IV. THEORY OF VARIABLE STARS (J. Cox, University of Colorado)

a) Introduction

In this report only certain aspects of the entire field of intrinsic variables will be discussed, and we shall include, for the most part, only work done in 1979 or later. These topics are Recent Books (Sec. b), Solar Oscillations (Sec. c), Long Period Variables (Miras) (Sec. d), R Coronae Borealis Stars (Sec. e), Hot and Cool White Dwarfs (Sec. f), Magnetic Cepheids (Sec. g), Time Dependent Convection (Sec. h), and Modal Selection (Sec. i). Other aspects of pulsating stars, such as Cepheid masses; Cepheid companions; atmospheric data relevant to helium-enrichment in Cepheid atmospheres; Beta Cephei theory; Delta Scuti theory; RR Lyrae and BL Herculis theory; the Blazhko effect; and resonances and bumps; may be found in the Commission 27 Reports.

Perhaps the only item that does not clearly fit into any one of the above categories and yet which should be mentioned is the discovery by D. W. Kurtz (preprint, 1981) of a new type of pulsating star. He has called these new variables rapidly oscillating Ap stars, and they have periods in the range 6 to 12 minutes.
Certain abbreviations, to be explained in Sec. b, will be used throughout this report.

b) Recent Books

Two monographs relevant to pulsating stars (primarily on the theory), have been published in the past two or three years. The first of these is *Nonradial Oscillations of Stars* (Tokyo: University of Tokyo Press), by W. Unno, Y. Osaki, H. Ando, and H. Shibahashi, published in 1979. The second is *Theory of Stellar Pulsation* (Princeton: Princeton University Press), by J. Cox, published in 1980. The first book is devoted exclusively to nonradial oscillations, the second to both radial and nonradial oscillations. The Japanese book is more mathematical than the other one.

A book on *An Introduction to Nonlinear Oscillations* (Cambridge: Cambridge University Press, 1980), by R. E. Mickens, has recently been published.


c) Solar Oscillations

The recent discovery of small solar oscillations has generated a large amount of literature, only a fraction of which can be summarized here (see the many references in the review articles to be mentioned below).


Particularly exciting are the observations carried out by Grec, Fossat, and Pomerantz (28.080.061) at the Earth's South Pole of 5-minute solar oscillations of high order but of low degree (i.e., small $l$, the latitudinal index of a spherical harmonic). These oscillations, no doubt $p$ modes, have been discussed by Christensen-Dalsgaard and Gough (28.080.062), who conclude that a solar interior of normal helium and metal content is consistent with the observations. They have also been discussed by, among others, Claverie, Isaak, McLeod, and van der Raay (*Nature*, 293, 443, 1981), who claim to have found rotational splitting of these frequencies. From these splittings the authors infer that the angular rotational velocity of the solar interior is 2 - 9 times larger than the surface value.

A new excitation mechanism, arising from nonlocal effects associated with
radiative transfer, has been found by Hill and Logan (preprint, 1980; see also Logan and Hill, *SH*, 301). The very interesting prospect of inferring the internal density distribution of the Sun from observations of p-mode, and only a few f- and g-mode, frequencies (helioseismology) has been discussed by Gough (*IAU Colloq. No. 66*, 1981).

d) Long Period Variables (Miras)

The many difficulties involved in these extremely complicated variable stars (to be abbreviated henceforth as LPV's) with greatly distended atmospheres are well described in the review article by Wing, *CPm*, 533. A shorter, but more up-to-date, review has been provided by Cahn, *SH*, 457. Still another review article is by Wood, *CTVSR*, 163.

Other review articles that bear on LPV's are by Payne-Gaposchkin (22.122.074); Merrill and Ridgway (26.114.055); Carbon (26.064.021); Zuckerman (Ann Rev A A, 18, 263, 1981); and Feast, (28.122.097). Also relevant in this context is (although this is not a review article) Celis, (28.122.056).

Observational support for the presence of shock waves in LPV's has been provided by Willson (17.122.034); Hinkle (21.122.017); Pilachowski, Wallerstein, and Willson (CPm, 577); Hinkle and Barnes (26.122.091); Hall, Hinkle, and Ridgway (CTVSR, 264); and Willson, Wallerstein, and Pilachowski (MNRAS, in press, 1981). The theoretical situation regarding shock waves in LPV atmospheres has been discussed by Willson and Hill (Proc. Los Alamos Solar and Stellar Pulsation Conference, 1976), Hill and Willson (25.064.049), and Wood (25.122.001). A PhD dissertation bearing on this topic has recently been completed by Pierce (1981).

The question of the evolutionary stage of the LPV's has been discussed by Willson (Effects of Mass Loss on Stellar Evolution, eds. Chiosi and Stalla, 353, 1981; Physical Process in Red Giants [PPHG], eds. Then and Renzini, 225, 1981; preprint, 1981); Wood and Zarro (Ap J, 247, 247, 1981); Wood (CTVSR, 163, 1979; PPHG, 205, 1981); and Tuchman, Sack, and Barkat (26.064.035). The effect of pulsation on mass loss has been discussed by Wood (25.122.001; CPm, 611).

The mode problem (fundamental vs. first overtone) has apparently still not been settled. In contrast to earlier thinking, fundamental mode pulsation has been argued for by Hill and Willson (25.064.049) and Willson (CTVSR, 199; 1981, above references), partly on the basis of smaller radii for these stars than had been accepted in the past. Wood (CTVSR, 163) argues for first overtone pulsation. The calculations of Wood and Zarro (1981, above reference) were not definitive in this respect. The linear, nonadiabatic pulsation calculations of Fox and Wood (preprint, 1981), are apparently the most complete and definitive to date. They show that nonadiabatic effects reduce the pulsation constant Q_0 for the fundamental mode, and increase Q_0 for the first overtone, thus rendering it even more difficult to distinguish observationally between the two modes. These calculations also suggest that the fundamental mode is favoured at the higher luminosities, the first overtone at the lower. On the other hand, the observational results of Robertson and Feast (MNRAS, 196, 111, 1981) suggest exactly the reverse, the higher luminosity LPV's being apparently first overtone pulsators. Some critical comments regarding the Hill-Willson fundamental mode arguments are presented by Wood (PPHG, 205).

Finally, arguments are given by Wood, Bessell, and Fox (preprint, 1981) for a division of LPV's in the Magellanic Clouds into asymptotic giant branch stars and core helium burning stars.
e) R Coronae Borealis (R CrB) Stars

Recent review articles on the R coronae Borealis (R CrB) stars are by Feast (CTVSH, 246) and King (SH, 519) (further references may be found in these articles). King concentrated to a large extent on the three R CrB stars that exhibit Cepheid-like pulsations. The large nonadiabaticity of these pulsations creates mode identification problems, strange modes, and other problems, which have been discussed by Cox, King, Cox, Wheeler, Hansen, and Hodson, SH, 529.

f) Hot and Cool White Dwarfs

Review papers (primarily observational) on the properties of the recently discovered variable DA white dwarfs, also called the ZZ Ceti stars (cf. McGraw, PhD Thesis, University of Texas at Austin, 1977, which contains a very thorough discussion of these stars), are by McGraw (25.126.017; this is not formally a review paper, but is essentially one nevertheless); Nather, 22.126.037; Robinson, CESPL, 423; Robinson, NNSP, 444; Robinson, WOVDZ, 343; and McGraw, SH, 601. Excellent theoretical discussions of the oscillatory properties of white dwarfs are by Van Horn, CESPL, 453; Hansen, in NNSP, 444; and Dziembowski, WOVDZ, 359.

The question of the excitation mechanism for the observed (200-1000) s oscillations (which are apparently g modes) of the ZZ Ceti stars is of great interest. Earlier discussions bearing on this question may be found in a number of papers in WOVDZ.

One of the most exciting recent findings in connection with this question is that the chemical stratification of the outer layers of white dwarfs (discussed in numerous papers in WOVDZ) seems to play a major role. It has recently been suggested by Winget, Van Horn, and Hansen (Ap J Let 245, L33, 1981) that a coincidence between the thickness of the hydrogen layer and the radial wavelength of one of the high-order g modes can trap this particular mode. Such a mechanism might explain why only certain modes, and not others, are observed in the ZZ Ceti stars.

Three recent studies show that the basic excitation mechanism for the oscillations of the ZZ Ceti stars is probably the kappa mechanism operating in the hydrogen ionization zone in the hydrogen layer of these stars. These studies are: Winget, Van Horn, Tassoul, Hansen, Fontaine, and Carroll (Ap J Let, in press); Winget, PhD thesis, Univ. of Rochester, 1981; and Dolez and Vauclair (A & A, in press). Somewhat earlier, the driving brought about by the kappa mechanism in the Ne ionization zone in the helium layer of these stars was studied by Dziembowski and Koester (A & A, 97, 16, 1981). The hydrogen driving was not found by the last authors, presumably because their models were not sufficiently cool. Otherwise the agreement among the above four sets of calculations, which all involved linear, fully nonadiabatic, nonradial oscillation theory, was satisfactory.

g) Magnetic Cepheids

It was suggested by Stothers (26.122.071) that Cepheids might have a tangled magnetic field, which would supply part of the pressure in the envelope. Such a tangled magnetic field is one means by which the mass discrepancy in "bump" and "beat" Cepheids might be resolved (e.g., A. Cox, 28.122.102).

New measurement techniques (Borra, Fletcher, and Poeckert, Ap J, 247, 569, 1981; Borra and Landstreet, 27.116.031) permit values of the longitudinal component of a magnetic field, averaged over the stellar disk, to be determined to within a few tens of gauss. These results are, according to the authors, consistent with the existence of tangled magnetic fields in Cepheids of the strength suggested by
A recent paper by Stothers, bearing on tangled magnetic fields in pulsating stars, is in MNRAS, 197, 351, 1981.

h) Time-Dependent Convection

Some attempts to treat time-dependent convection in a linear theory have been by Baker and Gough (26.122.069); Gonczi and Osaki (27.062.052); Saio (28.065.036); Gonczi (A & A, 96, 138, 1981); and Gonczi (preprint, 1981). However, the most definitive calculations which show that convection probably determines the red edge of the Cepheid and RR Lyrae instability strips are probably the nonlinear pulsation calculations of Deupree (27.122.003).

i) Modal Selection

The question of modal selection in pulsating stars, of interest in connection with the vexing problem of double-mode pulsation (for a review, see J. Cox, SH, 389), has recently been discussed by Simon (SH, 437; Ap J, 247, 594, 1981). He has discussed the work integral in connection with the iterative theory. He concludes, among other things, that the manner of treatment of artificial viscosity in numerical calculations might be important. In an earlier discussion, he suggested (25.122.063) that double-mode pulsation in Cepheids may involve a resonance among the fundamental, first, and third overtones. Faulkner and Shobbrook (26.122.011) have suggested that double-mode pulsation may result from a continuous switching back and forth between the fundamental and first overtone. A promising two-time formalism has been applied to the problem by Oded and Buchler (preprint, 1981).