Supersonic Compressible Convection

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Abstract

Stars like our sun display highly turbulent flows in their surface layers. Much of the vigorous motion in the solar surface results from thermal convection which transports nearly all the solar energy flux throughout a zone extending almost a third of the way into the interior. The density changes by six orders of magnitude from the tenuous gas near the surface to the far denser fluid at the base of the convection zone. With such a variation in density, fluid motions that span a fraction of the zone depth will be strongly influenced by the effects of compressibility. Another possibility is that such compressible, convective flows may even attain speeds in excess of the local sound speed. We have been studying the behavior of these nonlinear motions through supercomputer simulations and have discovered that supersonic flows accompanied by fluttering shock systems are a striking new form of convection.

Introduction

Vigorous motions can be established in layers of fluid by heating them from below. Such thermal convection occurs readily in nature, and has a major role in transporting the heat throughout the fluid, thereby contributing to the energy balance of planetary atmospheres, oceans and stars. Applications of the theory of convection are found in meteorology, geophysics and astrophysics.

There are two theoretical frameworks in which convection has been studied. In the simplest, one assumes that the fluid properties, like density, do not vary significantly across the vertical extent of the layer. This type of convection can be described by a reduced set of hydrodynamical equations obtained through the Boussinesq approximation [Spiegel and Veronis, 1960]. Boussinesq convection describes most forms of convection observed in laboratory experiments and in many geophysical situations. When the fluid properties vary significantly across the depth of the layer, as is the case in most astrophysical applications, the complete hydrodynamical equations must be used to describe the compressible motions. Because of the difficulty in arranging laboratory experiments where the density changes considerably across the fluid layer, the study of compressible convection has relied almost entirely on numerical simulations.

One of the parameters determining the nature of convection is the Rayleigh number, $R$, which is a dimensionless measure of the destabilizing effects arising from the temperature difference between the top and bottom of the layer. As the Rayleigh number increases, the convective flows develop increasing complexity and change from laminar to turbulent. One of the central objectives of convection theory is to provide a description of the flows at very high Rayleigh numbers. Increases in the speed and memories of computers have allowed the study of compressible convection to progress from simple two-dimensional models to more realistic three-dimensional models at moderate Rayleigh numbers and to two-dimensional models at high Rayleigh numbers.

In this article we describe the results of one such study where we explore the regime of Rayleigh

Compressible convection spanning a range of densities within a deep fluid layer may attain speeds in excess of the local sound speed. Studies here of the behavior of such nonlinear convection through supercomputer simulations reveal supersonic flows accompanied by fluttering shock systems.
numbers from $10^4$ to $10^7$ in a two-dimensional model of compressible convection with very high spatial resolution. Our results reveal that at Rayleigh numbers greater than $10^6$ the convective velocities can exceed the speed of sound. Although the possibility of supersonic convection had been conjectured, until recently this phenomenon could not be studied directly due to the lack of computational power. We find that the resulting flows display rapidly varying shock systems which contribute to a rich time dependence in the convection.

Model

We consider an idealized model of convection where a horizontal layer of perfect gas is confined by impenetrable boundaries above and below. The upper boundary is kept at constant temperature whereas a constant flux of heat is imposed at the lower one. As is common in astrophysical models of convection, the two boundaries do not exert any tangential stresses on the fluid. In order to simplify the model we assume that the shear viscosity and the thermal conductivity are constant. The fluid is initially in hydrostatic balance with a density at the bottom that is eleven times greater than at the top. The motions are then triggered by small random perturbations of the temperature field, and the evolution of the system is followed until a statistically steady state is reached.

Method

The state of the system is described at any instant by the velocity, density and temperature fields. Updated or future values for these fields are obtained from present values by integrating numerically the continuity equation and the equations of conservation of momentum and energy. Nonlinear advective terms in the equations are treated by an explicit scheme whereas the linear diffusive terms are treated implicitly. This kind of mixed scheme has been used widely and is known to give both high efficiency and accuracy. Because of the intrinsic inhomogeneity introduced by the stratification, we found it convenient to adopt two distinct techniques to describe the horizontal and vertical structure of the field variables. Horizontally the variables are treated pseudo-spectrally by an expansion in sines and cosines while the vertical variations are described by finite differences.

At the highest Rayleigh number studied we find that the solution is well resolved on a grid with 512 x 128 mesh-points with an aspect ratio of 4 to 1 (in comparing the horizontal domain to the vertical). At this resolution one hour of CPU time on the CYBER 205 at JVNC corresponds to a few turnover times, this being the characteristic time scale over which appreciable changes evolve in the system. We analysed the results at the University of Colorado using interactive graphics (IDL) on a Sun workstation.

Results

Convection arises in cases where the Rayleigh number exceeds about $10^6$ [Gough et al. 1976]. The fluid motions are in the form of circulating rolls extending from the upper to the lower boundary. As $R$ increases the convection becomes more vigorous and pronounced asymmetries appear. At $R$ between $10^4$ and $10^5$ the motions exhibit concentrated downflows alternating with broader, gentler regions of upflow (Hurlburt et al. 1984, 1986). In this regime the flows are steady and the rolls are typically twice as wide as they are deep. As the Rayleigh number nears $10^6$ the flow velocity in the upper part of the layer approaches the local speed of sound. For $R > 10^6$ the horizontal flows near the top of the layer become supersonic and a dramatic change occurs in the convection shocks appear in the upper part of the layer, the size of the rolls changes from 2:1 to 1:1, and the flow becomes vigorously time-dependent.

Figures 1 and 2, on the next two pages, show the distribution of entropy, vorticity and Mach number at two times in a simulation carried out at $R = 5 \times 10^6$. The accompanying arrow plots indicate the local velocity field. At the earlier time (Figure 1), the Rayleigh number has just been increased and the solution is adapting to the new time-dependent state involving supersonic flows and shocks. The upper boundary layer, which appears as dark in the entropy distribution has just become unstable and is beginning to collapse, forming two vortices which will later develop into a new convective cell. At the later time (Figure 2), the solution has evolved towards a new state with narrower cells as is evident in the arrow plots. In both figures, supersonic flows occur near the upper boundary where they can be seen as the regions of red to yellow in the Mach number distributions. The flows in these regions are predominantly horizontal and are decelerated in the shocks which form on either sides of the downflows. The shocks are revealed in the figures as abrupt transitions from yellow (Mach
Figure 1a: Supersonic convection achieved in a layer heated from below. Shown are the entropy and velocity fields as functions of horizontal and vertical position. The color table for the entropy goes from dark blue (low entropy) to yellow (high entropy). The dark region at the top is the thermal boundary layer. The uniform color throughout the interior shows that the layer has relaxed thermally to an almost adiabatic (constant entropy) state.

Figure 1b: Shown are the vorticity and Mach number (fluid velocity divided by the sound speed) as functions of position for the same time as in Figure 1a. In the Mach number display red corresponds to a transonic flow whereas bright yellow is a Mach 2 flow.
Figure 2a: The supersonic convection again showing the entropy and velocity field at a later time. The convection has now evolved to a state with narrower convective cells.

Figure 2b: The vorticity and Mach number for the convection shown in Figure 2a. The motions possess two distinctive, concentrated downflows and broader upflows.
number > 2) to dark blue (Mach number < 1) and as sharp, vertical and bright features in the entropy fields.

A remarkable property of these shocks is their non-stationary nature. Figure 3 shows the evolution of a shock system with time. Initially the shocks form in the vicinity of the downflows, appearing there as the dark, vertical features in the entropy, and then propagate upstream. As they approach the upflow sites the shocks weaken and eventually disappear. During their lifetime, however, the shocks interfere with the energetic balance of the thermal boundary layer which thickens downstream of the shocks. This thickening eventually leads to an instability which causes the collapse of the boundary layer and the formation of a new shock system. The propagation, weakening and reformation of shocks can also be followed in Figures 4-7 (on the next two pages), which show perspective views of the entropy fluctuations at times corresponding to those in Figure 3. The bright peaks in these figures correspond to the local generation of entropy by viscous stresses at the shocks. The dark valleys in these figures correspond to cold, dense, low-entropy fluid descending at the downflow sites. In one of our calculations we could follow the formation and propagation of shocks over several cycles indicating that the existence of supersonic flows is a robust feature of convection at these Rayleigh numbers.

Conclusions

The existence of convectively driven supersonic flows offers some intriguing possibilities in astrophysics. Observations of spectral lines from the surface of late-type stars often suggest flow velocities of the emitting gas exceeding the speed of sound [cf. Linsky 1984]. Although these results have been known for some time no detailed hydrodynamical explanation of the flow velocities has been proposed. Our results [see also Woodward et al. 1987] suggest that the observed velocities might have a convective origin. Before a definite model can be proposed, however, further calculations are necessary to extend the present results to a more realistic three-dimensional environment. Studies in that direction have already been undertaken by the authors and are in progress on the ETA10 supercomputer at JVNC.

References


Figure 4: Perspective view of the entropy fluctuations as a function of position. The bright yellow peaks correspond to the shocks where entropy is being generated by viscous stresses. The dark blue valleys correspond to cold, dense, low entropy fluid descending at the downflows.

Figure 5: As in Figure 4 but at a slightly later time. The shocks have propagated upstream leaving behind a layer of dense, low entropy fluid. These successive views correspond to the same intervals of time as shown in Figure 3.
Figure 6: Entropy fluctuations at yet a later time. The shocks have propagated almost to the upflow sites and are weakening considerably. The left shock has almost disappeared.

Figure 7: The final view of entropy fluctuations in this sequence. A new shock system has formed.
Figure 1: A portion of a parameter space illustration depicting a family of degree four rational functions. An object similar to the Mandelbrot set is present. Black regions correspond to members of the family which exhibit non-trivial periodic behavior, blue to stable families, and gold to transitional regions where family members undergo bifurcation.