ASTR 3740 Relativity & Cosmology Spring 2023. Problem Set 5. Due Wed 15 Mar

1. Gravitational lensing

It is possible to show that, in the weak field limit, light passing a spherical mass M at impact parameter y is deflected by angle

$$\Delta\phi = \frac{4GM}{yc^2} \ . \tag{1.1}$$

It is necessary to use general relativity to show this, because general relativity predicts a deflection angle twice as large as a more naive calculation. Physically, in general relativity, not only energy but also momentum gravitates, and photons have momentum equal to their energy.
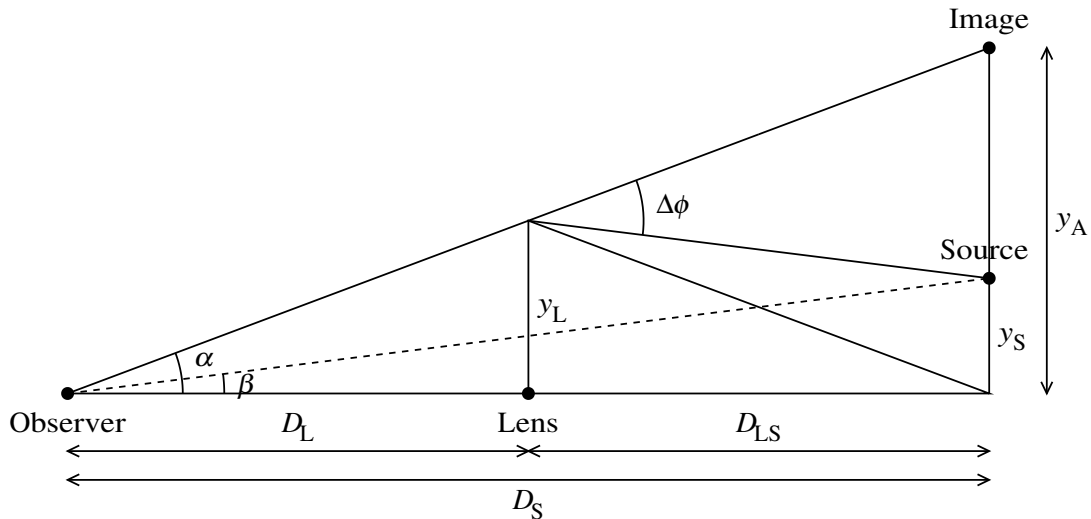
(a) Lensing equation

Figure 1: Lensing diagram.

Argue that the deflection angle $\Delta\phi$ is related to the angles α and β illustrated in the lensing diagram above by

$$\alpha D_{\mathrm{S}} = \beta D_{\mathrm{S}} + \Delta\phi D_{\mathrm{LS}} \ . \tag{1.2}$$

Hence or otherwise obtain the "lensing equation" in the form commonly used by astronomers

$$\beta = \alpha - \frac{\alpha_{\mathrm{E}}^2}{\alpha} \ , \tag{1.3}$$

where

$$\alpha_{\mathrm{E}} = \left(\frac{4GM}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}}} \right)^{1/2} \ . \tag{1.4}$$
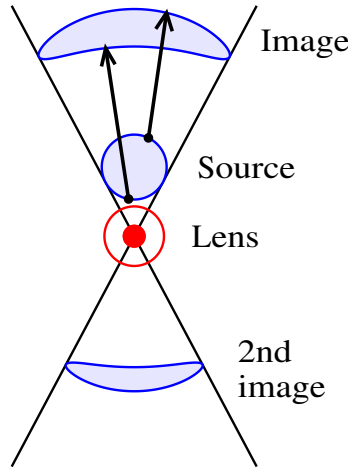
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Figure 2: The appearance of a lensed source. The lens in this case is a black hole, whose physical size is the filled circle, and whose apparent (lensed) size is the surrounding unfilled circle.

(b) Solutions

Equation (1.3) has two solutions for the apparent angles α in terms of β. What are they? Sketch both solutions on a lensing diagram similar to Figure 1.

(c) Einstein ring around the Sun?

The case $\alpha = \alpha_{\mathrm{E}}$ evidently corresponds to the case where the source is exactly behind the lens, $\beta = 0$. In this case the lensed source appears as an "Einstein ring" of light around the lens. Could there be an Einstein ring around the Sun, as seen from Earth? Google the numbers you may need.

(d) Einstein ring around Sgr A*

What is the maximum possible angular size of an Einstein ring around the $4 \times 10^6 \, \mathrm{M_\odot}$ black hole at the center of our Milky Way, 27,000 lightyears away? Might this be observable?

2. Deflection of light by the Sun

Calculate the predicted deflection angle $\Delta\phi$ in arcseconds for light that just grazes the limb of the Sun. Google the numbers you may need for the Sun.