ASTR 5110 Atomic and Molecular Processes Fall 2022. Problem Set 8. Due Wed 26 Oct.

1. X-ray spectrum of Capella

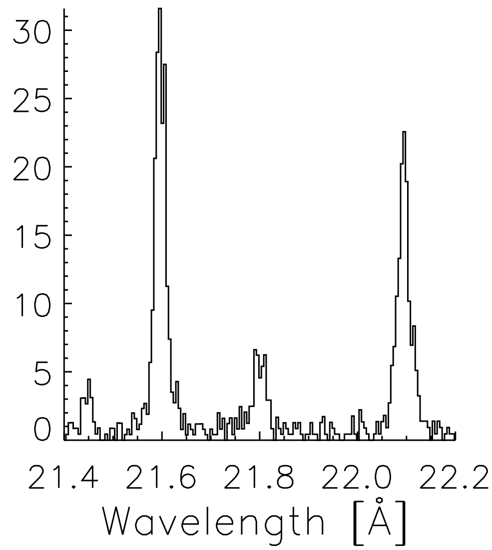

Figure 1: X-ray spectrum of Capella from the Chandra satellite, showing emission lines of He-like O VII 2.16, 2.18, 2.21 nm (Canizares et al. 2000, ApJL 539, L41).

(a) The three lines are the resonance, intercombination, and forbidden lines between the $n = 1$ and 2 energy levels of O VII. A resonance line is an allowed line that connects to the ground state. Draw a Grotrian diagram marking the lines. What are the terms and levels? What are the selection rules for each of the lines? [Hint: Look up the levels on the NIST Atomic Spectra Database.]

(b) For convenience, label the $1s^2\,{}^1S_0$, $2s\,{}^3S_1$, and $2p\,{}^3P_1$ levels 1, 2, and 3 respectively. Assume that levels 2 and 3 are populated by recombination/collision/radiative decay at some constant rates C_i, that they decay radiatively to ground with A-values A_i, and that the 2 level is excited into the 3 level by collisions with electrons at rate $n_e C_{23}$. In other words, the number densities n_i in levels 2 and 3 vary with time t as

$$\frac{dn_2}{dt} = C_2 - n_2 A_2 - n_2 n_e C_{23} \ , \tag{1a}$$

$$\frac{dn_3}{dt} = C_3 - n_3 A_3 + n_2 n_e C_{23} \ . \tag{1b}$$

Assume that the populations are in steady-state. Derive an expression for the ratio of emission line strengths in the form

$$\frac{n_2 A_2}{n_3 A_3} = \frac{R}{1 + n_e/n_c} \ . \tag{2}$$

You should obtain an expression for R in terms of the C_i, and an expression for the critical density n_c in terms of R, C_{23}, and the A_i. [Note: The real situation is more complicated than stated, but this version captures the main features that determine the ratio (2) of forbidden to intercombination line strengths. Among other things, the only level in ^3P that contributes to the intercombination line is the $^3P_1$ level; the other levels in ^3P, namely $^3P_0$ and $^3P_2$, decay by allowed radiative transitions to $^3S_1$, so effectively contribute to C_2. One process neglected in the given version of the problem is collisional de-excitation $3 \to 2$ (at rate C_{32}), which is the process inverse to collisional excitation $2 \to 3$ (at rate C_{23}). Physically, radiative decay from the 2 level is slow compared to radiative decay from the 3 level, so collisions out of the 2 level can be important while collisions out of the 3 level can be neglected. The solution set will include both collisional excitation and de-excitation $2 \leftrightarrow 3$, and it will be found that it is indeed legitimate to include the former but neglect the latter.]

(c) Evaluate the critical density n_c if $C_2/C_3 = 4.5$, $A_2 = 1.06 \times 10^3\,\mathrm{s}^{-1}$, $A_3 = 4.45 \times 10^8\,\mathrm{s}^{-1}$, and $C_{23} = 8 \times 10^{-15}\,\mathrm{m}^3\,\mathrm{s}^{-1}$ (Porquet & Dubau 2000, A&AS 143, 495 astro-ph/0002319). (I notice that the NIST Atomic Spectra Database does not have a value for the (semi-forbidden) transition A_3, which is totally weird.)

(d) Estimate the electron density n_e from the spectrum at the top of the previous page. Describe and justify your procedure.

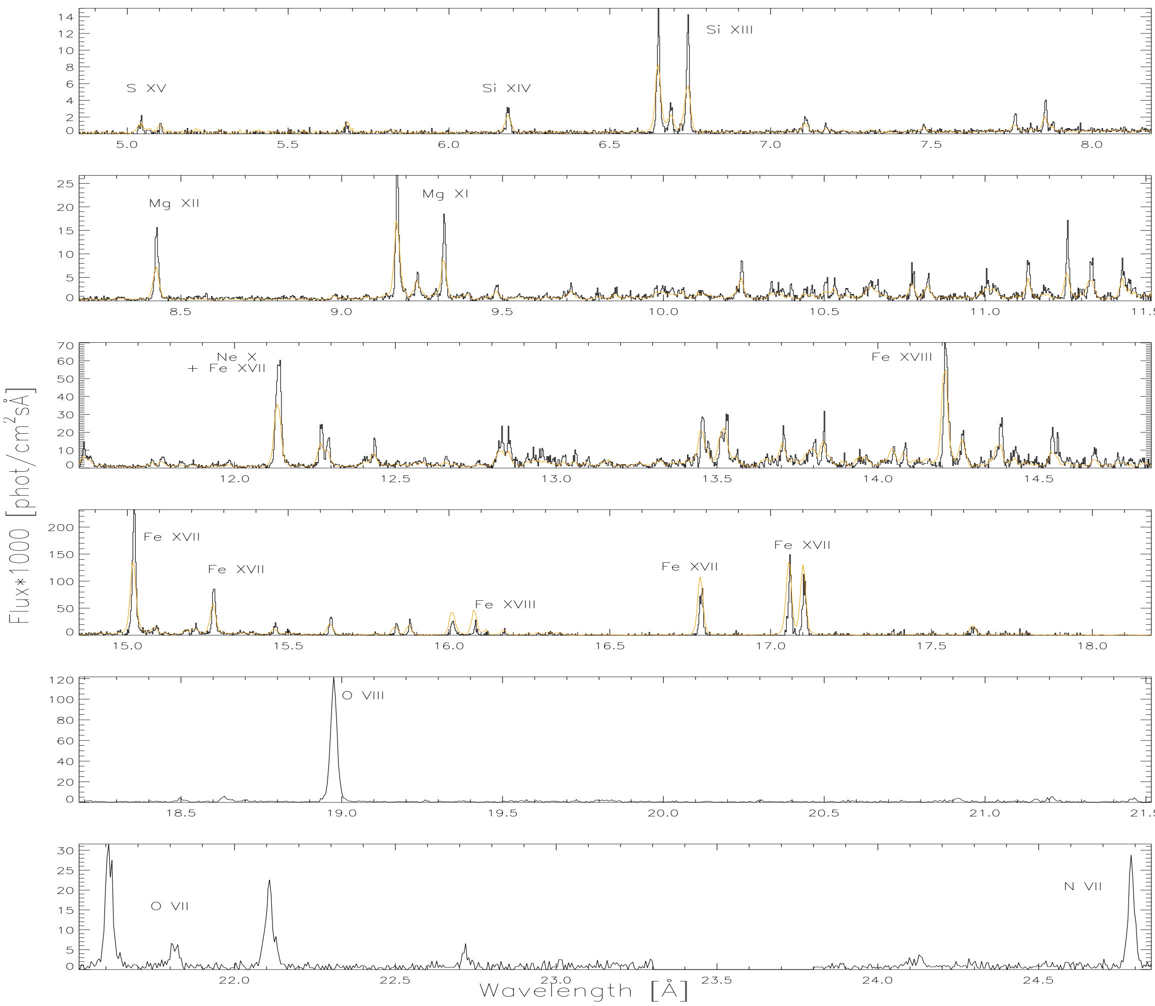

Figure 2: Complete x-ray spectrum of Capella from the Chandra satellite (Canizares et al. 2000 http://snl.mit.edu/pub/papers/2000/Canizares-ADN-2000.pdf). The spectrum shows Ly α ($n = 2$ to 1) lines of H-like N, O, Ne, Mg, and Si, and He α ($n = 2$ to 1) lines of O, Ne, Mg, Si, and S. The spectrum also shows "L-shell" ($n = 3$ to 2) transitions in Ne-like Fe XVII and F-like Fe XVIII.