Please interrupt, ask questions. Practice for Comps 2. 141

IONIZATION EQUILIBRIUM

=: steady state populations of ions

X, X^+, X^{++} or X^{2+}, ...

Three generic types of equilibria:

(1) Thermodynamic

(2) "Collisional" or coronal equilibrium

(3) Photoionization equilib.

Q: When not eq.? A: Time dep. Ex? CMB post-rec, shocks

(1) TE.

Valid when?

Density is high enough that

$$X + e \underset{\text{3-body recombination}}{\overset{\text{coll. ionization}}{\rightleftarrows}} X^+ + 2e$$

Q: Rates dep on n?

A: $\underset{\rightarrow}{n_x n_e C} = \underset{\leftarrow}{n_{x^+} n_e^2 \alpha}$

ie coll ion and its inverse dominate. Q: What relates proc & inv? A: Detailed balance

Under same circumstances

$$X + e \underset{\text{coll. deexc}}{\overset{\text{coll. exc}}{\rightleftarrows}} X^* + e$$

are already in TE. Why? Both 2-body procs.

Do colliders have thermal pop? Yes.

LTE

Local TE = ions & levels in TE, but radiation not in TE.

LTE often good approxn in stellar/planet atmospheres where opt depth $\lesssim 1$.

Q: Ex? Ans: CMB pre-rec, ★ interiors

Q: What is eqn describing TE of ions in non-relativistic $\underset{= \text{non-degen}}{\underline{\text{Boltzmann}}}$ case?

A: Saha eq

$$\frac{n_{x^+} n_e}{n_x} = \frac{g_{x^+} \overset{\propto ?\,2}{g_e}}{g_x} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_x / kT}$$

ie $$\frac{n_{x^+}}{n_x} = \frac{\text{func of } T}{n_e}$$

Q: What about eq describing TE of ions when relativistic and/or degenerate?

A: Apply free energy minimization

Minimize

$$F = U - TS$$

wrt constituents N_x

at fixed T and $N_{\text{conserved}}$.

(2) Collisional / coronal equilibrium

Ionization & recombination dom by 2-body collision processes of form

$$X + e \longrightarrow X^+ + \text{etc}$$

$$X^+ + e \longrightarrow X + \underbrace{\text{etc}}_{\text{see below}}$$

"Coll" equilibrium holds in hot, low density environments such as

- solar corona • supernova remnants
- hot, x-ray emitting gas in galaxy clusters – note such hot gas dominates baryonic matter: galaxies minor contrib.

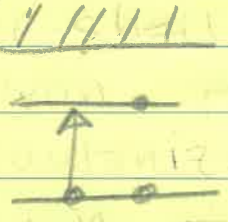
Ionization processes dominated by:

(a) Direct coll ionization

$$X + e \rightarrow X^+ + 2e$$

(b) "Auger" ionization

(i)



coll. exc of inner-shell elec

(ii)

autoionization of excited state

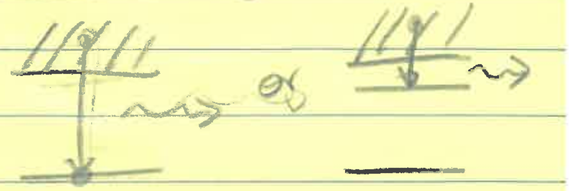
$$X + e \rightarrow X^* + e$$

$$X^* \rightarrow X^+ + e$$

Recombination process dominated by:

(a) Radiative recombination

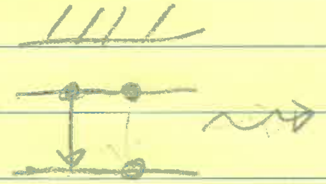
$$X^+ + n_e \rightarrow X + \gamma$$



(b) $\overset{=2}{\text{Di}}$electronic recombination

(i)

(ii)



electron capture
= inv of autoionization

rad decay

$$X^+ + n_e \rightarrow X^*$$

$$X^* \rightarrow X + \gamma$$

Darling example of $C^{++} + n_e \rightarrow C^+ + 2\gamma$

Collision ionization balance eqs

$$\frac{dn_x}{dt} = -n_x n_e \left(C_x(\text{direct}) + C_x(\text{Auger}) \right) + n_{x^+} n_e \left(\alpha_x(\text{rad}) + \alpha_x(\text{diel}) \right)$$

$= 0$ in equilibrium

Rates C_x & α_x depend on T_e, not density

Hence

$$\frac{n_{x^+}}{n_x} = \frac{C_x}{\alpha_x} = \text{func of } T_e$$

Typically $C_x \approx c_x e^{-\underbrace{\chi_x}_{\text{ionization potential}}/kT}$ why?

$$\alpha_x \approx 10^{-\text{few}} c_x$$

so $\frac{n_{x^+}}{n_x} \approx 1$ when kT = fraction of χ_x

Rate coeffs often fitted to simple analytic formulae (eg Aladdin database).

(3) Photoionization equilibrium

Ionization dominated by photoionization;
Recomb dominated by same 2-body
coll processes as in coll equilib.

Phot equilib holds in low density
environments where there is a
strong source of ionizing (UV, x-ray)
radiation eg.?

- H II regions around hot O B stars
- Planetary nebulae [O III]
- x-ray ionized nebulae around x-ray sources, such x-ray binary
- Broad Line Region BLR ← near BH (why?)
  Narrow " " NLR ← far from BH
  around Active Galactic Nuclei AGN

(14·6) →

Photionization processes dominated by

Photoionization

$X + \gamma \rightarrow X^+ + e$

inv of rad. recomb

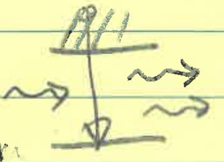
Q: If photoion & rad. rec dominate, what do you expect?

A: Photoionizing intensity → blackbody at kinetic temperature T_e of gas?

Q: Does that happen?

A: No because of

Yes in opt thick lines. Then need stim.



Spectra of photoionized regions

Dominated by emission lines

(i) Recomb lines of H

Ly, H, Pa, ..., radio rec. lines

↑ often opt thick (Ly)

↑ sometimes opt thick (H)

Line luminosities set by $\frac{\#\text{ photoionizations}}{\text{time}}$

(ii) Emission lines that radiate energy dumped into electrons by photoion

- Forbidden lines

	[O I]	[N II]	[O II]	[O III]	[Ne III]	[Ne V]
ion pot:	13.6 eV	29.6	35.1	54.9	63.4	128.2

less ← ionized

- Other lines of moderate excitation energy

Mg II C III] C IV O VI 2s - 2p

Temperature

$\approx 1-2 \times 10^4$ K ← If you see this temp, good chance you are looking at photoionized gas - Osterbrock.

Because Lyα is excited at $T \gtrsim 10^4$ K, Lyα cooling acts like thermometer.

How to measure T?

[N II] $\frac{6548+6583}{5755}$ $2p^2$ [O III] $\frac{4959+5007}{4363}$ (8.4)

AGN

www.ua.edu/keel/agn/spectra.html

ngc4151sp.html

Photoionization balance eqs

Photoionization rate of ion X

$$\Gamma_x \equiv \frac{\text{photoionizations}}{\text{time . ion}} = \int_{\nu_x}^{\infty} \underbrace{\frac{F_\nu}{h\nu}}_{\text{photon flux from ionizing source}} \underbrace{a_\nu}_{\text{photoionization cross-section}} \, d\nu$$

ionization frequency → ν_x

$$F_\nu = \int \underset{\substack{\uparrow \\ \text{intensity}}}{I_\nu} \; \underset{\substack{\uparrow \\ \text{angular directions}}}{d\Omega} \qquad \frac{\text{energy}}{\text{time . area . freq}}$$

Ion balance eqs

$$\frac{dn_x}{dt} = \underbrace{-n_x \Gamma_x}_{\text{photoion}} + \underbrace{n_{x^+} n_e \alpha_x}_{\text{recomb}}$$

thermally averaged rate $\alpha_x(T_e)$

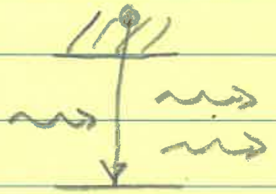
(14.9) → $= 0$ in equilib.

Q: Can rad rec rate α_x be related to photoionization x-section a_ν?

$$X + \gamma \underset{\text{rad rec}}{\overset{\text{photoion}}{\rightleftarrows}} X^+ + e$$

A: Yes, by detailed balance.

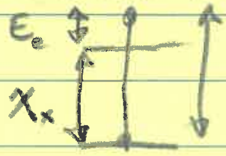
Have to include stim recomb



$$X^+ + e + \gamma \longrightarrow X + 2\gamma$$

$$\underbrace{n_x \frac{4\pi B_\nu}{h\nu} a_\nu}_{\text{Photoion}} = \underbrace{n_{x^+} \frac{dn_e}{d\nu} \alpha_\nu}_{\text{rad rec}} + \underbrace{n_{x^+} \frac{dn_e}{d\nu} \frac{4\pi B_\nu}{h\nu} a'_\nu}_{\text{stim rec}}$$

Why $\frac{dn_e}{d\nu}$? Energies must balance



$h\nu = \chi_x + E_e$

$$\alpha_\nu = \frac{4\pi B_\nu}{h\nu}\left(\frac{n_x}{n_{x^+}\, dn_e/d\nu}\, a_\nu - a_\nu'\right)$$

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$ Planck

$$\frac{n_{x^+} n_e}{n_x} = \frac{g_{x^+} g_e}{g_x}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_x/kT}$$ Saha

$$\frac{dn_e/d\nu}{n_e} = \frac{4}{\sqrt{\pi}}\, e^{-(v/v_{th})^2}\, \frac{v^2}{v_{th}^3}\, \underbrace{\frac{dv}{d\nu}}_{=\,c/\nu}$$ Maxwellian

$$v_{th} = \sqrt{2kT/m_e}$$

So $$\frac{n_x}{n_{x^+}\, dn_e/d\nu} = \frac{g_x}{g_{x^+}}\frac{h^2}{8\pi m_e^2 c}\frac{h\nu}{E_e}\, e^{(\chi_x+E_e)/kT}$$

$$\Rightarrow \underset{\text{stim rec}}{a_\nu'} = \frac{g_x}{g_{x^+}}\frac{h^2}{8\pi m_e^2 c}\frac{h\nu}{E_e}\underset{\text{photionization}}{a_\nu}$$

and

$$\boxed{\begin{aligned}\underset{\text{rad rec}}{\alpha_\nu} &= \frac{8\pi\nu^2}{c^2}\underset{\text{stim rec}}{a_\nu'}\\ &= \frac{g_x}{g_{x^+}}\frac{(h\nu)^3}{m_e^2 c^3 E_e}\underset{\text{photion}}{a_\nu}\end{aligned}}$$

$$\underset{\text{therm averaged rec rate}}{n_e\,\alpha_x(T)} = \int \underset{\text{Maxwellian}}{\frac{dn_e}{d\nu}}\,\alpha_\nu\, d\nu$$

In equilib

$$\frac{n_{x^+}}{n_x} = \frac{\Gamma_x}{n_e \alpha_x} .$$

Consider for example photoionizing source of luminosity L. At distance d from source

$$F_\nu = \frac{L_\nu}{4\pi d^2}$$

$$= \frac{L \phi_\nu}{4\pi d^2} \qquad \phi_\nu = \text{source spectrum}$$

$$\int \phi_\nu \, d\nu = 1$$

So $\Gamma_x = \dfrac{L}{4\pi d^2} \displaystyle\int \frac{\phi_\nu}{h\nu} a_\nu \, d\nu$

hence

$$\frac{n_{x^+}}{n_x} = \underbrace{\frac{L}{n_e d^2}}_{\text{called}} \frac{1}{4\pi \alpha_x} \int \frac{\phi_\nu}{h\nu} a_\nu \, d\nu$$

"photoionization parameter" ξ

$$\xi \equiv \frac{L}{n_e d^2}$$

is key parameter of photoionization models.

14·10

Web

Star forming regions/nebulae

Orion spectrum

Planetary nebulae

" spectrum

Active galaxies

spectrum