

## ASTR 3740 Relativity & Cosmology Spring 2025. Project 2. Fri Feb 28.

### The River Model of Black Holes

Your group should come up with a single set of answers for the whole group. When you are done, upload your group solution to canvas. On the solution, please include the name of the Scribe, and of all other members of the group.

According to the River Model of Black Holes, the behavior of objects near black holes is precisely as if space were falling like a river into the black hole. For spherical black holes, this model was discovered by Allvar Gullstrand and Paul Painlevé in 1921.

In the model, space falls inward through a flat background at the Newtonian escape velocity

$$v = \left( \frac{2GM}{r} \right)^{1/2}$$

hitting the speed of light at the horizon.

What does the river model predict for the answers to the following questions:

1. (4 points) What radius does the river model predict for the horizon of a black hole?
2. (4 points) Suppose that you are a light beam exactly at the horizon. What would happen to you if were pointed directly outward? What would happen to you if you were pointed mostly but not exactly outward? In what way, if any, does this behavior differ from what Newtonian gravity (John Mitchell's 1784 picture) would predict?
3. (4 points) Suppose that you are a light beam orbiting the black hole in a circular orbit. On this orbit, the so-called "photon sphere," are you at the horizon, inside the horizon, or outside the horizon?
4. (4 points) How does the river model account for redshifting and freezing at the horizon? [Hint: Think about time dilation: the redshift factor is precisely the rate at which an observer sees an emitter's time go by.]
5. (4 points) Make a connection between the appearance of the sky if you hover just above the horizon of a black hole, and special relativistic beaming.
6. (4 points) Qualitatively, what would the river model predict for the radial and transverse tidal forces experienced by an infalling observer?
7. (3 points) One of the fundamental propositions of Special and General Relativity is that you cannot tell how fast a piece of space is moving — only relative motions are measurable. What then does it mean to say that space is falling into a black hole?
8. (3 points) In the river model, the flow of space accelerates inward to the black hole. If the river were moving uniformly instead of accelerating, would there be any gravity?