SQUEEZED QUANTUM STATES IN

THEORY OF COSMOLOGICAL PERTURBATIONS

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ABSTRACT

It is shown that the notion of squeezed quantum states arises naturally in the
theory of quantized cosmological perturbations. Gravity-wave and matter-density
perturbations are studied in some detail. A new phenomenon -- desqueezing of
matter-density perturbations -- is also considered, and its relevance to the so-called
Sakharov's oscillations is clarified.

1. INTRODUCTION

The quantum origin of the primordial cosmological perturbations seems to be a very natural
and attractive idea. In this paper we study the evolution of gravity-wave and matter-density
cosmological perturbations in the framework of quantum mechanics. We demonstrate the close
relationship between the theory of cosmological amplification of zero-point quantum oscillations
and the theory of squeezed quantum states. We show that the initial vacuum state of the quantized
perturbations inevitably evolves into a squeezed quantum state in the course of cosmological
expansion [1]. This process can be described with the help of the Schrödinger equation for a
harmonic oscillator with varying parameters. Variation of the parameters is caused by the very fact
of the universe expansion. The produced strongly squeezed quantum states correspond to classical
random fluctuations with very large variances in amplitude and very small variances in phase.

The concrete properties of squeezed quantum states for matter-density perturbations may
be different from those for gravitational waves. We will show that in the case of matter-density
fluctuations one can have an additional effect. It turns out that the squeeze parameter $r$ attributed
to fluctuations with some wave numbers $n$ can decrease at late stages of cosmological evolution.
Since the quantum state of these fluctuations remains a squeezed one, though its squeeze parameter
decreases, we call this effect "desqueezing." This effect takes place for some definite intervals of \( n \) and leads to an oscillatory behavior of the function \( r(n) \). We relate this quantum effect to the effect of the so-called Sakharov's oscillations [2,3] known in classical theory of matter-density perturbations.

In this paper we first consider the gravitational waves (Sec. 2) and then the matter-density fluctuations (Sec. 3).

2. GRAVITATIONAL WAVES

For simplicity we restrict ourselves to a spatially flat Friedmann-Rebertson-Walker (FRW) universe. This space-time metric can be written in the form:

\[
ds^2 = a_t^2 \rho a^2 \left( d\eta^2 - dx^1^2 - dx^2^2 - dx^3^2 \right),
\]

where \( a(\eta) \) is the dimensionless scale factor, \( \ell_p \) is the Planck's length, and \( \sigma = \) constant (here and below \( h=c=1 \)). If necessary, one can consider a FRW universe with a finite spatial volume caused by a nontrivial topology. In this case the constant \( \sigma \) is \( \sigma = [(3/4\pi)\rho J d^3x]^{1/2} \). However, the assumption of finite volume is only important when one treats the quantum properties of the space-time itself with the help of the Wheeler-DeWitt equation. While considering the quantum fluctuations on a given classical background space-time, this assumption is not necessary.

A gravitational-wave perturbation \( h(\vec{x}, \eta) \) (for simplicity we consider only one polarization component) can be written in the form:

\[
h(\vec{x}, \eta) = \frac{1}{a} \sum_{\vec{\pi}}' \sum_{J=1}^2 \mu_{\vec{\pi}, J}(\eta) U_{\vec{\pi}, J}(\vec{x}),
\]

where \( \vec{\pi} = (n^1, n^2, n^3) \) is the wave vector, and the functions \( U_{\vec{\pi}, J}(\vec{x}) \) obey the equation:

\[
\Delta U_{\vec{\pi}, J} = -n^2 U_{\vec{\pi}, J},
\]

where \( \Delta \) is the Laplace operator, \( n^2 = n^1^2 + n^2^2 + n^3^2 \), \( J = 1, 2 \) denotes two linearly independent solutions. Here and below the sign \( \sum'_{\vec{\pi}} \) means that the sum is taken over one-half of \( \vec{\pi} \)-space, for example, over those \( \vec{\pi} \) which satisfy the inequality \( \vec{\pi} \cdot \vec{x} \geq 0 \).

The functions \( \mu_{\vec{\pi}, J}(\eta) \) satisfy the equation [4]:
\[
\mu_{\lambda J}'' + \left( n^2 - \frac{\alpha''}{\alpha} \right) \mu_{\lambda J} = 0 ,
\]

(2.3)

where \( ' \equiv d/d\eta \). The function \( V(\eta) = \alpha''/\alpha \) plays the role of a potential barrier in Eq. (2.3).

The real field \( h(\vec{r}, \eta) \) can be decomposed into a set of mode solutions by using both the complex and real functions \( U_{\vec{n^*}, J}(\vec{r}) \). As a consequence, one deals with the complex solutions to Eq. (2.3) in the first case and with the real solutions in the second case. This leads, correspondingly, to the use of two-mode or single-mode squeezed states which transform into each other by \( \pi/4 \) rotation in the complex plane of the basic functions \( U_{\vec{n^*}, J} \) [6]. Below we will follow [5] and use the real functions \( U_{\vec{n^*}, J}(\vec{r}) \) in order to deal with the simpler technique of single-mode squeezed states.

It is obvious that all physical results expressed in terms of quantum-mechanical mean numbers do not depend on one or another choice of the basic functions \( U_{\vec{n^*}, J} \).

For every \( (\vec{n}, J) \) mode Eq. (2.3) can be derived from the Hamilton function

\[
H = \frac{p^2}{2M} + \frac{1}{2} M \Omega^2 y^2 ,
\]

(2.4)

where \( y = \mu/a \), \( M = a^3 \), \( \Omega = n/a \), \( p \) is the momentum canonically conjugated to the coordinate \( y \). Thus, we have a Hamiltonian for a harmonic oscillator with varying mass \( M \) and frequency \( \Omega \) (for more details see [1,5]). The associated Schrödinger equation,

\[
- \frac{1}{i} \frac{\partial \psi}{\partial \eta} = H \psi ,
\]

(2.5)

admits a wide class of solutions \( \psi(\eta, y) \) having the form:

\[
\psi(\eta, y) = c(\eta) \exp \left[ -B(\eta)y^2 \right] ,
\]

(2.6)

where the complex function \( B(\eta) \) is determined by solutions to the classical Eq. (2.3) according to the relation

\[
B(\eta) = \frac{i}{2} \frac{a^2}{\mu/a} \left( \frac{\mu/a}{\mu/a} \right)' .
\]

It is supposed that at the initial moment of time \( \eta = \eta_0 \), or \( \eta \to -\infty \), the quantum state was vacuum, so that \( B(\eta_0) = 1/2 \) \( n a^2 \). The function \( B(\eta) \) contains all the information about parameters of a single-mode squeezed state produced from the vacuum state by the action of the squeeze operator [6].
\[ S_1(r, \varphi) = \exp \left[ \frac{r}{2} (e^{-2i\varphi} b^2 - e^{2i\varphi} b^{-2}) \right] \]

where \( b^+ \) and \( b \) are the creation and annihilation operators for gravitons, \( r \) is the squeeze parameter and \( \varphi \) is the squeeze angle. Real parameters \( r(\eta) \) and \( \varphi(\eta) \) are related to the complex function \( B(\eta) \) according to [6]:

\[ B = \frac{M\Omega \cosh r + e^{2i\varphi} \sinh r}{2 \cosh r - e^{2i\varphi} \sinh r} \]

(2.7)

The squeeze parameter \( r \) defines also the mean number of quanta \( N \) in a given mode:

\[ \langle N \rangle = \sinh^2 r \]

We will consider a specific FRW-universe which has three consequential stages of expansion: inflationary (i), radiation dominated (e), and matter dominated (m).

The scale factor \( a(\eta) \) at these three stages is:

\[ a_i = -\sqrt{\kappa} \eta, \quad \eta_1 \leq \eta < 0, \]
\[ a_e = c_1(\eta + \theta_1), \quad \eta_1 \leq \eta < \eta_2 \]
\[ a_m = c_2(\eta + \theta_2)^2, \quad \eta \geq \eta_2 \]

where \( \kappa = \) constant, \( \eta_1 \) and \( \eta_2 \) are the instants of time when the stages join. The constants \( \kappa, \eta_1, \eta_2 \) can be chosen in such a way that the \( a(\eta) \) will give a reasonable cosmological model [5]. The constants \( c_1, c_2, \theta_1, \theta_2 \) are determined by continuous joining of \( a \) and \( a' \) at \( \eta_1 \) and \( \eta_2 \):

\[ \theta_1 = -2\eta_1, \quad \theta_2 = \eta_2 - 4\eta_1 \]

The basic (normalized) solutions \( \xi \) and \( \xi^* \) to Eq. (2.3) can be presented in the universal form valid for the three stages (i), (e), (m):

\[ \xi = \left( 1 - i \frac{q}{(\eta + \theta)} \right) e^{-i(\eta + \theta) \eta} \]

(2.8)

where \( q_i = q_m = 1, q_e = 0, \theta_i = 0, \theta_e = \theta_1, \theta_m = \theta_2 \). The general solution to Eq. (2.3) reads as \( \mu(\eta) = \gamma(1) \xi + \gamma(2) \xi^* \), where \( \gamma(1) \) and \( \gamma(2) \) are complex constants. According to our prescription, one has \( r = 0 \) (vacuum) at \( \eta = \eta_b \). Bearing in mind this condition one can calculate the squeeze parameter \( r(\eta) \) at any later moment of time, including the present time \( \eta_0 \). Large
squeezing \( r(\eta_0) \geq 1 \) takes place only for waves which encountered the potential \( V(\eta) \) and underwent a superadiabatic amplification regime in the course of cosmological expansion, i.e. for waves which satisfied the condition:

\[
n^2 << \left| a''/a \right|
\]  

(2.9)

at some interval of time. For some waves (with the present frequencies ranging from \( 10^{-16} \) Hz to \( 10^{+8} \) Hz), this condition was valid only at the (i) stage. For longer waves (with the present frequencies ranging from \( 10^{-18} \) Hz to \( 10^{-16} \) Hz), this condition was fulfilled at both (i) and (m) stages.

By joining continuously \( \mu \) and \( \mu' \) at \( \eta_1 \) and \( \eta_2 \) one can eventually calculate \( r(\eta_0) \) [5]:

\[
\begin{align*}
  r(\eta_0) &= \ln[n^{-2}\eta_1^{-2}] \quad , \quad 10^{-16}\text{Hz} \leq \nu_0 \leq 10^{+8}\text{Hz} \quad , \\
  r(\eta_0) &= \ln[(3/8)n^{-3}\eta^{-2}_1(\eta_2-2\eta_1)^{-1}] \quad , \quad 10^{-18}\text{Hz} \leq \nu_0 \leq 10^{-16}\text{Hz} \\
\end{align*}
\]

(2.10)

For all wavelengths, \( r(\eta_0) \) can be approximately presented as the logarithm of the ratio of the scale factors at two crucial instants of time: when a given wave returns under the Hubble radius at the (e) or (m) stage and when the wave leaves the Hubble radius at the (i) stage. The numerical values of \( r(\eta_0) \) follow from (2.10). The parameter \( r(\eta_0) = 1 \) for \( \nu_0 \approx 10^8 \) Hz and \( r(\eta_0) \) increases toward smaller frequencies, so that \( r(\eta_0) \) reaches \( r(\eta_0) \approx 10^2 \) for \( \nu \approx 10^{-16} \) Hz. Thus we conclude that relic gravitons should now be in strongly squeezed quantum states. From the observational point of view this means that relic gravitational waves form a highly wide-band stochastic signal with very specific statistical properties, to be discussed briefly below.

We consider the waves shorter than the present-day Hubble radius. For every \((\vec{n},J)\) mode the present-day behavior of the function \( \mu_{\vec{n},J}(\eta) \) can be written in the form

\[
\mu = A \cos(-n\eta + \phi)
\]

where \( A \) and \( \phi \) are random numbers. It can be shown [1], that for the strongly squeezed states under discussion, the variance of \( A \) is very large while the variance of \( \phi \) is negligibly small. The Gaussian distribution for \( \phi \) looks like a \( \delta \)-function and is almost identical for all \((\vec{n},J)\)-modes with the same \( n \). One implication of this fact is that the stochastic background of relic gravitational waves forms a collection of standing waves rather than a collection of traveling waves. One can see this by inspecting Eq. (2.2) and remembering that \( \phi_{\vec{n},J} \) is almost fixed and equal for all \((\vec{n},J)\) modes with the same \( n \). For a given \( n \) this enables one to take the temporal dependence from under the summation sign over \( \vec{n} \) in Eq. (2.2), that is to demonstrate that the \( n \)-th harmonic of the field \( h(\vec{x},\eta) \) is a product of a function of time and a function of spatial coordinates, i.e., it presents a
standing wave. Its statistical properties are determined by the random distributions of the amplitudes $A_n$. (About gravitons produced in cosmological models see also some early and more recent papers [7].)

Before going to the matter-density perturbations we would like to make a comment. As was mentioned above, the very long gravitational waves, with present-day frequencies $10^{-18} \text{ Hz} \leq \nu_0 \leq 10^{-16} \text{ Hz}$, have encountered two potential barriers, the first at the (i) stage and the second at the (m) stage. However, the time-separation between the barriers was smaller than the periods of the waves, so that none of the waves had enough time to complete even one oscillation. As a result, the two barriers effectively acted as one. The final squeeze parameter as a function of $n$, $r(n)$, and the power spectrum of the perturbations are being described by slowly varying monotonic functions. (They correspond to the so-called Harrison-Zeldovich spectrum.) In contrast to this, in the case of matter-density perturbations, we will be dealing with two barriers sufficiently separated so that the different standing sound waves could have experienced a different number of oscillations between the barriers, and their final amplitudes are strongly dependent on their phases. As a result, some of the waves can deamplify and their power spectrum components go to zero. In terms of quantum mechanics this leads to the phenomenon of "desqueezing". We will concentrate on this phenomenon in Sec. 3.

3. MATTER-DENSITY PERTURBATIONS

As in Sec. 2, we will discuss a simplified cosmological model in which the radiation dominated stage (e), governed by matter with the equation of state $p = \frac{1}{3} \epsilon$, goes over into the matter-dominated stage (m), governed by matter with the equation of state $p = 0$, at some moment of time $\eta = \eta_2$. We will follow the evolution of the matter-density perturbations $\delta \epsilon/\epsilon$ and the associated perturbations of the gravitational field (the metric tensor perturbations $\delta g_{\mu\nu}$). The previous history of the perturbations, prior to the beginning of the (e) stage at $\eta = \eta_1$, determines their state at $\eta = \eta_1$. For consistency we will assume that at the preceding (i) stage there existed zero-point quantum fluctuations of some scalar field which have been amplified and then have transformed into $\delta \epsilon/\epsilon$ at $\eta = \eta_1$. Such an assumption is an ingredient of a "complete cosmological theory" [8] and it was developed in many papers. We will use some formulas from the work [9] based on some previous publications (in particular, see the early papers [10]).

In general, the linearized perturbations of the gravitational and non-gravitational (matter) fields are governed by a set of coupled differential equations (see for example [11]). However, for such a highly symmetric background space-time as a FRW universe, Eq. (2.1), and for the scalar type perturbations which we are interested in now, the equations can be reduced to a single equation for a single fundamental field $v(x, \eta)$. The zero-zero component of the metric tensor field $\Phi(x, \eta)$ and
the matter-density perturbations $\delta \epsilon / \epsilon$ can be expressed in terms of $v(\vec{x}, \eta)$ as follows [9]:

$$\Delta \Phi = -2 \pi^{1/2} l_p \frac{\beta}{\alpha c_s^2} \left( \frac{v}{Z} \right)' ,$$

$$\frac{\delta \epsilon}{\epsilon} = \frac{2}{3 \alpha^2} \left( \Delta \Phi - 3 \alpha \Phi' - 3 \alpha^2 \Phi \right) ,$$

where $\alpha = a'/a$, $\beta = a^2 - \alpha'$, $c_s = (dp/d\epsilon)^{1/2}$ is the velocity of sound, $Z = \alpha \beta^{1/2} / \alpha c_s$, and $\Delta$ is the Laplace operator.

The field $v(\vec{x}, \eta)$ is subjected to quantization. Similar to what we were doing in Sec. 2 we will present $v(\vec{x}, \eta)$ in the form

$$v(\vec{x}, \eta) = \sum_{\vec{\eta}} \sum_{J=1}^{2} \chi_{\vec{\eta}, J}(\eta) U_{\vec{\eta}, J}(\vec{x})$$

and will use the real functions $U_{\vec{\eta}, J}(\vec{x})$. The time-dependent functions $\chi_{\vec{\eta}, J}(\eta)$ satisfy the equation

$$\chi_{\vec{\eta}, J}'' + \left( n^2 - V(n, \eta) \right) \chi_{\vec{\eta}, J} = 0$$

(3.1)

where the potential barrier is described by the function

$$V(n, \eta) = 1 - c_s^2 n^2 + \frac{Z''}{Z}$$

(3.2)

As might have been expected, Eq. (3.1) is very similar to Eq. (2.3) for gravitational waves. The nonvanishing function $Z''/Z$ makes it clear that other perturbations, not only gravitational waves, do also obey the non-conformally-invariant equations [10,12]. However, in contrast to Eq. (2.3) for gravitational waves, the function $V(n, \eta)$, Eq. (3.2), depends on the wave number $n$. This is a consequence of the fact that a new parameter has appeared in the problem -- the velocity of sound $c_s$, not equal to the velocity of light $c$, $c=1$.

The function $V(n, \eta)$ has different behavior at different stages. At the (m) stage one has $c_s^2 = 0$ and $Z''/Z = 2(\eta + \theta)^2$. In this regime Eq. (3.1) describes the gravitational instability in a pressureless medium. At the (e) stage one has $c_s^2 = 1/3$ and $Z''/Z = 0$, so that $V(n, \eta)$ rescales the wave number $n$. At the (i) stage Eq. (3.1) governs the scalar field perturbations if one replaces $c_s^2$ $\to$ 1 and $Z \to a\psi_0'/\alpha$, where $\psi_0(\eta)$ is the unperturbed value of the scalar field (for more detail see
[9]). Since at the (i) stage \( Z'/Z \approx a'/a \), in this regime Eq. (3.1) coincides exactly with Eq. (2.3) for gravitational waves. As in the case for gravitons, the initial vacuum state of the scalar field perturbations will also transform into a strongly squeezed quantum state by the end of the (i) stage.

Let us turn to the quantum version of Eq. (3.1). For every \((\pi, J)\) mode this equation can be derived from the Hamiltonian

\[
H = \frac{p^2}{2} + \frac{1}{2}(e^2 n^2 - \frac{Z''}{Z}) \chi^2 ,
\]

where \( p \) is the momentum canonically conjugated to \( \chi \). The Hamiltonian (3.3) describes a harmonic oscillator with varying frequency. The associated Schrödinger equation is

\[
-\frac{1}{i} \frac{\partial \psi}{\partial \eta} = H \psi .
\]

We will again seek solutions to this equation in the form \( \psi(\eta, \chi) = c(\eta) \exp \{-B(\eta)\chi^2\} \). The function \( B(\eta) \) can be written as

\[
B(\eta) = \frac{n}{2} \frac{\xi^* - \gamma \xi}{\xi^* + \gamma \xi} ,
\]

where \( \gamma \) is a complex constant and \( \xi, \xi^* \) are basic solutions to Eq. (3.1), \( \chi(\eta) = \gamma \xi + \xi^* \). We assume that \( \xi \) is a positive \( n \)-frequency solution to Eq. (3.1) at some moment of time \( \eta = \eta_1 \), so that

\[
\xi'(\eta_1) = -in\xi(\eta_1) .
\]

The basic solutions \( \xi \) have the following explicit form at the (e) and (m) stages:

\[
\xi_e = \frac{1}{2}(\sqrt{3} - 1)e^{in(\eta + \theta_2 e^{i\sqrt{3}})} - \frac{1}{2}(\sqrt{3} + 1)e^{i2(\eta + \theta_1 - \eta_1)e^{i\sqrt{3}}} ,
\]

\[
\xi_m = \frac{\sqrt{4 + n^2(\eta + \theta_2)^2}}{3} \left[ \frac{\eta + \theta_2 + (\eta + \theta_2)^2}{(\eta + \theta_2)^2} \right]^{1/2} \left[ \frac{1 - in(\eta + \theta_2)}{2 + in(\eta + \theta_2)} \right] .
\]

It is convenient to choose \( \eta = \eta_1 \) at the (e) stage and \( \eta = \eta_0 \) at the (m) stage, where \( \eta_0 \) is the present time. The functions \( \chi(\eta) \) and \( \chi'(\eta) \) valid at the (e) and (m) stages should be continuously joined at \( \eta = \eta_2 \). This defines the Bogoliubov coefficients \( u, w \) relating the quantum states at \( \eta = \eta_1 \)
and \( \eta = \eta_0 \). The value of \( \gamma \) at the (e) stage, i.e. \( \gamma_e \) determines the squeeze parameter \( r \) at \( \eta = \eta_1 \):

\[
\sinh^2 r(\eta_1) = \frac{\gamma_e \gamma_e^*}{1 - \gamma_e^* \gamma_e}.
\]

Knowing \( u \) and \( w \), one can find \( \gamma \) at the (m) stage, that is \( \gamma_m \):

\[
\gamma_m = \frac{w^* + \gamma_e u}{u^* + \gamma_e w}
\]

and, finally, the squeeze parameter \( r \) at \( \eta = \eta_0 \):

\[
\sinh^2 r(\eta_0) = \frac{\gamma_m^* \gamma_m}{1 - \gamma_m^* \gamma_m}
= \frac{w^* w}{1 - \gamma_e^* \gamma_e} \left[ 1 + \gamma_e \gamma_e^* + 2 \left( \frac{1 + w^* w}{w^* w} \right)^{1/2} (\gamma_e \gamma_e^*)^{1/2} \cos \delta \right] + \sinh^2 r(\eta_1),
\]

where \( \delta = \arg \gamma_e + \arg u + \arg w \).

Now we can discuss Eq. (3.6). The most interesting feature of this formula is the \( \cos \delta \) term. This term varies with \( n \) so that it "modulates" \( r(\eta_0) \) and can make it smaller than \( r(\eta_1) \) at some frequencies, for which \( \cos \delta \approx -1 \). The depth of the modulation depends on the numerical values of \( |\gamma_e|^2 \) and \( |w|^2 \). The value of \( \gamma_e \) is determined by the evolution of \( v(\vec{x}, \eta) \) field at the (i) stage, starting from its vacuum state at \( \eta = \eta_B \). Similarly to the graviton case, the \( v(\vec{x}, \eta) \) field and, therefore, the \( \delta e/e \) fluctuations, should be in a strongly squeezed state by the end of the (i) stage, that is, \( 1 - |\gamma_e|^2 \ll 1 \). In addition, one has \( |w|^2 \gg 1 \) for all typical situations. By combining these conditions one can show that the two terms in Eq. (3.6) can almost cancel each other for those \( n \) which give \( \cos \delta \approx -1 \). In other words, the depth of the modulation can approach 100%. Thus, for some modes, the final \( r(\eta_0) \) can be much smaller than the initial \( r(\eta_1) \). We call this phenomenon "desqueezing". In terms of the mean number of particles \( <N> \) and the power spectrum of the perturbations this means that at some frequencies \( <N> \) is much smaller than at others and the power spectrum has zeros at those frequencies.

We see the relationship between this phenomenon and Sakharov's oscillations in classical theory of density perturbations. In our model, the density perturbations have been squeezed at the (i) stage and have acquired definite phases before entering the oscillating regime at the (e) stage.
(the regime of sound waves, Eq. (3.4) for \( n(\eta + \theta_1) / \sqrt{3} >> 1 \)). In other words, the density perturbations have been presented in the form of standing sound waves prior to their entering the (m) stage. At the (m) stage these perturbations have encountered the barrier \( V(n, \eta) \). Under the barrier, the amplitude of a given mode with a given frequency \( n \) will increase or decrease depending on the phase of the mode. (If one was dealing with random phases and had averaged over phases this would always lead to a net amplification, so that a "typical" wave with a given \( n \) would be always amplified [4].) Since the phases of the density perturbations are fixed, some modes will lose their energy. The power spectrum will have an oscillatory shape which is known as Sakharov's oscillations. We would like to notice here that it is crucial to have the strongly squeezed quantum states (or standing sound waves) at the (e) stage before they encounter the barrier \( V(n, \eta) \) at the (m) stage. Only in this case some incoming quantum states can be desqueezed and the Sakharov's oscillations can develop. However, the initial squeezing can not be produced just at the beginning of the (e) stage, despite the possibility of presenting, artificially, the solution \( \xi_e \), Eq. (3.4), in the form of "growing" and "decaying" components, for \( \eta \) satisfying the condition \( n(\eta + \theta_1)/\sqrt{3} << 1 \).

It is clear from the form of the potential \( V(n, \eta) = 2/3 n^2 \) at the (e) stage that \( n^2 > V(n, \eta) \) for every \( n \)-mode. Thus, there is no reason for generating the squeezed states at the (e) stage; one can only achieve a negligibly small amount of the over-barrier squeezing while the mode propagates all over the (e) stage. We believe that Sakharov's oscillations, first discovered in Refs. [2,3] as a result of numerical calculations, are accounted for by the choice of the initial conditions at the beginning of the calculations at the (e) stage which the authors of the calculations considered to be the most "reasonable".

The problem of desqueezing occurs in theoretical quantum mechanics. If there are two barriers \( V(\eta) \), the evolution operator will be a product of two squeeze operators with parameters \( r_1 \) and \( r_2 \), corresponding to the first and the second barriers. The resulting operator is again a squeeze operator with the parameter \( r \) which can vary between \( |r_1 - r_2| \) and \( r_1 + r_2 \) [6]. It is clear that under certain conditions the desqueezing phenomenon occurs. In the limiting case of an infinite number of barriers \( V(\eta) \), that is in the case of a time-periodic potential, one can even get an analogue of Bloch's waves [13] which describe the wave function of an electron in crystals with spatially periodic potentials.

In conclusion we would like to call attention to the fact that the notions and ideas of "ordinary" quantum mechanics are quite useful and applicable to the problems of quantum gravity and cosmology.
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REFERENCES


