Experimental Details The experimental configuration consists of the cavity (mode waist of 71 \(\mu\)m and length of 1.9 cm) with a quantization axis defined by a 2.7 G magnetic field along the cavity axis. By using an optical cavity, we close the feedback loop, avoid undesired single-pass propagation effects through the ensemble, and coerce the atoms to superradiantly emit light into a single spatial mode. The atoms are trapped by a 1-D intracavity optical lattice at 823 nm and laser cooled to approximately 40 \(\mu\)K, putting them in the Lamb-Dicke regime along the cavity axis but not perpendicular to the axis. The cooling is not applied during steady state superradiance.

Primary Experimental Configuration Steady state superradiance was primarily observed in an experimental configuration that is shown in Supplementary Figure 1. All data displayed in the main text was measured in this configuration with two exceptions described in the secondary configuration section. The 795 nm dressing laser is applied along the cavity axis, and by non-resonantly driving the cavity, we inject enough intensity to drive the superradiant emission into a resonant cavity mode. The dressing laser is linearly-polarized and appears as a superposition of \(\sigma^+\) and \(\sigma^-\). The Clebsch-Gordan coefficients on the emission path from \(|e\rangle \equiv |5^2S_{1/2}, F = 2, m_f = 0\rangle\) to \(|g\rangle \equiv |5^2S_{1/2}, F = 1, m_f = 0\rangle\) through intermediate states \(|i_{\pm}\rangle \equiv |5^2P_{1/2}, F' = 2, m_f' = \pm 1\rangle\) lead to a cancellation of all emission except for linearly-polarized light, rotated 90\(^\circ\) from the polarization of the dressing laser. The polarization orthogonality allows the emitted light to be polarization separated from the dressing light that also emerges from the cavity. The dressing laser is typically tuned 1.1 GHz to the blue of the \(|e\rangle \rightarrow |i_{\pm}\rangle\) atomic transition. The cavity mode is typically tuned 1.1 GHz to the blue of \(|g\rangle \rightarrow |i_{\pm}\rangle\) transition. The repumping lasers are \(\pi\)-polarized and tuned to the frequency between the ground states and the optical excited state \(|5^2P_{3/2}, F' = 2\rangle\) such that the single state dark to the repumping is \(|e\rangle\). The F1 repumper moves atoms primarily from the ground \(|F = 1\rangle\) state to the ground \(|F = 2\rangle\) state, and the F2 repumper pushes population to \(|e\rangle\), as the Clebsch-Gordan coefficient for the transition \(|5^2S_{1/2}, F = 2, m_f = 0\rangle \rightarrow |5^2P_{3/2}, F' = 2, m_f' = 0\rangle\) is zero.

In this configuration, the nodes and antinodes of the standing waves of the the dressing laser and the cavity emission mode do not coincide due to the difference frequency of 6.834 GHz. We define a spatially averaged decay rate \(\gamma_{eg} = \langle \gamma_{eg}(z) \Omega_{eg}^2(z) \rangle / \langle \Omega_{eg}^2(z) \rangle\) weighted by the coupling strength to the emission cavity mode \(\Omega_{eg}^2(z)\).

Secondary Experimental Configuration Steady-state superradiance was observed in a second experimental configuration that might be applied for magnetometry, shown in Supplementary Figure 2. Because the configuration is first-order sensitive to magnetic field broadening and Doppler decoherence, it is used to compare the measured linewidth to the predicted linewidth due to these sources in the main text. The quantization axis is defined by a magnetic field along the cavity axis. The \(\pi\)-polarized 795 nm dressing laser is applied perpendicular to the cavity axis, a direction in which the atoms are not in the tightly-confined Lamb-Dicke regime. The state \(|e\rangle \equiv |5^2S_{1/2}, F = 2, m_f = -2\rangle\) is the only state dark to the repumping light. This is achieved by applying two different frequency 780 nm lasers at 45\(^\circ\) to the cavity axis, such
that they consist of a linear combination of $\pi$ and $\sigma^-$ polarization. The F1 repumper, tuned to the $|5^2S_{1/2}, F = 1\rangle \rightarrow |5^2P_{3/2}, F' = 2\rangle$ transition, primarily moves atoms from the ground $|F = 1\rangle$ state to the ground $|F = 2\rangle$ state, and the F2 repumper, tuned to the $|5^2S_{1/2}, F = 2\rangle \rightarrow |5^2P_{3/2}, F' = 1\rangle$ transition, moves atoms to $|e\rangle$. Light emitted into the standing wave cavity mode through the intermediate state $|i\rangle \equiv |5^2P_{1/2}, F' = 2, m_f = -2\rangle$ can only be $\sigma^+$ or $\sigma^-$ polarized. This polarization constraint, combined with cavity resonance frequency conditions, selects $|g\rangle \equiv |5^2S_{1/2}, F = 1, m_f = -1\rangle$ as a unique ground state for superradiant emission. The dressing laser is typically tuned 1.1 GHz to the blue of the $|e\rangle \rightarrow |i\rangle$ atomic transition. The cavity mode is typically tuned 1.1 GHz to the blue of the $|g\rangle \rightarrow |i\rangle$ transition.

**Atom Loss** Atom loss is observed via in situ dispersive cavity shift measurements and direct fluorescence imaging on a CCD. The loss is not fundamentally linked to the superradiant process, as atom loss is observed even in the absence of superradiance. Temperature measurements indicate that atoms are not being heated out of the trap, implying that the atoms are likely lost due to light-assisted collisions with rates dependent on the repumping light intensity. The atom loss

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**Supplementary Figure 1: Primary experimental configuration – dressing beam along cavity axis.**

- **a:** The physical arrangement of the apparatus. The cavity is vertically oriented, and the quantization axis defined by the vertical magnetic field $B$. The atoms (red) are confined to the cavity axis by an intracavity optical lattice (not shown). The linearly-polarized D1 Raman dressing beam is in red, and the $\pi$-polarized D2 repumping beams are green (F2) and purple (F1). The emitted light, blue, goes into the cavity mode with a linear polarization that is rotated 90° from the dressing beam.

- **b:** The energy level diagram for the D1 Raman beam (red) and the superradiantly emitted light (blue). The cavity mode resonance frequency is denoted with a blue dashed line.

- **c:** The energy level diagram for the D2 repumping beams F2 (green) and F1 (purple). The repumping dark state is labeled with a gray circle.
The physical arrangement of the apparatus. The cavity is vertically oriented, and the quantization axis defined by the vertical magnetic field $B$. The atoms (red) are confined to the cavity axis by an intracavity optical lattice (not shown). The linearly-polarized dressing beam is in red, and the repumping beams are green (F2) and purple (F1). The circularly-polarized emitted light, blue, goes into the cavity mode (polarization denoted by the thin arrows).

The energy level diagram for the D1 Raman dressing beam (red) and the superradiantly emitted light (blue). The cavity mode resonance frequency is denoted with a blue dashed line.

**Supplementary Figure 2** | Secondary experimental configuration – dressing beam perpendicular to cavity axis.

**a.** The physical arrangement of the apparatus. The cavity is vertically oriented, and the quantization axis defined by the vertical magnetic field $B$. The linearly-polarized dressing beam is in red, and the repumping beams are green (F2) and purple (F1). The circularly-polarized emitted light, blue, goes into the cavity mode (polarization denoted by the thin arrows).

**b.** The energy level diagram for the D1 Raman dressing beam (red) and the superradiantly emitted light (blue). The cavity mode resonance frequency is denoted with a blue dashed line.

**c.** The energy level diagram for the D2 repumping beams F2 (green) and F1 (purple). The repumping dark state is labeled with a gray circle. The polarization of the repumping beams is a linear combination of $\pi$-polarization (thick lines) and $\sigma^-$ polarization (thin lines).

Mechanism, while important, is not essential to the underlying physics and can be eliminated in future work by using higher-dimensional lattices.

**Photon Number and Phase Uncertainty** The intracavity photon number $M_c = \langle a(t)^\dagger a(t) \rangle_t$ is the time averaged expectation value of the cavity mode number operator $a(t)^\dagger a(t)$. We calculate the intracavity photon number via $M_c = 2\bar{M}_d/q\kappa_0$, where $\bar{M}_d$ is the detected photon flux (photons/second) from one end of the symmetric cavity, $q \approx 0.6$ is the quantum efficiency from the cavity mirror to the heterodyne detector, and $\kappa_0/2\pi = 5.3(3)$ MHz is the cavity linewidth due to mirror transmission alone. The fundamental phase noise due to the photon shot noise of this photon flux is calculated as $\Delta \phi = 1/\sqrt{4\bar{M}_d(T)}$, where $\bar{M}_d(T)$ is the average number of photons measured using homodyne detection in a time interval $T$. In the main text, $\Delta \phi$ is compared to the standard quantum limit $\Delta \phi_{SQL} = 1/\sqrt{N}$ for an ensemble of $N$ atoms.

**Phasor Correlation** We define a phasor correlation function $C_p = \langle \text{Re} \left[ \frac{e^{i\Delta \phi_k}}{|e^{i\Delta \phi_k}|} \right] \rangle_k$, where $\Delta \phi_k = \phi_k - \phi_0$ is the phase difference just before and after the dark time $T_{\text{dark}}$ in a single trial $k$. In the case that the phase measurements have no ambiguities due to measuring modulo $2\pi$, this function $C_p$ is a Lorentzian.
\[
C_p = \langle \cos[\phi_1^k - \phi_0^k - (\phi_1^k - \phi_0^k)_k] \rangle_k.
\]
This definition is chosen such that if there is no noise in the phase difference, \(C_p \to 1\), while if \(\phi_1\) is random with respect to \(\phi_0\), \(C_p \to 0\). The local oscillator frequency was not equal to the atomic precession frequency in our experiment, so some average relative phase \(\langle \phi_1^k - \phi_0^k \rangle_k \neq 0\) is accumulated during the dark time \(T_{\text{dark}}\). The normalized average phase difference phasor \(\frac{\langle e^{-i(\Delta \phi_k)} \rangle_k}{\langle |e^{-i(\Delta \phi_k)}| \rangle_k}\) accounts for this constant phase offset.

**Lorentzian Fits** Here we provide details of the Lorentzian fits described in the main text and used to probe more fundamental sources of phase noise. A significant signal from Lorentzian contributions to the power spectrum may remain at large offset frequencies from the carrier, while predominantly low-frequency, technical noise is expected to fall off much more rapidly. A Lorentzian fit that excludes low offset frequencies can recover the more fundamental Lorentzian contributions.

Prior to performing Lorentzian fits, we remove low frequency spectral components believed to arise from changes in atom number. A change in atom number leads to a dispersive change in the cavity frequency, causing a pulling of the oscillation frequency. Atom number variation from one trial to the next causes variation in the center oscillation frequency from one trial to the next, while atom loss during a single trial caused frequency chirping. Note that these mechanisms are not expected to be present if an actual narrow optical transition (i.e. not a Raman transition) were to be employed.

The average frequency chirp is removed by calculating the average phase of the emitted light for each time \(t\), \(\langle \phi_k(t) \rangle_k\). Here \(\phi_k(t)\) is the measured phase during the \(k^{\text{th}}\) trial, each 5.5 ms long. To calculate the average, 200 trials are used. This average is then subtracted from each measured phase yielding \(\phi_k'(t) = \phi_k(t) - \langle \phi_k(t) \rangle_k\).

We account for the fluctuation in the center frequency between trials by subtracting a linear fit from each \(\phi_k'(t)\), resulting in \(\phi_k''(t) = \phi_k'(t) - (m'_k t + b'_k)\). This is equivalent to moving the center frequency of each trial to 0 Hz before calculating the average power spectral density. The two-sided power spectrum, on which the Lorentzian fits are performed, is calculated for each trial from \(e^{i\phi_k''(t)}\) using a 4th order Blackman-Harris windowing function to avoid power spectral leakage of the carrier to large offset frequencies.