ASTR 5770 Cosmology Fall 2025. Problem Set 9. Due Wed Nov 19

1. Program the equations for the simplest set of cosmological assumptions (20 points)

This is essentially Exercise 30.11 in the book.

Write computer code that integrates numerically the evolution equations (30.53)–(30.55) in the text. In the final exam, you will build on this code to compute the matter power spectrum (essentially, Exercise 30.16 in the book) so you should try to write the code in a well-structured fashion. It is theoretically and numerically advantageous to treat $\delta_c - 3\Phi$ and $\Theta_0 - \Phi$ as dependent variables, rather than δ_c and Θ_0 . I found it convenient to use $\ln a$ as the integration variable, and to work in units $a_{\rm eq} = H_{\rm eq} = 1$. Assume adiabatic initial conditions, $\zeta_c = \zeta_r$ (see §30.10), and without loss of generality normalize to unit initial amplitudes, $\zeta_c = \zeta_r = 1$. Do the computation for a selection of wavenumbers k. Plot $\Theta_0 - \Phi$ and -2Φ together to bring out the fact that the former oscillates about the latter, as expected from Problem Set 8. A numerical issue you may encounter is that your integration routine may get stuck trying to integrate the oscillating radiation monopole and dipole once the mode is well inside the horizon, $k\eta \gg 1$. One strategy is to stop following the photon moments after a certain time. Another convenient strategy is to introduce an artificial damping term, by changing the radiation dipole equation (30.54b) to

$$\dot{\Theta}_1 + \frac{k}{3}(\Theta_0 + \Psi) = -2k \,\kappa \,\Theta_1 \,\,, \tag{1.1}$$

where κ is a dimensionless damping coefficient that becomes large when the fluctuation is well inside the horizon, $k\eta \gg 1$,

$$\kappa = \epsilon k \eta \,\,, \tag{1.2}$$

with ϵ some suitably small number (I chose $\epsilon = 10^{-3}$). (To see why the damping term works as claimed, combine the radiation monopole and dipole equations into a second order differential equation, and read §32.5. The introduction of damping anticipates, but is not an adequate substitute for, the physical processes of damping addressed in Chapter 32.)