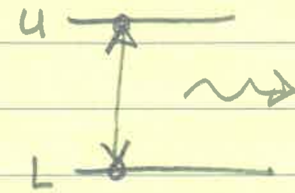
Two-level atom

Consider simplified case
of 2-level atom
collisionally excited
and de-excited by electrons,
and able to radiate spontaneously.



Prototype of more complicated systems:
1. Several levels - eg 3 level PS 7 Q2.
2. Inclusion of radiative absorption
   & stimulated emission.

First, just collisions.
Assume coll with electrons, Why?
Distrn of elecs will be? Maxwellian. Why?

Notes $\boxed{\text{coll.ps}}$ on exc, de-exc

2-level atom with radiative decay

rate $\uparrow$ $= n_e \, n_L \, C_\uparrow$

rate $\downarrow$ $= \underbrace{n_e \, n_u \, C_\downarrow}_{\text{coll}} + \underbrace{n_u \, A_\downarrow}_{\text{rad}}$

In equilibrium these are equal

$\Rightarrow \quad n_e n_L C_\uparrow = n_e n_u C_\downarrow + n_u A_\downarrow$

$= n_u (n_e C_\downarrow + A_\downarrow)$

$$\Rightarrow \quad \frac{n_u}{n_L} = \frac{n_e C_\uparrow}{n_e C_\downarrow + A_\downarrow}$$

$$= \frac{C_\uparrow / C_\downarrow}{1 + \dfrac{A_\downarrow}{n_e C_\downarrow}}$$

$$= \frac{\dfrac{g_u}{g_L} e^{-E_{uL}/kT}}{1 + \dfrac{n_c}{n_e}} \qquad E_{uL} \equiv E_u - E_L$$

where $n_c = \dfrac{A_\downarrow}{C_\downarrow}$ is critical density

Departure coefficient

$$\left.\frac{n_u}{n_L}\right|_{TE} = \frac{g_u}{g_L} e^{-E_{uL}/kT}$$ depends on T but not density n.

$$b_{uL} \equiv \frac{n_u/n_L}{n_u/n_L|_{TE}} = \frac{1}{1+n_c/n_e}$$

$$\rightarrow \begin{cases} 1 & n_e \gg n_c \quad \text{high density} \\ n_e/n_c & n_e \ll n_c \quad \text{low} \quad " \end{cases}$$

Dependence of emission on density

$$\# \frac{\text{phots emitted}}{\text{time . Vol}} = n_u A_\downarrow$$

$$\propto n_u$$

$$n_u = \frac{n_u}{n_u+n_L} n \quad , \quad n \equiv n_u + n_L$$

$$= \frac{n}{1 + n_L/n_u}$$

$$= \frac{n}{1 + \frac{g_L}{g_u} e^{E_{uL}/kT}\left(1+\frac{n_c}{n_e}\right)}$$

$$\rightarrow \begin{cases} \dfrac{n}{1+\frac{g_L}{g_u}e^{E_{uL}/kT}} \propto n & n_e \gg n_c \\ \dfrac{n\, n_e/n_c}{\frac{g_L}{g_u} e^{E_{uL}/kT}} \propto n\, n_e & n_e \ll n_c \end{cases}$$

$$\# \frac{\text{phots}}{\text{time}} = A_{ul}\int n_u\, dV \propto \begin{cases} \int n\, dV & n_e \gg n_c \\ \underbrace{\int n^2\, dV}_{\text{"emission measure"}} & n_e \ll n_c \end{cases}$$

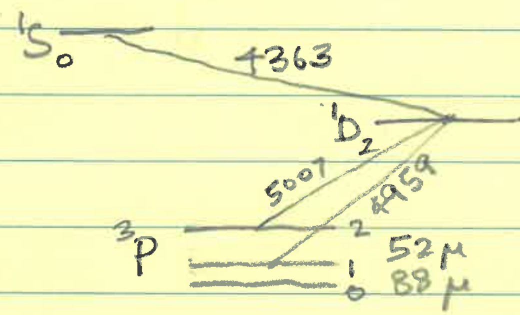
Temperature & density diagnostics

Temperature (of what? colliders - usually elecs)
Ratios of strengths of emission lines provide good temperature diagnostic if?

(1) lines from same ion
(2) excitation energies differ substantially, so Boltzmann factor is large
(3) for optical/UV lines, wavelengths are close, so that "reddening" by dust absorption is not a problem.

Ex/ $\underset{2p^2}{[OIII]}$ $\dfrac{4363}{4959+5007}$

is classic T diagnostic for ionized nebula



$\underset{2p^4}{[OI]}$ $\dfrac{5577}{6300+6364}$ PS 7

Density (of what? colliders, usually elec)

Ratio of strengths of emission lines good n_e diagnostic if

(1) lines from same ion

(2) similar excitation energies, so not temperature sensitive

(3) different critical densities

(4) for optical/UV lines, wavelength are close to avoid reddening ambiguity.

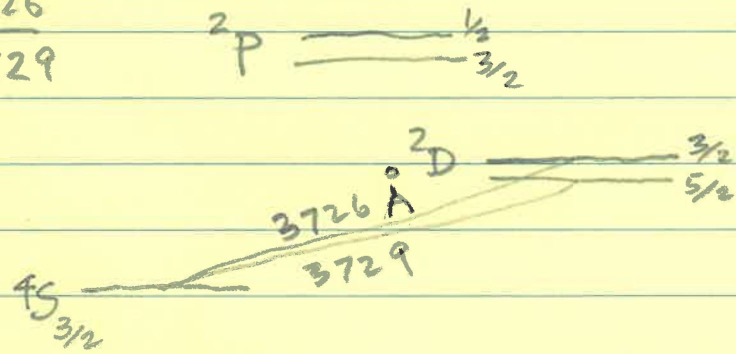
Classic examples are optical line pairs – see Osterbrock (1989) "Astrophysics of gaseous nebulae"

With modern IR astronomy, pairs of IR fine structure lines good

Ex/ optical $2p^3$ or $3p^3$ atoms

[O II] $\frac{3726}{3729}$

$2p^3$

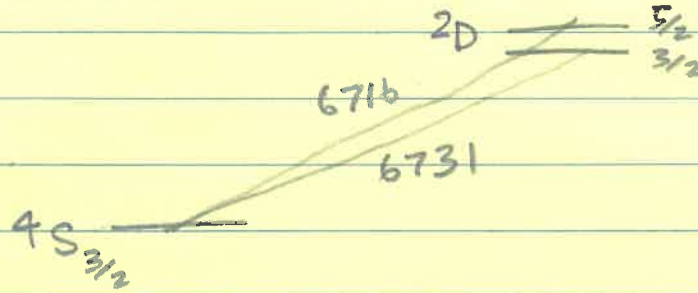


violates Hund's rule...

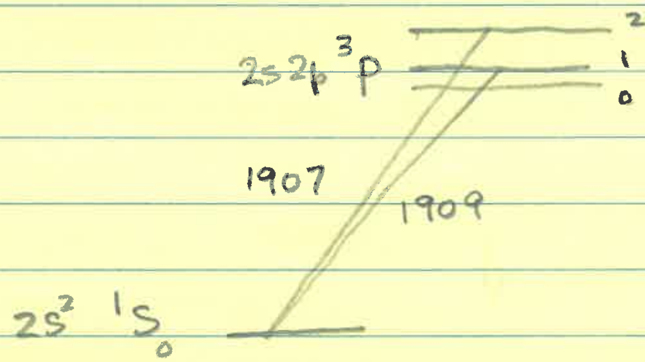
sensitive at $n_e \sim 10^2 - 10^4$ cm^{-3}

[S II] $\frac{6716}{6731}$

$3p^3$



$\frac{\text{[C III] 1907}}{\text{C III] 1909}}$



sensitive at $n_e \sim 10^3 - 10^5\ cm^{-3}$

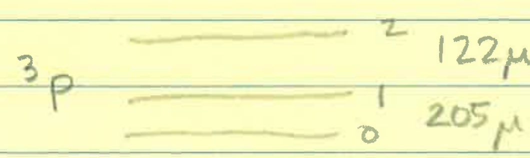
Ex IR lines

[O III] $\frac{52\mu}{88\mu}$ $\quad n_e \sim 10^3 - 10^5\ cm^{-3}$

$2p^2$

[N II] $\frac{122\mu}{205\mu}$

$2p^2$



[S III] $\frac{18.7\mu}{33.5\mu}$

[O I] $\frac{63\mu}{145\mu}$

$2p^4$