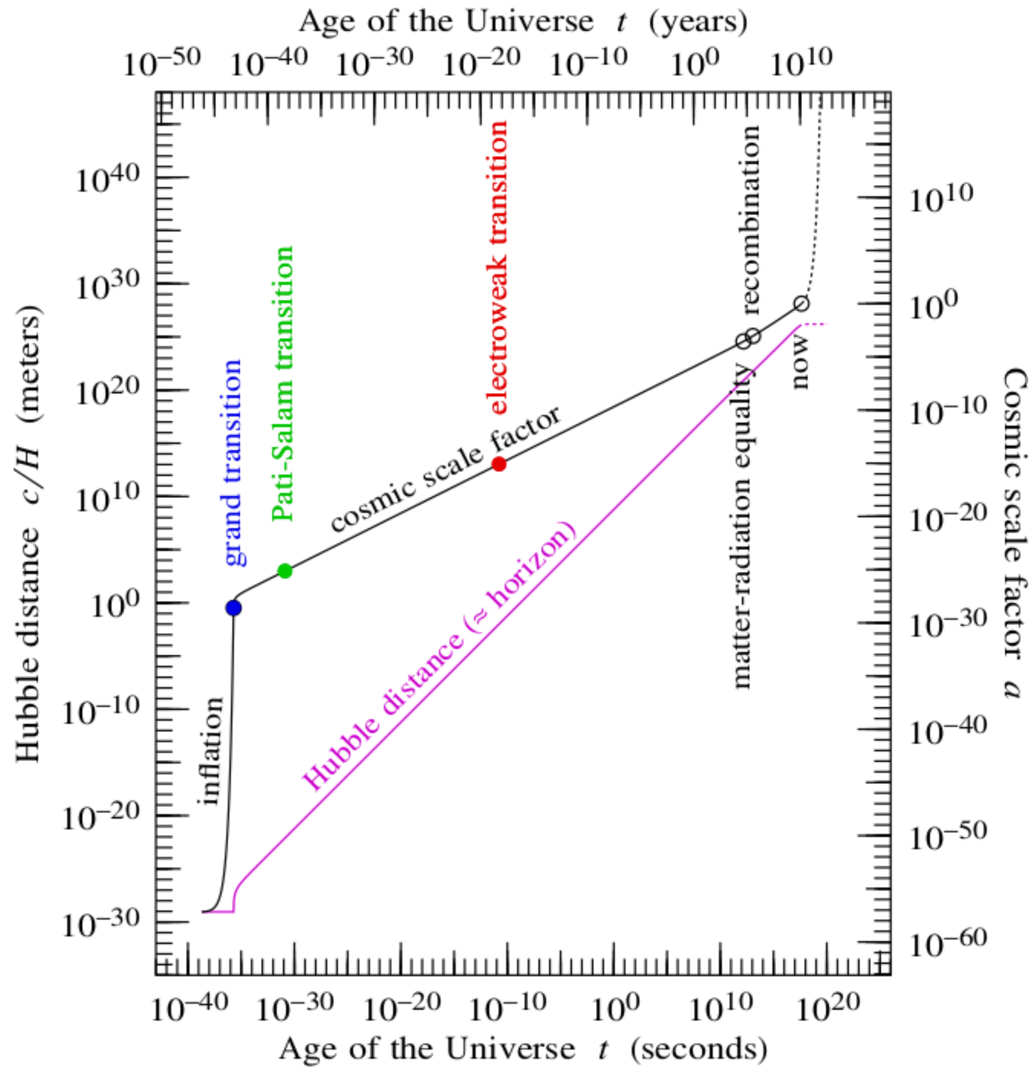
12/15. The tachyon

As expected in a bosonic string theory, the ground eigenstate of the theory is tachyonic, with mass squared

$$m^2 = -2/L^2 \ .$$

The ground state has the energy of the string scale L, which is thought to be that of grand unification. The tachyonic ground state is generally considered reason to discard bosonic string theory as unphysical.

However, the Spin(11,1) grand Higgs field $\langle \boldsymbol{T} \rangle$ matches the properties of the tachyonic ground state: it is a spacetime scalar, it has zero standard-model charge, and it is unstable by the Higgs mechanism. $\langle \boldsymbol{T} \rangle$ does have t charge, which allows it to generate a Majorana mass for the right-handed neutrino, but it has zero $yzrgb$ standard-model charge.

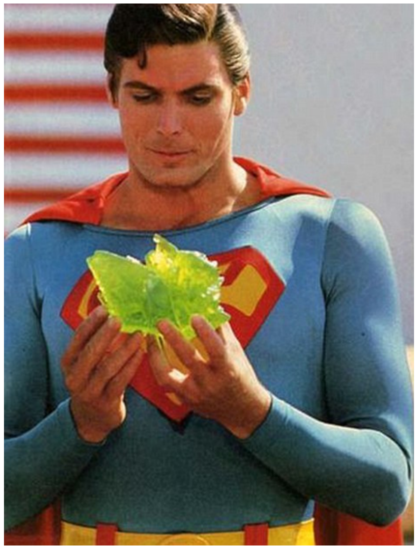


In the standard model of cosmology, vacuum energy has decayed through successive Higgs-mediated symmetry breakings to the present time, where the vacuum, the dark energy, has a tiny but nonzero positive density. The dark energy is apparently stable over cosmological timescales, but it is not known whether it is stable in an absolute sense.

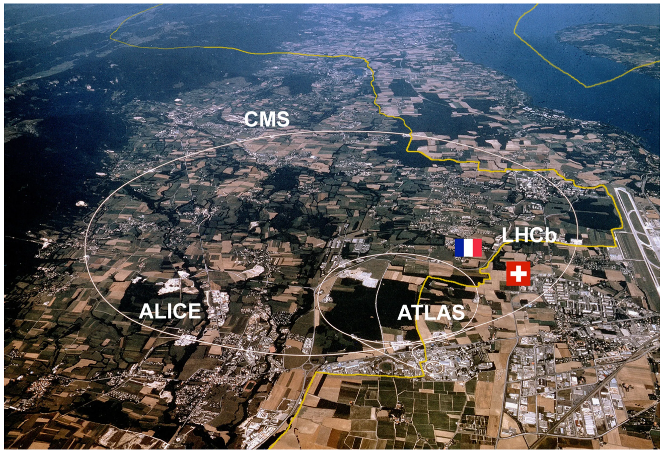
13/15. The weakness of string theory revisited

Standard arguments for supersymmetry (super = fermionic):

1. Nature has fermions.
   Polchinski (1995) taught us that fermions can live on D-branes.
2. Evades Coleman-Mandula (1967) no-go theorem.
   Spin(11,1) obeys theorem: Dirac and internal algebras always commute.
3. Gets 3 coupling parameters of standard model to meet at grand unification.
   Spin(11,1): 3 couplings meet in 2 separate steps.
4. The ground state of bosonic strings is tachyonic, therefore unstable.
   In cosmology, high energy vacua are unstable by the Higgs mechanism. Are these the tachyons?
5. Fermion-boson infinities cancel each other in scalar boson masses.
   Can standard particle qft be applied to the electroweak Higgs boson?
6. Magically cancels gauge anomalies in string theory.
   Spin(11,1): gauge anomalies vanish because the theory is chirally balanced.



Superman + kryptonite (1978)

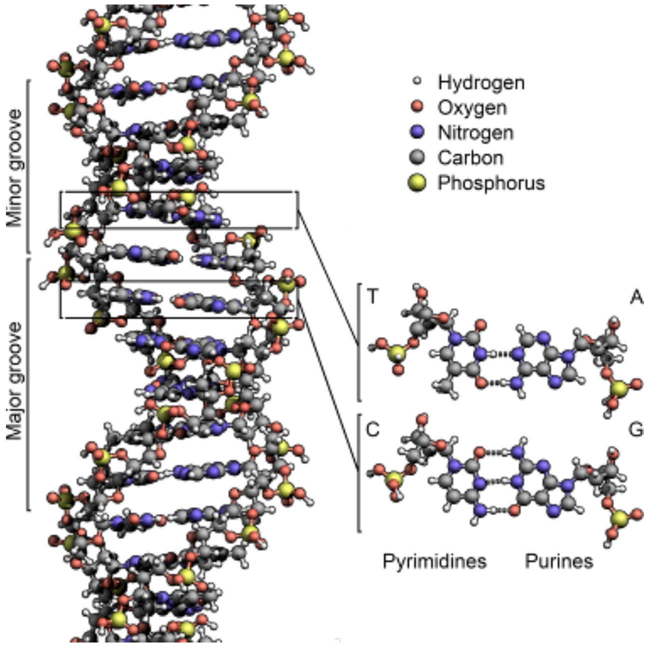


LHC − superpartners (2017)

Problems with supersymmetry:

1. Predicts symmetries that are not observed.
   Spin(11,1) is not supersymmetric.
2. Preempts symmetries that are observed.
   Spin(11,1) possesses the symmetries that are observed.

14/15. The multiverse just got a whole lot bigger



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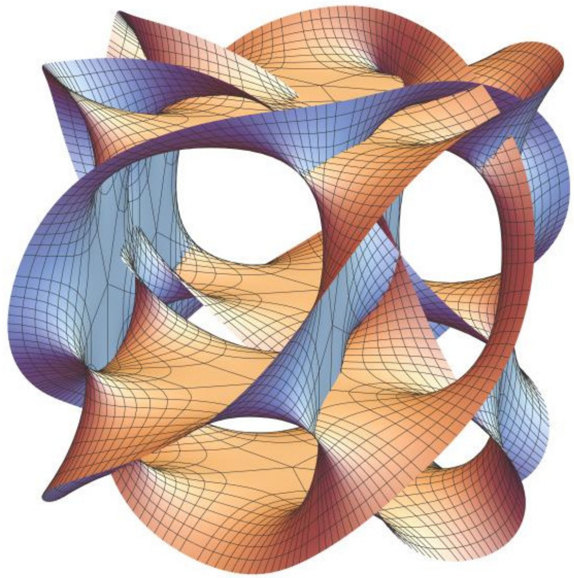
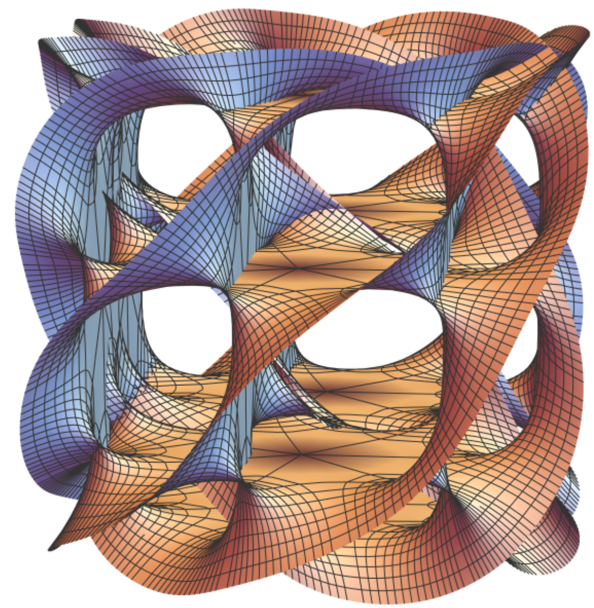


Calabi-Yau three-fold

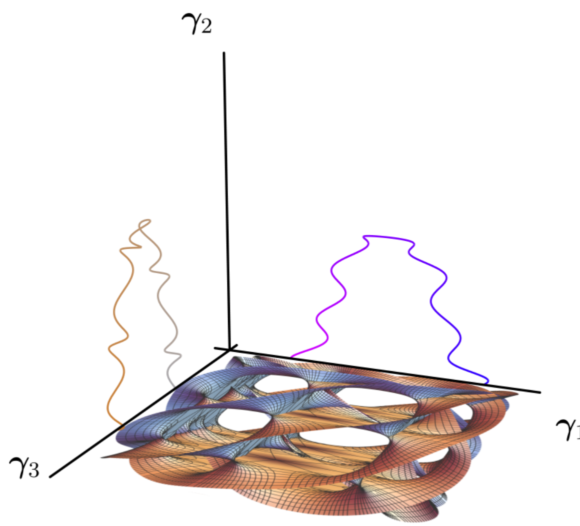


Calabi-Yau five-fold
[Andrew Hanson (2011) Mathematica code]()

- The complicated DNA of earthly life is a language written with 4 letters, T, A, C, G.
- It may be that the complicated DNA of the Universe is encoded in the labyrinthal convolutions of the folded-up dimensions and the stringy fields that coil them.
- If $\mathrm{Spin}(11,1)$ is correct, then the letters of the DNA of our Universe are the 6 bits $tyzrgb$.

15/15. Spin(11,1) string theory is:

1. Motivated by the fact that Spin(10) chirality equals Dirac chirality. From this, all else follows.
2. Not a direct product of internal and spacetime manifolds.
3. 11+1 dimensional, nontrivially accommodating 10 compact internal dimensions and 3+1 large spacetime dimensions.
4. Weak and color gauge fields are carried by bosonic open strings whose ends attach to D4- and D6-subbranes of the internal fermionic D10-brane.



5. The complex fermionic D10-brane becomes a Calabi-Yau five-fold on Pati-Salam symmetry breaking to the standard model.
6. A 26d bosonic string theory compactified to 12d on a 14d maximal self-dual torus of SU(8)×SU(8) (like heterotic theory compactified to 10d on the 16d maximal self-dual torus of $E_8 \times E_8$).
7. Not supersymmetric, so does not predict unobserved superpartners.
8. Anomaly-free, modular-invariant.
9. Are there Calabi-Yau five-folds with Euler characteristic ±6, giving three generations?
10. Tachyonic. It decays through tachyonic Higgs fields down to the present time, where the vacuum energy (dark energy) is apparently stable over cosmological timescales.
11. Spin(11,1) may already be ruled out because the predicted energy scale of grand unification, 10^{15} GeV, is less than the lower limit of 4×10^{15} GeV inferred for Spin(10) models by King et al. (2021) from the Super-Kamiokande lower limit of 1.6×10^{34} yr on the proton lifetime. A more careful calculation needs to be done that follows the symmetry breaking chain of Spin(11,1) rather than Spin(10).