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Reducing collective quantum state rotation errors with reversible dephasing

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We demonstrate that reversible dephasing via inhomogeneous broadening can greatly reduce collective quantum state rotation errors, and observe the suppression of rotation errors by more than 21 dB in the context of collective population measurements of the spin states of an ensemble of 2.1 \times 10^5 laser cooled and trapped ^{87}\text{Rb} atoms. The large reduction in rotation noise enables direct resolution of spin state populations 13(1) dB below the fundamental quantum projection noise limit. Further, the spin state measurement projects the system into an entangled state with 9.5(5) dB of directly observed spectroscopic enhancement (squeezing) relative to the standard quantum limit, whereas no enhancement would have been obtained without the suppression of rotation errors. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4905148]

Decoherence destroys entanglement, degrades precision measurement signals, and limits a wide range of coherent processes from lasing to operating quantum gates.\textsuperscript{1,2} Therefore, most technologies relying on real or synthetic atoms try to minimize decoherence resulting from loss, relaxation, and inhomogeneous broadening. Recently, however, specifically engineered forms of decoherence have been used to enhance certain processes. Dissipative decoherence, for example, can remove information from a system leading to stabilization of polar molecules from lossy collisions\textsuperscript{3} or generation of entanglement.\textsuperscript{4–6} Also, non-dissipative, reversible, decoherence in the form of inhomogeneous broadening can be used to stabilize coherent operations allowing, for example, storage of non-classical light signals,\textsuperscript{7} or as we show in this letter, insensitivity to errors in collective quantum state rotations.

Precision measurements using one or many atoms require precise rotations of the atoms’ quantum state. These rotations, achieved by applying a coherent field at or near the atomic transition frequency, are used to excite an atomic transition,\textsuperscript{8,9} map the evolution of a quantum phase into a measurable quantity,\textsuperscript{10,11} or simply transfer state populations for precision readout.\textsuperscript{12} Imperfections in these rotations lead to classical uncertainty in the atoms’ quantum state, which can dominate fundamental quantum uncertainty and limit precision measurements.

In this letter, we propose and experimentally demonstrate an approach to suppress rotation errors using reversible inhomogeneous broadening, an alternative to the composite coupling pulses that are often used to correct state rotation errors.\textsuperscript{13–20} We first theoretically show how collective rotations of many qubits can be performed with greatly reduced errors if controlled inhomogeneous broadening of the transition is applied prior to the desired rotation. We also show that collective coherence is restored by reversal of the inhomogeneous broadening after the rotation. In some sense, our technique is the converse of the traditional spin echo.\textsuperscript{21–23}

Instead of a rotation removing inhomogeneous broadening, controlled inhomogeneous broadening is used to correct errors in a rotation.

We apply dephased rotations in a specific experiment, demonstrating a maximum suppression of technical noise of greater than 21 dB when rotating the internal states of laser-cooled and trapped \(^{87}\text{Rb}\) atoms. Dephased rotations aid the generation and observation of entangled, spin-squeezed states with a directly observed enhancement in quantum phase estimation 9.5(5) dB below the standard quantum limit (SQL) for an unentangled ensemble, one of the largest such enhancements in atomic systems reported to date.\textsuperscript{24–28} In the absence of any reversible inhomogeneous broadening, either incidental or deliberate, we estimate that little to no squeezing would have been observed in this experiment due to imperfections in the required quantum state rotations.\textsuperscript{29}

Dephased rotations are a general concept and could be applied to a variety of applications, having several advantages over traditional composite rotation sequences. Composite sequences rely on cancellation between the errors of each individual rotation. However, cancellation fails if the errors fluctuate on time scales comparable to the time required for the composite pulse sequence. Furthermore, increasing the rate of rotations to enhance the correlation in errors may actually be detrimental depending on the form of the noise spectrum.\textsuperscript{30} Finally, composite pulses require precise control over the phase of the coupling field, and the most effective composite sequences require many pulses, increasing the time required for a measurement sequence. The approach presented here to a large degree avoids these requirements. We note that intense efforts to apply composite pulses to reduce rotation-added noise were largely unsuccessful in our experiment. In this paper, we only consider \(\pi\)-pulses, but our technique may be applicable to other types of rotations by implementing reversible inhomogeneous rotations around arbitrary axes.

We describe our system of \(N\) 2-level atoms as spin-1/2 particles using a collective Bloch vector \(J = J_x \hat{x} + J_y \hat{y} + J_z \hat{z} = \sum_{i=1}^{N} J_i\), where the \(i\)th Bloch vector \(J_i = \langle J_i \rangle\) is the expectation value of the quantum spin projection operator for the

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ith atom. The $\hat{z}$ projection of the collective Bloch vector $J_z \equiv \mathbf{J} \cdot \hat{z} = (N_i - N_f)/2$ is directly determined by measuring the number of atoms in spin up $N_i$ and down $N_f$. Precision measurements with 2-level systems are fundamentally limited by quantum uncertainty in the angles describing the orientation of the Bloch vector. This quantum uncertainty appears as quantum projection noise (QPN) in the measurement of the spin projection $J_z$. For unentangled atoms, the rms fluctuation for a coherent spin state (CSS) with $\mathbf{J} = N/2 \hat{z}$ is $\Delta J_{\text{QPN}} = \sqrt{N}/2$. The projection noise limits the estimate of the Bloch vector’s polar angle to an rms uncertainty of $\Delta \theta_{\text{QPN}} = 1/\sqrt{N}$, the so-called SQL. Due to this scaling, states with large $N$ are desirable for precise phase estimation, but in these states, classical rotation errors become more challenging to reduce below the smaller SQL.

The rotation of the $i$th Bloch vector through angle $\psi_i$ about an axis $\hat{n}$ is defined by the rotation matrix $\mathcal{R}_{\hat{n}}(\psi_i)$. If the rotation is uniform ($\psi_i = \psi$ for all $i$) then the result is a rigid rotation, in which the length of the Bloch vector is conserved. The errors we wish to suppress are those generated by uniform rotation errors associated with the coupling field, in particular, an arbitrary erroneous rotation through a small angle $\phi$ described by $\mathcal{R}_{\phi}(\phi)$. The suppression of the rotation errors will be achieved by introducing a brief, controlled inhomogeneous broadening of the energy difference between $|\uparrow\rangle$ and $|\downarrow\rangle$ before and after the imperfect rotation. The time-integrated effect of the broadening on the $i$th vector is characterized by the non-uniform rotation $\mathcal{R}_i(\psi_i)$. The amount of dephasing is quantified by the fractional reduction in the collective Bloch vector’s transverse projection $J_\perp = \sqrt{J_x^2 + J_y^2}$. Specifically, we define the transverse coherence after dephasing $C_d = J_{\perp,d}/J_{\perp,0}$, where the subscript $d$ refers to $J_\perp$ after dephasing and 0 refers to $J_\perp$ prior to dephasing. In the present work, the inhomogeneous broadening will be achieved through light shifts, but could also be realized through magnetic fields or electric fields. Whatever method is used, the key is that the dephasing must be reversible: at a later time, the opposite rotation can be realized $\mathcal{R}_i(-\psi_i)$ to fully or partially undo the dephasing. Here, the dephasing will be undone by using a $\pi$-pulse (e.g., $\mathcal{R}_z(\pi)$) followed by identical inhomogeneous broadening.

To theoretically show that dephased collective spin vectors are protected from small rotation errors, we analyze the rotation error of a nominal $\pi$-pulse, with fractional amplitude error $\epsilon$ and detuning error $\delta$ of the applied coupling field from the atomic transition, which is preceded and followed by dephasing steps. The final Bloch vector after such a sequence is $\mathbf{J}_f = \sum_{i=1}^{N_e} \mathcal{R}_i(\psi_i)\mathcal{R}_z(\beta)\mathcal{R}_i(\psi_i)\mathbf{J}_0$, where the subscript $F$ indicates a quantity after all rotations.

The effective rotation angle is a function of both $\epsilon$ and $\delta$ and can be written $\beta = \pi \sqrt{(1+\epsilon)^2 + \delta^2}$, where $\delta^* = \delta/\Omega$ and $\Omega$ is the on resonance Rabi frequency of the applied rotation. In the rotating frame of the applied field, the rotation axis depends on the detuning error, $\hat{y} \propto \Omega \hat{x} + \delta \hat{z}$. For an arbitrary initial Bloch vector, the rotation axis $\hat{z} = \hat{y}$ can be chosen without loss of generality.

As an example, we assume that the inhomogeneous phase rotation angles $\psi_i$ are drawn from a Gaussian distribution with mean of zero and rms value $\sigma$. The reduction in transverse coherence due to the applied inhomogeneous broadening in this case is approximately $C_d = e^{-\sigma^2/2}$ if $N \gg 1$. The complete sequence of applied broadening and imperfect rotations can then be averaged over all atoms to compute the final Bloch vector $\mathbf{J}_f$ with solution

$$
\begin{pmatrix} J_{xf} \\ J_{yf} \\ J_{zf} \end{pmatrix} \approx - \begin{pmatrix} J_{0x}(1 - \eta^2) + C_d \pi \epsilon J_{0x} \\ J_{0y}(1 - \eta^2) - C_d 2 \delta^* J_{0y} \\ J_{0z}(1 - 2\eta^2) - C_d (2\beta J_{0z} + \pi \epsilon J_{0z}) \end{pmatrix},
$$

where $\eta^2 \equiv \pi^2 \epsilon^2/4 + \delta^2$. We have assumed here that $\pi \epsilon$, $\delta^*$, and $C_d \ll 1$, and neglected all terms of third order in products of these quantities.

The key result is that all rotation errors that are first order in $\pi \epsilon$ and $\delta^*$ are reduced by a factor $C_d$. The cost of this error suppression is shortening of the Bloch vector, but only at second order in the rotation error $\eta$. The final transverse Bloch vector component $C_F = J_{yf}/J_{0y}$ is reduced as $C_F \approx 1 - \eta^2$, and the $\hat{z}$ projection of the Bloch vector is reduced to $J_{zf}/J_{0z} \approx 1 - 2\eta^2$.

Fig. 1 graphically demonstrates the reduced sensitivity of an arbitrary CSS to a rotation about an axis on the equator. In one case ((a) and (b)), a small rotation $\mathcal{R}_z(\pi/16)$ representing an error is applied with no dephasing. When dephasing is applied after the rotation, and reversed after the rotation ((a)–(e)), the Bloch vector’s sensitivity to the rotation is greatly reduced, with minimal loss in final coherence. The rotation error is rendered negligible compared to quantum noise (shown for $N = 120$ here).

The dephased rotation scheme exhibits an additional useful attribute for suppressing rotation errors in the generation and manipulation of spin-squeezed ensembles. Dephased rotations can significantly reduce the amount of anti-squeezing projected into the low noise squeezed quadrature by a rotation error. We show this theoretically in the supplementary material.31

We apply the proposed scheme to collective measurements of $N = 2.1 \times 10^9$ to $N = 5 \times 10^9$ $^{87}$Rb atoms laser-cooled and trapped inside an optical cavity of finesse $F = 660$ (see Ref. 24 for experimental details). The atoms are tightly confined by a 1D optical lattice formed by exciting a longitudinal TEM$_{00}$ mode of the cavity with wavelength $\lambda = 823$ nm. The atoms fill lattice sites long the central 2 mm of the cavity. The spin system is defined by hyperfine ground states $|\uparrow\rangle = |F = 2, m_F = 2\rangle$ and $|\downarrow\rangle = |F = 1, m_F = 1\rangle$. Coherent rotations between these states are performed by applying microwaves at the transition frequency 6.83 GHz. $N_1$ can be inferred by measuring the dispersive frequency shift of another TEM$_{00}$ cavity mode tuned $\approx 200$ MHz from resonance with the optical transition between $|\uparrow\rangle$ and an excited state $|\epsilon\rangle = |F' = 3, m_F = 3\rangle$ on the 780 nm D2 line.24

The probe light at $\lambda_p = 780$ nm that is used to measure the cavity frequency shift and infer $N_1$ also creates an inhomogeneous light shift that dephases the atoms. Since the standing waves of the lattice and probe are incommensurate ($\lambda_p \neq \lambda_1$), the atoms at different lattice sites experience different light shifts from the probe, leading to dephasing (shown in Fig. 2). We can also apply an additional dephasing laser tuned to resonance with yet another TEM$_{00}$ longitudinal
mode of the cavity. This dephasing beam is detuned ≈50 GHz from the 795 nm D1 optical transition and allows us to modify the amount of dephasing without modifying the signal to noise of the atom number probe or causing additional unwanted free-space scattering. Because the atoms are tightly confined with respect to the cavity axis, the same light shifts can be applied at a later time, after a π-pulse, to reverse the applied phase shifts.

We can measure $C_d$ due to dephasing from the probe and dephasing lasers by first preparing a coherent spin state along $\hat{x}$. We then apply either the probe or dephasing laser for a varying amount of time, after which we apply the rotation $R_y(\pi/2)$ about a random axis $\hat{z}$ lying in the $\hat{x}-\hat{y}$ plane. Finally, we measure the number of atoms $N_1$. When averaged over all rotation axis, the standard deviation of $N_1$ is proportional to $C_d$. In Figs. 2(b) and 2(c), $C_d$ and $C_F$ are plotted versus the average number of probe $M_p$ and dephasing $M_d$ photons transmitted through the cavity. $C_F$ is more perfectly restored using the dephasing beam due to its larger detuning from the optical transition.

In Fig. 3(a), we demonstrate reduced sensitivity to rotation noise arising from environmental noise sources using our dephased rotation scheme. Data showing reduced sensitivity to intentionally applied rotation errors can be found in the supplementary material. In our experiment, undesirable environmental rotation noise arises primarily from microwave amplitude noise and frequency fluctuations in the magnetic field-sensitive hyperfine transition. To demonstrate a reduction in sensitivity to environmental noise sources, $J_{yF}$ is measured after a large even number of π-pulses. With increased dephasing strength $M_d$, the rotation-added noise can be reduced below QPN even after eight π-pulses.

We now show how dephased rotations can be used in experiments to generate entangled, spin squeezed states by

FIG. 1. Reduced sensitivity to a rotation error using dephasing. A single representative collective Bloch vector prepared in the $\hat{x}-\hat{z}$ plane, along with the quantum uncertainty in its position, is shown (red arrow and noise distribution). Each sphere also has a series of colored lines denoting the tips of Bloch vectors that are at a constant $J_z$ in the initial configuration. The original rings of constant $J_z$ are shown in (b)–(e) as thin black lines for reference. A small rotation $R_y(\pi/16)$ representing an error is applied without (b) and with ((c)(e)) dephasing. By reversibly dephasing the Bloch vector to $C_d=0.14$, the impact of the rotation is greatly reduced. Rotation errors that would otherwise dominate can be suppressed well below the fundamental quantum noise.

FIG. 2. (a) The standing wave intensity of each beam is shown inside the cavity (blue mirrors). The atoms are trapped at antinodes of the 823 nm optical lattice (blue). The probe laser at 780 nm (red) and dephasing beam at 795 nm (green) cause dephasing due to their inhomogeneous light shifts. We detect the phase of the probe light to infer $N_1$. (b) and (c) The reduction in transverse coherence after dephasing $C_d$ (red and green squares) and rephasing $C_F$ (black circles) is measured versus the average number of transmitted photons from the probe beam (b) and dephasing beam (c).
correlated below

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noise with increased

work.24 Here, we primarily emphasize the role dephased collective measurements of $J_z$, which would increase sensitivity to low frequency noise.

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in Fig. 2. As a result, the state generated after the premeasure-

sequence with dephasing provides a directly observed

measurement background are shown versus $M_p$ (transmitted probe photons in a single measurement window). With the probe beam alone (i.e., $M_p=0$), the spin noise reduction $R$ (red data and fit) is one order of magnitude below the QPN at $2.1 \times 10^5$ atoms. Additionally, the rephasing nearly completely restores coherence, as demonstrated in Fig. 2. As a result, the state generated after the premeasurement can be viewed as a deterministically generated spin-squeezed state (i.e., no post-selection), conditioned on knowledge of the measurement outcome $J_z f$, on a given trial. After accounting for both the degree of spin-noise reduction $R$ and the loss of coherence $C_F$, the optimum measurement sequence with dephasing provides a directly observed enhanced phase resolution $9.5(5)$ dB below the SQL. In contrast, with any reversible dephasing, rotation-added noise would have precluded the observation of any enhancement beyond the SQL.

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making precise collective measurements of the spin projection $J_z$. These experiments are treated in detail in a related work.24 Here, we primarily emphasize the role dephased rotations can play, enabling large reductions in technical rotation noise and allowing resolution of the spin projection far below the quantum projection noise level. In our experiment, dephased rotations are highly advantageous to com-

composite pulse sequences because they do not require any control of the applied rotation axis and do not increase the duration of the measurement sequence, which would increase sensitivity to low frequency noise.

In order to create spin-squeezed states, two consecutive collective measurements of $J_z$, labeled $J_{z,p}$ and $J_{z,f}$ must be correlated below $\Delta J_z$QPN. The degree of spin noise compared to quantum projection noise is characterized by the spin noise reduction $R = [\Delta(J_z f - J_{z,p})]^2/\Delta J_z$QPN, where $\Delta(J_z f - J_{z,p})$ is the standard deviation in the differential quantity $J_z f - J_{z,p}$.

The measurement sequence for $R$ is shown in Fig. 3(b). The spin noise reduction has two contributions $R = R_{\text{bck}} + R_{\text{rot}}$. One, $R_{\text{bck}}$, we attribute to measurement imprecision of the experiment along with measurement-induced noise in $J_z f$. The other, $R_{\text{rot}}$, is rotation-added noise from the two $\pi$-pulses in the measurement sequence. In this experiment, $R_{\text{rot}}$ and $R_{\text{bck}}$ are uncorrelated, and $R_{\text{bck}}$, to good approximation, is independent of whether or not $\pi$-pulses are inserted into the sequence.32 For this reason, we can estimate $R_{\text{bck}}$ (black line in Fig. 3(c)) by performing the measurement sequence of 3(b) without the $\pi$-pulses. The experiment is then repeated with the $\pi$-pulses included, and any increase in $R$ is assigned as rotation-added noise $R_{\text{rot}}$.

In Fig. 3(c), the measured spin noise reduction and measurement background are shown versus $M_p$ (transmitted probe photons in a single measurement window). With the probe beam alone (i.e., $M_p=0$), the spin noise reduction $R$ (red data and fit) lies well above the measurement background $R_{\text{bck}}$ (black line). However, when the additional dephasing is applied with strength $M_d = 6.1(3) \times 10^5$, the observed $R$ (blue points and fit) is improved to values very close to the measurement background.

Fig. 3(d) (inset) displays the inferred rotation-added noise $R_{\text{rot}}$ with and without the additional dephasing applied (blue and red lines, respectively). The combined dephasing of the probe and dephasing beams allows a reduction of the rotation-added noise of greater than approximately 21 dB enabling up to $R = 13(1)$ dB of spin noise reduction below the QPN at $2.1 \times 10^5$ atoms. Additionally, the rephasing nearly completely restores coherence, as demonstrated in Fig. 2. As a result, the state generated after the premeasurement can be viewed as a deterministically generated spin-squeezed state (i.e., no post-selection), conditioned on knowledge of the measurement outcome $J_z f$, on a given trial. After accounting for both the degree of spin-noise reduction $R$ and the loss of coherence $C_F$, the optimum measurement sequence with dephasing provides a directly observed enhanced phase resolution $9.5(5)$ dB below the SQL. In contrast, without any reversible dephasing, rotation-added noise would have precluded the observation of any enhancement beyond the SQL.

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5. H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, “Entanglement generated by dissipation and steady...


29A different measurement sequence (Ref. 24) that avoids any rotations achieved a comparable amount of directly observed spin squeezing in the same system without relying on dephased rotations, but only by sacrificing a factor of two in fundamental measurement resolution. This additional factor of two was not realized in this work due to the probe laser’s frequency noise coupling more strongly into the measurement sequence of Fig. 3.


31See supplementary material at http://dx.doi.org/10.1063/1.4905148 for an experimental demonstration showing redaction of intentionally applied rotation errors as well as an analysis of the effect of dephased rotations on quantum noise.

32The noise sources contributing to \(R\) in our experiment are characterized in the supplementary material of Ref. [24]. The primary source of noise which correlates \(R_{\text{back}}\) and \(R_{\text{rot}}\) is noise from state-changing transitions. This causes a negligible over-estimation of rotation-added noise at high \(M_\phi\) in the sequence of Fig. 3(b) by a value approximately 27 dB below the projection noise level.