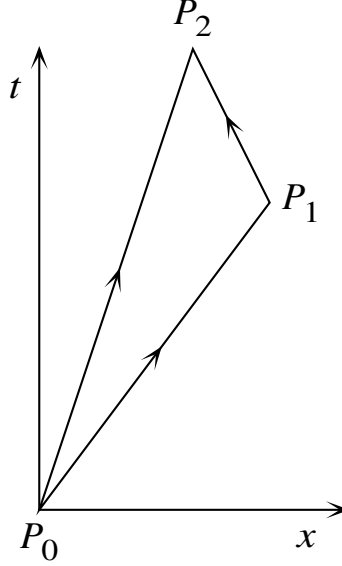


ASTR 3740 Relativity & Cosmology Spring 2025. Problem Set 3.  
Due Wed Feb 19

1. (15 points) The Longest Proper Time between Two Events is a Straight Line



Consider a person whose worldline goes from spacetime event  $P_0$  to spacetime event  $P_1$  at velocity  $v_1$  relative to some inertial frame, and then from  $P_1$  to spacetime event  $P_2$  at velocity  $v_2$ , as illustrated in the Figure. Assume for simplicity that the velocities are both in the (positive or negative)  $x$ -direction. Show that the proper time along a straight line from  $P_0$  to  $P_2$  is always greater than or equal to the sum of the proper times along the two straight lines from  $P_0$  to  $P_1$  followed by  $P_1$  to  $P_2$ . Hence conclude that the longest proper time between two events is a straight line. What does this imply about the twin paradox? [Hint: It is simplest to use rapidities  $\alpha$  rather than velocities. Let the segment from  $P_0$  to  $P_1$  be  $\{t_1, x_1\} = \tau_1\{\cosh \alpha_1, \sinh \alpha_1\}$ , and the segment from  $P_1$  to  $P_2$  be  $\{t_2, x_2\} = \tau_2\{\cosh \alpha_2, \sinh \alpha_2\}$ . The segment from  $P_0$  to  $P_2$  is the sum of these,  $\{t, x\} = \{t_1 + t_2, x_1 + x_2\}$ . Show that

$$\tau^2 - (\tau_1 + \tau_2)^2 = 4\tau_1\tau_2 \sinh^2\left(\frac{\alpha_2 - \alpha_1}{2}\right), \quad (1.1)$$

which is a minimum for  $\alpha_2 = \alpha_1$ .]

## 2. (15 points) Falling into a Black Hole

Explore the “Falling into a Black Hole” and “Inside Black Holes” websites at

<http://jila.colorado.edu/~ajsh/bh/>

and

<http://jila.colorado.edu/~ajsh/insidebh/>

Formulate a quiz question on black holes *different* from any of those at

<http://jila.colorado.edu/~ajsh/bh/quiz.html>

and answer it. 80% of your grade (12 points) on this problem will be on your quiz question, and 20% (3 points) on your answer to it. So be sure to state the question clearly and precisely, in a way that anyone browsing the web site would understand unambiguously. You may collaborate with others, but you should write your quiz question and answer by yourself.

The (anonymized) questions will be posted on the course website, and you will vote for the best questions.