Radiation pressure effects in early-type close binaries and implications for the solution of eclipse light curves

Horst Drechsel¹,², Stefan Haas³, Reinald Lorenz³, and Sebastian Gayler²

¹ Joint Institute for Laboratory Astrophysics, University of Colorado, Campus Box 440, Boulder CO 80309-0440, USA
² Dr. Remise-Sternwarte Bamberg, Astronomisches Institut der Universität Erlangen-Nürnberg, Sternwartstraße 7, D-96049 Bamberg, Germany

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Abstract. A new method is presented to include radiation pressure effects in the modelling of close binaries. The radiative interaction of the binary components increases with $T_{\text{eff}}^4$ and the inverse square of reduced separation, and is of particular importance for hot OB-type binaries in close configurations. Since no analytical representation of the modified potential field exists, a numerical procedure was developed to implement radiation pressure. The radiative forces are parametrized using the ratio of radiative relative to gravitational forces as normalizing factor for the calculation of the radiation pressure action on the irradiated photospheres. Both stars are considered in their correct 3D shapes. The local incident flux is obtained by an integration over the visible surface parts of the radiating component, and the effective radiation pressure action on the irradiated star is determined with respect to the local geometrical conditions (partial shadowing and 3D orientation of the surface). Principal effects of radiation pressure on the shape of the stars and the binary configuration are demonstrated by model calculations. There are several important implications for the binary structure: the geometry of the stellar surfaces is modified; the Lagrangian points are shifted, and the shape and extent of the Roche lobes are changed; the tendency to take up inner contact in $L_1$ is partly counteracted by radiative forces; outer contact components (with surfaces incorporating $L_2$ or $L_3$) may be formed above some critical radiation pressure strength; obvious consequences for the evolution of systems with hot and luminous components like WR or X-ray binaries exist. The modified Roche potential is used as an improved model for the calculation of eclipse light curves, based on the general logistics of the Wilson-Devinney method. The inverse problem is solved by applying the nonlinear simplex parameter optimization algorithm. The feasibility of the new method is demonstrated by photometric solutions of the OB systems IU Aur and AB Cru. The implementation of radiation pressure effects yields improved solutions compared with conventional methods, even in the case of small radiation pressure parameters (e.g., less than 1% for IU Aur). Therefore it appears promising to apply the method to hot O-type systems with non-negligible radiative effects, in order to derive more reliable absolute dimensions for this particularly important group of stars.

Key words: radiation mechanisms; miscellaneous – binaries: eclipsing – stars: early-type – stars: fundamental parameters

1. Introduction

The structure of close binary systems is usually described by the classical Roche model (cf. Kopal 1959) considering tidal forces as the only interaction effect. The Roche potential is composed of gravitational and centrifugal terms, and does not take into account other internal or external forces acting on stellar or circumstellar matter. Appropriate modifications for eccentric orbits and asynchronous rotation of the stars (Plavec 1958; Limber 1953; Kruszwieski 1966; Szabolics 1967) have rendered the Roche model a more general approximation applicable to the majority of close binaries. It provides a physically reasonable basis for the description of the geometrical structure and evolutionary processes of most systems of intermediate to late spectral type, which are not too strongly magnetized.

The interaction between radiation and matter, however, becomes especially important for early-type systems, because radiation pressure rapidly increases with the fourth power of the effective temperature. Irradiation effects are further enhanced in close binaries as the forces gain in strength with the inverse square of decreasing separation between the stars. It has been shown (Gayler et al. 1990; Drechsel et al. 1991; Haas et al. 1992) that the radiative forces exerted by the mutual irradiation of the binary components cannot be neglected for close luminous O- and B-type systems. There is observational evidence for a more or less continuous mass loss of the majority of early-type stars via stellar wind, which is mainly driven by their strong photospheric radiation fields. In close binaries of early spectral type

Send reprint requests to: H. Drechsel

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also colliding stellar winds from both components are well established. Hence it is obvious that radiation pressure forces will also have impact on the photospheric layers of the stars, given the delicate equilibrium of forces in near-contact systems.

Some previous attempts were made to treat the influence of radiation pressure on the equipotential structure of close binaries (e.g. Schueerman 1972; Kondo & McCluskey 1976; Vanbeveren 1977, 1978; Djurašević 1986; Zhou & Leung 1987). With the exception of the hydrodynamic and magnetic approach of Djurašević, the other authors derived equipotential surfaces by means of an effectively reduced gravitational potential. However, it will be shown that a more sophisticated treatment of the physical and geometrical conditions is necessary to avoid inconsistent and unrealistic results.

The inclusion of radiation pressure terms in the total potential makes an analytical representation of the equipotential surfaces impossible. This might be one of the reasons why modifications of the classical Roche model have so far not been attempted in a more rigorous way. Radiation pressure was now incorporated as a numerical extension of the classical restricted three body problem. If the strength of radiation pressure comes into a range where it cannot be neglected as a competing factor compared with gravity, several important deviations from the usual Roche configuration can be expected: the most evident consequences are a different overall geometry of the equipotential field, modified shapes of the photospheric layers as well as a shift of the positions of the Lagrangean points \( L_1 \), \( L_2 \), and \( L_3 \). This will have immediate consequences for the system configurations and the dynamics of mass transfer through \( L_1 \) or mass loss via \( L_2 \) or \( L_3 \), and can be of importance for the binary evolution.

Modern procedures for the solution of photometric light curves of eclipsing binaries are mostly based on the Roche model, and have also overcome its intrinsic approximations of circular orbits and synchronous rotation. Tidal and rotational distortions, reflexion effect, limb and gravity darkening are all treated in an adequate physical way. However, the interaction between radiation and matter was so far widely neglected in connection with the structure and dynamics of close binaries. We therefore used a radiation pressure-modified Roche model as a new basis to determine the geometry of the potential field and to calculate synthetic light curves of eclipsing binaries. Since the stellar surfaces have no analytical representation, they are computed by a numerical procedure. Also gravity darkening, which plays an important role for the surface brightness distribution of close binaries, has to be handled over a numerical calculation of the potential gradients at the stellar surfaces.

Theoretical light curves produced with due regard to radiation pressure effects are certainly more appropriate for an interpretation of the light changes of close eclipsing binaries with early spectral type than other approaches based on the classical Roche model. The modelling of numerical light curves can now be performed with proper adjustment of radiation pressure parameters among all other geometrical and physical quantities characterizing a given system, and information about the actual relevance of radiation pressure for an individual system can be extracted from the observed light curves. The photometric solutions will also yield more realistic parameters and configurations for close early-type systems. The new code was recently applied to the \( \beta \) system IU Aur by Drechsel et al. (1994) and to the \( \beta \) system AB Cru by Lorenz et al. (1994). Though radiation pressure forces in the moderately hot IU Aur system are relatively small, still definitely improved light curve solutions could be achieved compared with previous results. For the hotter AB Cru system, a good photometric solution was achieved with considerably larger radiation pressure parameters. The application of the radiation pressure-modified code to hot O-type systems is therefore very promising and will help to determine more accurate absolute stellar quantities of very early-type stars.

In Sect. 2, the numerical calculation of modified Roche potentials is described. Section 3 illustrates the effects of radiation pressure on the stellar surfaces and system configurations. The computation of synthetic light curves on the basis of the Wilson-Devinney (hereafter: WD) model (Wilson & Devinney 1971) combined with a non-linear parameter optimization method, the simplex algorithm (Knuth & Linnell 1987), is outlined in Sect. 4, with special emphasis on the modifications necessary for the treatment of radiation pressure effects. As an example, the application of the new code to the OB systems IU Aur and AB Cru is demonstrated in Sect. 5.

2. Modified Roche potential

The classical Roche model allows for no other than gravitational and centrifugal forces. The tidal interaction and the rotational flattening of the binary components are adequately described for binaries, in which the separation of the two stars is large compared with their radii, and is in general adequate for not too early-type systems. However, for close OB-type binaries the radiative interaction has to be accounted for. The above cited previous attempts to incorporate radiation pressure were based on the fact that both gravitation and the flux-proportional radiation pressure force vary with the inverse square of distance, which was simply used to exchange the pure gravity terms of the Roche potential by effectively reduced gravity terms. Nevertheless, these approaches did not consider the actual geometrical conditions. For instance, the radiation source was approximated as a point source located at the mass center of the star, and also no regard at all was paid to the shape and extent of the irradiated component. The potential field at the surface and in the shadow cone behind the irradiated star was computed under physically inconsistent assumptions:

- First, a global reduction of the gravitation terms due to radiation pressure acts isotropically with respect to the mass centers, but obviously there is no radiation received from the companion behind the horizon of the irradiated star.
- Second, the incident flux at any point on the surface of the irradiated star will be different from the one emitted by a point source located at the mass center of the radiating star, if the actual shape and extent of this star is properly taken into
account; the real flux has to be determined by a numerical surface integration.

- Finally, an overall constant value of \( \delta \) pays no attention to the varying angle \( \theta \) between the incident radiation and the surface normal of the irradiated parts of the companion’s photosphere. Actually the radiation pressure force follows a \( \cos \theta \) law, and is declining towards the horizon of the irradiated star. For a correct description of the radiation pressure effects, the geometrical shapes of both stars are crucial.

2.1. Parametrization of radiation pressure

For each stellar component, the dimensionless ratio of radiation pressure force over gravitational force per unit mass at a given position,

\[
\delta_{1,2} = \frac{F_{\text{rad}}}{F_{\text{grav}}} = \text{const.} \quad (0 \leq \delta_{1,2} < 1),
\]

can be used to characterize the strength and efficiency of its radiation pressure.

Only if gravitation and radiation pressure are acting in exactly opposite directions and no shadowing takes place, both forces are correctly expressed by means of an effectively reduced gravitational potential

\[
(1 - \delta) \frac{GM}{\rho}.
\]

This is a good approximation for any volume element of stellar matter with infinite mass and extent belonging to the radiating component itself. Especially the radiation pressure action on the surface layer of a star due to the radially symmetric net flux originating from its own core can be taken into account by the use of such a reduced potential. Meridional intersections of the equipotential surfaces

\[
\Omega_1 = \frac{1 - \delta_1}{r_1} + \frac{q}{r_2} + \frac{q + 1}{2} (x^2 + y^2) - qx = 3.75 \tag{1}
\]

of the primary component of a binary system with mass ratio \( q = M_2/M_1 = 1 \) for different values of the radiation pressure parameter \( \delta_1 \) and a fixed value of \( \Omega_1 = 3.75 \) are shown on the left side of the top part of Fig. 1. Here the Roche potential is expressed in the original notation of Kopal (1959) using cartesian coordinates, with the origin in the mass center of \( M_1 \):

\[
r_1 = \sqrt{x^2 + y^2 + z^2} \quad \text{and} \quad r_2 = \sqrt{(A - x)^2 + y^2 + z^2}
\]

are the radial distances from the centers of \( M_1 \) and \( M_2 \), which are separated by \( A \), which is usually set equal to 1 to normalize the potential. The equipotential surfaces of the irradiated secondary component on the right side have been obtained in a different way: they cannot be represented by a simple reduction of the gravitational term, but have to be calculated by a more complex numerical procedure involving the modified Roche potential \( \Omega_2^{\text{mod}} \) of Eq. 5 \( (\Omega_2 = 3.75 \text{ and } \delta_2 = 0) \).

![Fig. 1. Meridional intersection of equipotential surfaces of a binary system with mass ratio \( q = 1 \) for different \( \delta_1 \) values (\( \delta_2 = 0 \) in all cases), illustrating the effects of inner radiation pressure. The top part shows the shrinking of a fixed equipotential surface (\( \Omega_1 = 3.75 \)) with increasing \( \delta_1 \); the bottom part demonstrates the influence of increasing inner radiation pressure on the extent of the Roche lobe of the primary. The deformation of the irradiated star is also evident.]

To demonstrate the consequences of the so-called inner radiation pressure, \( \delta_2 \) was assumed to be zero, and the more complicated influence of irradiation by the companion was suppressed. For \( \delta_1 = 0 \), \( \Omega_1 = \Omega_2 = 3.75 \) then corresponds to the classical inner critical Roche surface. The increase of the radiation pressure parameter \( \delta_1 \) causes the equipotential surface \( \Omega_1 \) to shrink to smaller volumes, or in other words — a given stellar surface is represented by an equipotential value, which is the smaller the larger \( \delta_1 \) will be. The increasing deformation of the secondary with larger values of \( \delta_1 \) is also evident. The different surface shapes of the secondary in the top part of Fig. 1 were calculated for the indicated \( \delta_1 \) values, \( \delta_2 = 0 \), and a value of \( \Omega_2 = 3.75 \), which is equal to the potential of the inner critical Roche boundary of a system with \( q = 1 \) and \( \delta_{1,2} = 0 \). The back side of the secondary coincides with this Roche surface, though \( \delta_1 \neq 0 \). This is due to the blocking of incident radiation by the star itself (\( \delta^*_1 = 0 \) in the shadow region); details about the computation of surfaces irradiated by the companion follow later.
The bottom part of Fig. 1 illustrates the effects of an increased inner radiation pressure of the primary on the shape and extent of its own inner critical Roche volume. It is seen that the Roche lobe will also shrink with increasing values of $\delta_1$. Its equipotential value decreases from $\Omega_1 = 3.75$ for $\delta_1 = 0$, over 3.44 for $\delta_1 = 0.15$, 3.13 for $\delta_1 = 0.3$, to 2.67 for $\delta_1 = 0.5$. In all cases, $\delta_2$ was assumed to be zero, and the secondary surface always corresponds to $\Omega_2 = 3.80$.

Radiation pressure is caused by the interaction between electromagnetic radiation and stellar matter. Its amount depends on the momentum transfer of $h\nu/c$ per photon, which is absorbed or scattered in the irradiated photosphere, and is a very complicated function of the local physical and chemical conditions. A large fraction of the momentum transfer is certainly due to absorption processes in prominent UV resonance lines. In this sense the problem is related to the radiative acceleration of stellar winds.

The radiation pressure force $F_{\text{rad}}$ per unit mass is obtained from the radiation pressure $P_{\text{rad}}$ acting on a unit surface element:

$$P_{\text{rad}} = \frac{1}{c} \int \int I_\nu \cos^2 \phi \ dn \ d\omega,$$

where $c$ is the velocity of light, $\phi$ is the angle between the surface normal and the incident radiation, $d\omega$ is the opening of the radiation cone, and $I_\nu$ the intensity in the frequency interval $dn$ around $\nu$. Using a plane parallel radiative transfer equation, we have

$$F_{\text{rad}} = -\frac{1}{\rho} \frac{dP_{\text{rad}}}{dr} = \frac{1}{c\rho} \int \int k_\nu I_\nu \cos \phi \ dn \ d\omega,$$

where $\rho$ is the mass density and $k_\nu$ the monochromatic absorption coefficient.

With such a relation, the radiation pressure parameter $\delta = F_{\text{rad}}/F_{\text{grav}}$ could easily be expressed as a function of the integral stellar mass and luminosity and of a mean absorption coefficient $\bar{k}$. However, the accuracy of such a correlation would be questionable due to the considerable uncertainties involved in a quantitative specification of the different absorption and scattering processes, which are relevant for the irradiation of partially ionized photospheric layers from outside. Similarly, an accurate measure of the accelerating force of stellar winds is still subject of refined theories currently under development. No attempt was therefore made to incorporate radiation pressure in our model in an explicit way on the basis of stellar atmosphere calculations. Also, a coupling between radiation pressure and the stellar mass and luminosity would cause numerical difficulties when used in connection with the light curve solution procedure, which sensitively reacts on any correlation of free parameters, given the relatively large number of adjustable quantities. For these reasons, we rather preferred to treat the efficiency of radiation pressure as an independent parameter, i.e. to characterize it by means of $\delta_{1,2}$ values for both stellar components, in order to gain information about the principal impact of radiation pressure on the structure of hot close binary systems.

2.2. Computation of modified Roche equipotentials

The radiation pressure parameters $\delta_1$ and $\delta_2$ can account for the interaction of stellar matter with the inner isotropic radiation field, which leads to an effective reduction of the surface gravity:

$$F_{\text{grav}}^\text{eff} = F_{\text{grav}} - F_{\text{rad}} = (1 - \delta)F_{\text{grav}}.$$

The $\delta$ values are essentially constant over the stellar surfaces, because gravitation and radiation pressure forces can both be treated there as central forces, at least to a very good approximation. The consequences of irradiation of the companion, however, cannot be dealt with in a similar easy way due to the more complex geometrical conditions.

The effective reduction of the gravitational attraction exerted by the companion at a certain location on the surface of the irradiated star will be instead described by the use of $\delta^*$ parameters

$$0 \leq \delta_1^* = f(\theta_2, \phi_2, r_2) \leq \delta_1 \quad \text{and}$$

$$0 \leq \delta_2^* = f(\theta_1, \phi_1, r_1) \leq \delta_2,$$

which are no constants, but analytically unknown functions $f$ of position on the irradiated photospheres (polar coordinates are more appropriate here). The variation of $\delta_{1,2}^*$ between the maximum value of $\delta_{1,2}$ at the intersection points of the line connecting both mass centers with the stellar surfaces ($\theta = 0^\circ$) and the minimum value of zero at the stellar horizons ($\theta = 90^\circ$) has two reasons:

1. First, it is caused by the diminishing incident flux as the horizons are approached: there is a central part of the irradiated surface, where the flux can be obtained by an integration over those fractions of the surface of the irradiating star, which are visible from a given point on the irradiated surface; beyond this unrestricted area exists a penumbra region, where the flux integration for any point inside must account for the eclipse of a certain surface area of the irradiating star by parts of the irradiated star itself; and of course there is a complete shadow zone behind the boundary of the penumbra region, where no flux at all is received from the companion.

2. The second reason for the variation of the $\delta^*$ values is the increase of the angle $\theta$ between the surface normal and the direction of the integral flux vector from $0^\circ$ (perpendicular incidence) up to $90^\circ$ at the horizon, causing a respective decrease of the radiation pressure force proportional to $\cos \theta$.

For an adequate treatment of these effects, the actual 3-dimensional shape of both the irradiated and the irradiating components must be evaluated; the assumption of point sources would obviously yield unrealistic results.

In the Roche model, the stellar surfaces are obtained as closed equipotential surfaces for certain potential values $\Omega_{1,2}$ and mass ratios $q$. When radiation pressure is included, a modified potential field has to be used. With the above defined $\delta$ and
\(\delta^*\) parameters, the modified Roche potential of the primary star, expressed in spherical coordinates, is:

\[
\Omega_1^{\text{mod}}(\theta_1, \phi_1, r_1) = \frac{1 - \delta_1}{r_1} + q \frac{1 - \delta_1^*(\theta_1, \phi_1, r_1)}{\sqrt{1 - 2 \lambda r_1 + r_1^2}} - \lambda r_1 + \frac{q + 1}{2} r_1^2 (1 - \nu^2)
\]

(4)

and for the secondary star:

\[
\Omega_2^{\text{mod}}(\theta_2, \phi_2, r_2) = \frac{1 - \delta_2}{r_2} + \frac{1 - \delta_1^*(\theta_2, \phi_2, r_2)}{\sqrt{1 - 2 \lambda r_2 + r_2^2}} + \frac{q + 1}{2} r_2^2 (1 - \nu^2) - \lambda r_2 + \frac{1 - q}{2}
\]

(5)

where \(\lambda = \sin \theta \cos \phi, \nu = \cos \theta\), and the origins are placed in the centers of \(M_1\) and \(M_2\), respectively.

Once the functions \(\delta_1^* = f(\theta_2, \phi_2, r_2)\) and \(\delta_2^* = f(\theta_1, \phi_1, r_1)\) can be specified, the stellar surfaces are determined pointwise with the numerical Newton-Raphson method: for given values of \(q\) and \(\Omega_0\), the local radius \(r_0(\theta, \phi)\) for any direction \((\theta, \phi)\) is iteratively obtained as the zero coefficient of the function \(g\) defined as

\[g_i(\theta_1, \phi_1, r_1, \Omega_0, q) = \Omega_0 - \Omega_i(\theta_1, \phi_1, r_1, q)
\]

with \(i = 1, 2\).

2.2.1. Integration of incident flux

For the numerical determination of the radial distance \(r\) of any surface point \(P(\theta, \phi, r)\) from the mass center, it is necessary to specify the local value of \(\delta^*(\theta, \phi, r)\). This can be achieved by the integration of the actual incident flux in \(P\), which has to be normalized with a reference flux incident in a perpendicular direction and characteristic for each point \(P\). The exact calculation of the actual and reference fluxes, however, requires the knowledge of the 3-dimensional shape of both stars under the influence of radiation pressure, which is a priori unknown and can only be determined iteratively. To a first approximation, both stars are therefore assumed as triaxial ellipsoids defined by

\[
\frac{x^2}{r_{\text{point}}^2} + \frac{y^2}{r_{\text{side}}^2} + \frac{z^2}{r_{\text{pole}}^2} = 1,
\]

(7)

with semi-axes \(r_{\text{point}}, r_{\text{side}}, \text{and } r_{\text{pole}}\) identical with the Roche radii in the directions \((\theta = 90^\circ, \phi = 0^\circ), (\theta = 90^\circ, \phi = 90^\circ), \text{and } (\theta = 0^\circ, \phi = 0^\circ)\), respectively. These radii are readily obtained without any flux integration, because the value of \(\delta^*\) in the point \((\theta = 90^\circ, \phi = 0^\circ, r = r_{\text{point}})\) is exactly equal to \(\delta\) due to the perpendicular incidence of the radiation and the absence of any eclipse effects; in the points \((90^\circ, 90^\circ, r_{\text{side}})\) and \((0^\circ, 0^\circ, r_{\text{pole}})\), the angle of incidence, \(\delta\), is so close to \(90^\circ\) that the actual incident flux, which is proportional to \(\cos \delta\), becomes negligible, and hence \(\delta^* = 0\). For any point \(P(\theta_0, \phi_0, r_0)\) on the irradiated ellipsoid the incident and reference fluxes can then be integrated to determine the local value of \(\delta^*(\theta_0, \phi_0, r_0)\) in a way described below. Substituting these \(\delta^*\) values in the modified potential equations (4) and (5), an improved radial position \(P_i(\theta_i, \phi_i, r_i)\) follows by applying the Newton-Raphson method. In general, this procedure converges so rapidly that in many cases only one iteration step is necessary to determine the points \(P_i(\theta_i, \phi_i, r_i)\), \((i = 1, 2)\) on the radiation pressure-modified surfaces.

For the calculation of the incident flux, the ellipsoidal surface of the irradiating star is subdivided into a grid of differential elements using cylindrical coordinates \((x, \sigma, \varphi)\) related with rectangular coordinates through:

\[x = x; \quad y = \sigma \sin \varphi; \quad z = \sigma \cos \varphi;\]

the origin is at the center of the irradiating star, the \(x\)-axis is the connecting line between the mass centers, and \(\sigma\) is the cylindrical radius. A similar integration technique was used by Mochnacki & Doughty (1972a) and later also by Kitamura & Yamasaki (1984) in their detailed treatment of the reflexion effect in close binaries, as well as by Binnendijk (1977) in his dumbbell model for W UMa-type contact binaries. A good illustration of the geometrical definitions is, e.g., given in Fig. 1 of the paper of Kitamura & Yamasaki (1984).

The size \(dA\) of a surface element centered on the grid point \(Q(x, \sigma, \varphi)\) is

\[dA = \frac{\sigma \ dx dc}{m_1 \sin \varphi + n_1 \cos \varphi},\]

(8)

where \(m_1\) and \(n_1\) are the direction cosines of the surface normal in \(Q\) with respect to the \(y\)- and \(x\)-axes.

The differential flux \(dS\) emitted from the element \(dA\) and received by a unit area centered on the point \(P\) with polar coordinates \((\theta, \phi, r)\) on the irradiated surface can then be written as

\[dS(P) = I(\xi) \frac{\cos \xi \cos \eta}{r^2} dA,\]

(9)

\(\xi\) and \(\eta\) are the angles between the connecting line \(QP\) and the surface normals in \(Q\) and \(P\), respectively, and \(\rho\) denotes the separation of \(Q\) and \(P\). \(I(\xi)\) is the intensity of radiation emitted from \(Q\) in the direction of \(P\), with due regard to the local limb darkening:

\[I(\xi) = I(\xi = 0) (1 - u + u \cos \xi),\]

(10)

with the usual limb darkening coefficient \(u\). \(I(0)\) can be normalized to 1, because the calculation of \(\delta^*\) requires the use of flux ratios only.

The amount and direction of the total flux vector \(S\) of radiation incident in \(P\) is then obtained by an integration over all surface elements \(dA\), which are visible from \(P\):

\[S(P) = \int_{A} I(\xi) \frac{\cos \xi \cos \eta}{\rho^2} e \ dA,\]

(11)

i.e. all surface elements are evaluated for which at the same time \(\cos \xi > 0\) and \(\cos \eta > 0\); \(e\) is the unit vector in the direction of
$P$ for each of the different surface elements of the irradiating component producing a differential flux contribution to the total flux incident in $P$; the direction of the total flux vector is generally slightly different from the direction of the connecting line between $P$ and the mass center of the irradiating star, depending on the position of $P$ and the degree of deformation of the stars. However, it was shown (Drechsel et al. 1990; Gayler 1991) that the radiation pressure effects are described without a relevant loss of accuracy, if only the flux component in the direction of the connecting line between the mass center and $P$ is used.

2.2.2. Calculation of $\delta^*(\theta, \phi, r)$

At any given point $P$ within the irradiated surface area the gravitational force of the companion is reduced by the radiation pressure of this star to an effective force (per unit mass)

$$F_{\text{eff}}(P) = F_{\text{grav}}(P) - F_{\text{rad}}(P) = \left[1 - \delta^*(P)\right] \frac{GM}{r^2},$$

(12)

where $r$ is the distance of $P$ from the mass center of the irradiating star with mass $M$. The amount of reduction depends on the position of $P$, and is measured by the $\delta^*$ parameter, which is proportional to the actual flux $S(P)$ incident on a unit area fraction of the local stellar surface centered on $P$. $S(P)$ is normalized with a reference flux $\bar{S}(P)$ incident on a unit area around $P$ perpendicular to the direction of $\bar{S}$. Further, the integration of $\bar{S}(P)$ has to be carried out with disregard of any eclipse effects relevant for the integration of the actual flux $S(P)$. Such a normalization flux is chosen in order to relate the $\delta^*$ function on the irradiated star with the inner radiation pressure parameter $\delta$ of the irradiating component. The radiation pressure force $F_{\text{rad}}(P)$ due to the reference flux $\bar{S}(P)$ is acting on a unit area perpendicular to the direction of $\bar{S}$ and is therefore proportional to the $\delta$ parameter of the irradiating star:

$$F_{\text{rad}}(P) = \delta F_{\text{grav}}(P).$$

(13)

Combining Eqs. (12) and (13) gives

$$\delta^*(P) = \frac{F_{\text{rad}}(P)}{F_{\text{rad}}(P)} \delta = \frac{S(P)}{\bar{S}(P)} \delta,$$

(14)

hence we can derive $\delta^*$ for any point $P$ on the irradiated surface by an integration of the actual and reference fluxes. Since the reference flux integration can only be carried out for a predefined orientation of the irradiated surface (entering over the $\cos \eta$ factor in Eq. (11)) $\bar{S}(P)$ has to be determined in an iterative way: the surface normal is first assumed to point towards the mass center of the irradiating star, and the integration of $\bar{S}$ yields the direction of the corresponding total flux vector; as a next step the opposite direction of the flux vector is taken as improved orientation of the reference surface normal, and so on. This procedure generally converges fairly well after a few steps; it is stopped when the improved direction differs from the previous one by less than $1^\circ.5$, which guarantees sufficiently accurate $\delta^*$ values.

The $\delta^*$ parameters can now be inserted in Eqs. (4) and (5) to derive the modified Roche potentials. This way the radiation pressure effects can be handled with only two additional parameters $\delta_1$ and $\delta_2$, but with due regard to the actual geometric conditions. Keeping the number of free parameters as small as possible is important with respect to the already large number of parameters necessary to specify the physical properties and geometrical structure of close binaries in the frame of the classical Roche model. Any increase of the number of adjustable quantities leads to more severe numerical problems for the light curve analysis, and will affect the uniqueness and stability of the achieved solutions.

The $\delta$ parameters characterize the efficiency of the interaction between radiation and stellar matter. They depend on the energy distribution of the radiation field: an increased UV flux enhances both the inner radiation pressure as well as the interaction with the irradiated photosphere, as it is the main reason for the difference in wind mass loss rates. On the other hand, also the chemical and physical properties of the outer stellar atmospheres have an influence on the effective opacity and optical depth. This complex correlation, however, must not be considered here in full detail, because the optical depths in the photospheres will in any case be so large that the mean free path length of the photons incident from outside is of the same order as the photospheric scale height of the irradiated star. In the frame of our mechanical model, the momentum transfer can therefore be regarded as an "on the spot" event, i.e. the penetration depth is negligible compared with the stellar radius.

In the frequent case of very similar chemical and physical conditions of the atmospheres of both binary components, the normalization of $\delta^*_i$ ($i = 1$ or 2), which is used to describe the radiation pressure action on the irradiated star, with the inner radiation pressure parameter $\delta_i$ of the irradiating component is fully justified: $\delta_i$ and $\delta^*_i$ are correlated over the characteristic spectral energy distribution of the same star. On a microscopic scale, the efficiency of radiation pressure depends on the chemical and physical atmospheric characteristics over the local mean opacity or mean free path length of the incident photons. The radial net momentum transfer per photon is to a certain degree influenced by the relative importance of electron scattering compared with the bolometric absorption opacity. Except for very low densities like in the outer atmospheres of supergiants, the cumulative metallic absorption coefficient however dominates the total opacity, especially in the mid and far UV close to the maximum of the energy distribution of early-type stars. So even in the case of different chemical and physical parameters of the two stars, the net momentum transfer per unit bolometric flux in radial direction will not be much different, since the inner radiation pressure in the atmosphere of the irradiating star has a very similar efficiency as the radiation pressure exerted on the irradiated photosphere of the companion.

The shape of the irradiating star contains very little information about the strength of radiation pressure, because the surface potential $\Omega_\delta$ and $\delta_i$ are anticorrelated: a somewhat larger $\Omega_\delta$ value has — to a high degree — the same effect on the extent and shape of the radiating star like a slightly less efficient radiation...
pressure interaction (smaller $\delta_1$), and vice versa. This, however, means no complicating parameter correlation between $\delta_1$ and $\Omega_1$, since $\delta_1$ also has characteristic and uncorrelated effects on the shape of the irradiated star. The value of $\delta_1$ can therefore be determined in the numerical light curve solution procedure in an unambiguous way.

3. Effects of radiation pressure on binary structure

There are important implications of radiation pressure for O- and early B-type binaries in close configurations. Compared to the classical Roche model, two main differences are encountered:

- first, the stellar shapes are influenced by radiative interaction, and
- second, the system configuration is changed due to the shift of the positions of the Lagrangean points and the altered shapes and extents of the Roche lobes.

3.1. Deformation of stellar surfaces

The influence of radiation pressure on the irradiated component was already evident from Fig. 1. Depending on the value of $\delta_1$, the surface layers of the irradiated component are obviously deformed. To demonstrate this effect more clearly, model calculations were carried out for various values of $\delta_1$ and $\delta_2$. In the first few cases, $\delta_2$ was assumed to be zero, in order to illustrate the effects on the irradiated star without interference with the more complex bearings of mutual irradiation.

The left part of Fig. 2 shows a sequence of meridional intersections of modified Roche equipotentials for a model system with mass ratio $q = 1.0$ and $\delta_1$ values of 0, 0.15, 0.30, and 0.50. In all cases, the surface potential of the secondary was kept fixed at a value of 3.75, which corresponds to the potential of the inner critical Roche volume in the classical radiation pressure-free case (shown in the top part of Fig. 2). While $\delta_1$ is increased from 0 over 15 %, 30 %, up to 50 % (from top to bottom), the potential of the primary, $\Omega_1$, is always chosen such that this star fills its maximum possible volume. It is clearly seen how the Roche lobe shrinks under the influence of increased inner radiation pressure. This also implies that the position of the inner Lagrangean point $L_1$ gradually shifts toward the mass center of the primary, reeding from the companion. An increased radiation pressure efficiency of the primary causes a more pronounced deformation of the secondary at its facing side; such flattening also extends the distance between the two stellar surfaces.

A more realistic impression of the actual geometric conditions and the radiative deformation of the stellar surfaces is given by 3D simulations as shown on the right side of Fig. 2. Here we have depicted a sequence of aspects of a model system with mass ratio $q = 1.0$ and with the same parameters as those used for the meridional intersections. The system is viewed under an arbitrary angle of 65°, so that the poles of the stars become visible. Though $\delta_1$ values of 30 % or larger are probably unrealistic for even the hottest O-type systems, such large values were assumed here for a better illustration of the principal effects. Close binaries with compact components may, however, contain X-ray emitting sources with a sufficiently high luminosity to enter this parameter range.

Figure 3 shows another sequence for a model system with $q = 0.5$. Again, $\delta_1$ was set to zero, in order to avoid an interference with effects due to mutual irradiation. $\delta_1$ is increased from 0 over 10 %, 20 %, to 30 %. The surface potential of the primary was kept fixed at an arbitrary value of $\Omega_1 = 4.0$, corresponding to a star well inside its Roche lobe for all chosen $\delta_1$ values. The potential of the secondary, $\Omega_2$, was taken such that this component always fills its maximum possible volume, depending on the value of $\delta_1$. It can be seen that the increase of $\delta_1$ lets the Roche volume of the secondary expand up to a certain point, where the compression of the star at its irradiated facing side counteracts a further enlargement of its volume. Yet, the most noticeable effect of an increased $\delta_1$ is the switching from an inner contact configuration to an outer contact component at some critical value of $\delta_1$ between 10 % and 20 %. The configuration change happens when the potential value of the inner Lagrangean point $\Omega(L_1)$ as a function of $\delta_1$ becomes smaller than the constant value $\Omega(L_2) = 2.5773$ of the outer Lagrangean point; since $L_2$ is located within the shadow zone provided by the secondary star, its potential is independent of $\delta_1$. Again, the shrinking of the primary equipotential surface $\Omega_1 = 4.0$ with increasing $\delta_1$ is evident. The corresponding sequence of 3D aspects for the same parameter set is shown on the right side of Fig. 3, illustrating the geometrical deformation of the irradiated star.

Another sequence for a model system with mass ratio $q = 0.5$ is shown in Fig. 4. In this case, both radiation pressure parameters are simultaneously different from zero ($\delta_1 = 0.15$; $\delta_2 = 0, 0.15, 0.30, 0.50$). The primary potential $\Omega_1$ is fixed at a sub-contact value of 2.8, while $\Omega_2$ is always set to the minimum value allowing for a maximum extent of the secondary. With increasing $\delta_2$, the Lagrangean points $L_1$ and $L_2$ are displaced in opposite directions, squeezing the secondary into smaller and smaller Roche lobes. Again the secondary switches over from an inner to an outer contact component, this time caused by the increased $\delta_2$ value. The deformation of both stars due to the mutual irradiation is also evident.

3.2. Implications for close binary configurations

Radiation pressure not only influences the surface shapes of the binary components, but also affects the potentials and positions of the Lagrangean points. Given sufficiently large $\delta$ values, restrictions for possible system configurations will emerge, with particular consequences for contact or near-contact systems.

In the radiation pressure-free case, for any mass ratio $q \leq 1$ we have:

$$\Omega(L_1) > \Omega(L_2) \geq \Omega(L_3).$$

This is no longer true for the modified Roche model. Depending on the amount of $\delta_1$ and $\delta_2$, one has to distinguish several cases:
Fig. 2. Meridional intersections (left side) of a model system with $q = 1$ for various $\delta_1$ values ($\delta_2 = 0$); the radiation pressure-free case is shown on top. $\Omega_2 = 3.75$ in all cases, which corresponds to the inner critical Roche volume in the absence of radiation pressure; $\Omega_1$ is always at its minimum value to let the star fill its Roche lobe. The shrinking of the primary Roche lobe and the shifting of $L_1$ and $L_3$ are obvious. The 3D aspects on the right side give a better impression of the geometrical conditions and the deformation of the irradiated component.

Fig. 3. Same as Fig. 2 for a model system with mass ratio 0.5; $\delta_1$ was increased in slightly different steps, $\delta_2 = 0$. $\Omega_1$ is fixed at 4.0 for a star well inside its Roche lobe for all $\delta_1$ values; $\Omega_2$ was always assumed such that the secondary fills its maximum possible volume. An important consequence of increasing $\delta_1$ is the switching of the secondary from an inner to an outer contact configuration.
Fig. 4. Same as Fig. 2 for a system with $q = 0.5$, but with both $\delta$ parameters different from zero; $\Omega_t$ is fixed at a sub-contact value of 2.8, $\Omega_s$ is set to allow for the maximum extent of the secondary; the shift of the Lagrange points, the change of the contact configuration as well as the radiative deformation of both components are evident.

i): $\delta_{1,2}$ small:
$\Omega(L_1) > \Omega(L_2)$ and $\Omega(L_1) > \Omega(L_3)$

ii): $\delta_1$ large, $\delta_2$ small:
$\Omega(L_2) > \Omega(L_1) > \Omega(L_3)$

iii): $\delta_1$ small, $\delta_2$ large:
$\Omega(L_3) > \Omega(L_1) > \Omega(L_2)$

iv): $\delta_{1,2}$ large:
$\Omega(L_2) > \Omega(L_1)$ and $\Omega(L_3) > \Omega(L_1)$

The classical case ($\delta_{1,2} = 0$; marked by plus signs) and the cases of simultaneously small (e.g. $\delta_{1,2} = 0.1$; asterisks) and large (e.g. $\delta_{1,2} = 0.5$; crosses) radiation pressure parameters are contained in Figs. 5 and 6, where the effective potential is plotted as a function of $x$ (cartesian coordinate along connecting line of mass centers) for the arbitrary mass ratios $q = 1.0$ and 0.25, respectively. The above cases ii) and iii), where one star has a small and, at the same time, the other one a large $\delta$ value, are illustrated in Fig. 7 for an optional $q$ value of 0.8.

Case i) is similar to the radiation pressure-free case, and all classical system configurations can occur: if $\Omega_t$ and $\Omega_s$ are both larger than the inner critical value $\Omega_c$, the system is detached; semi-detached configurations are encountered, if $\Omega_1 = \Omega_c$ or $\Omega_2 = \Omega_c$; for contact systems, $\Omega_1 = \Omega_2 \leq \Omega_c$.

In case ii), the secondary can never reach an inner contact configuration, because $\Omega(L_2) > \Omega(L_1)$, i.e. the evolutionary expansion of this star will lead to an outer contact system, and matter will be lost from the binary via the outer Lagrange point $L_2$; however, mass transfer from the more massive primary to the secondary is possible like in the classical case.

Case iii) is the mirror image situation, where the primary would always evolve into outer contact before reaching the inner Lagrange point, preventing the onset of mass transfer via $L_1$. So in both cases ii) and iii), contact systems cannot be formed. Case iii) could be relevant for Wolf-Rayet (WR) binaries, in which the mass ratio was reversed during the first phase of rapid mass transfer from the originally more massive O-type star to its companion. After the loss of a major fraction of its ZAMS mass, the O star can become a very hot and luminous WR star with a correspondingly large $\delta$ value. The radiation pressure action on its companion would render it impossible that this star ever reaches inner contact, and hence the second mass transfer stage could not occur. Due to the reversal of the mass ratio in the first mass transfer phase, the denomination of primary and secondary components according to their larger and smaller masses has to be exchanged, and the above mentioned primary (WR star) is actually the less massive component, with a larger $\delta$ value than the more massive star. Similarly, the case of considerably different $\delta$ values for both components could also...
influence evolutionary processes of X-ray binaries and progenitor systems, in which luminous radiation sources are present.

Finally, case iv), which is illustrated in Figs. 5 and 6 as circles ($\delta_{1,2} = 0.3$) or crosses ($\delta_{1,2} = 0.5$) applies to systems, in which both $\delta$ values are large. Then $\Omega(L_1)$ is smaller than $\Omega(L_2)$ and $\Omega(L_3)$, and neither semi-detached nor contact systems are possible. Such systems will remain detached, because the evolution of both components will let them expand into outer contact and trigger mass loss of the binary system through $L_2$ or $L_3$.

The ($\delta_1, \delta_2$) pairs, for which transitions between the above mentioned cases will occur, cannot be derived analytically. They can only be approximated by numerical calculations for various values and combinations of $\delta_1$ and $\delta_2$. The problem also always depends on the mass ratio $q$, and the critical ($\delta_1, \delta_2$) pairs cannot be stated in a general way.

4. Light curve analysis code

Eclipse light curves are such complex functions of a large number of free parameters, that no analytical solutions exist. Numerical procedures have been in use since the early 1970s, e.g. the WD approach or Wood’s WINK code (Wood 1972), which have been applied most frequently due to their rather physical binary models. The generation of a synthetic light curve means calculation of light values $I(\phi)$ for a sufficient number of orbital phase angles $\phi$. To determine such light values, essentially three different steps have to be taken:

- the 3-dimensional shape of the stellar photospheres has to be specified;
- line-of-sight fluxes emerging from surface elements of both stars visible at a certain phase angle under a certain aspect angle must be calculated: this is done by subdividing the stellar surfaces into a grid of finite elements and computing the flux of each with proper account of the local physical conditions, e.g. temperature and surface brightness under the influence of gravity darkening, limb darkening, and reflection effect;
- the local fluxes have to be integrated to yield the total flux in the line-of-sight direction with due regard of orbital inclination, phase angle, and eclipse effects.

Including the radiation pressure parameters $\delta_1$ and $\delta_2$, the WD model requires no less than 17 independent parameters for the description of a single monochromatic light curve. Since
the luminosities of the binary components and of possible third light sources as well as the limb darkening coefficients are wavelength-dependent, the total number of free parameters is increased by 5 for each additional simultaneously analyzed light curve, e.g. the adjustment of a set of UBV light curves requires the specification of 27 independent quantities characterizing the geometrical and physical properties of a given system. The general logistics and a successful concept of numerical light curve analysis were described by Wilson & Devinney (1971 and subsequent papers).

We adopted the WD approach as principal light curve analysis concept for our problem. However, the geometrical structure of the binary system is no longer represented by the classical Roche model, but is now based on radiation pressure-modified Roche equipotentials, i.e. the 3-dimensional shape of the stellar surfaces is numerically computed taking into account the radiative effects. The efficiency of the radiative forces is for each component characterized by its $\delta$ parameter, and radiative deformation of the stars with immediate consequences for the eclipse light changes can be accounted for. Since the modified potential has no analytical representation, the potential gradient at the stellar surfaces also needs to be determined by a numerical procedure, which will be described in Sect. 4.1. The knowledge of the local potential gradients is necessary for the computation of gravity darkening. The problem of determining the geometrical and physical properties of a binary system from the shape of its eclipse light curve is highly non-linear, and an efficient mathematical optimization procedure is required, which iteratively modifies the adjustable quantities in a way to minimize the residuals between the observed and calculated light curves. For the method used here see Sect. 4.2.

4.1. Gravity darkening in the radiation pressure-modified case

The surface brightness distribution of the components of close binary systems depends on the mutual irradiation of the two stars (reflexion effect) and on the variation of the local surface gravity, and is hence more complicated than in the rotationally symmetric case of rotating single stars. While the reflexion effect is taken into account like in the original WD approach, i.e. over the albedo-dependent local heating of surface elements due to the irradiation by the companion, gravity darkening has to be treated in a different way: the ratio of the local bolometric flux relative to the flux emergent from the pole, which is used as normalization point, is proportional to the gradient of the local surface potential to the power $g \cdot (\nabla \Omega_{loc})^2$, with the usual gravity darkening exponent $g$. While the partial derivatives of the potential with respect to the position coordinates $\partial \Omega / \partial x$, $\partial \Omega / \partial y$, $\partial \Omega / \partial z$ — are obtained analytically in the classical Roche model, this is no longer possible in the radiation
Fig. 7. Same as Fig. 5 for the mass ratio \( q = 0.8 \). Two combinations of strongly different values for \( \delta_1 \) and \( \delta_2 \) were assumed: \( \delta_1 = 0.4, \delta_2 = 0.01 \) (plus signs) and \( \delta_1 = 0.01, \delta_2 = 0.4 \) (circles).

pressure-modified case for those surface areas, which are irradiated by the companion. In these regions the radiation pressure parameter \( \delta^* \), which enters the expression for the local potential (Eqs. (4) and (5)), varies with position and has to be determined numerically. The value of \( \delta^* \) is not only position-dependent over the surface, but also variable along the potential gradient vector, and again there is no analytical representation of this effect. A numerical procedure has to be applied to derive the direction and amount of the local potential gradient vector.

By definition, the gradient vector \( \nabla \Omega \) is perpendicular to the equipotential surface and points in the direction of steepest potential increase. Hence, the unit gradient vector \( \hat{\mathbf{g}} \) is given by the local surface normal pointing inside the star, and can be readily obtained as follows: for each point \( P(\theta, \phi, r) \) on the surface two other points \( P_1(\theta_1, \phi, r_1) \) and \( P_2(\theta, \phi_2, r_2) \) with infinitesimal distance from \( P \) are chosen to construct a surface element (triangle \( PP_1P_2 \)) as local approximation of the stellar surface. Its normal vector \( \hat{\mathbf{g}} \) follows from the cross product of the vectors \( \mathbf{I}_1 = PP_1 \) and \( \mathbf{I}_2 = PP_2 \):

\[
\hat{\mathbf{g}} = \frac{\mathbf{I}_2 \times \mathbf{I}_1}{|\mathbf{I}_2 \times \mathbf{I}_1|}.
\]

With known \( \hat{\mathbf{g}} \) the amount of \( \nabla \Omega \) can be found. For this purpose, an auxiliary surface with infinitesimally higher potential \( \Omega' \) than that of the stellar surface \( \Omega \) is introduced. Using an iterative procedure to find the point \( D \) of intersection of the line defined by \( \hat{\mathbf{g}} \) with this auxiliary surface allows to determine the amount of the potential gradient:

\[
|\nabla \Omega| = \frac{\Omega' - \Omega}{|DP|}.
\]

Possible locations of the intersection point \( D \) are restricted to the intersecting line of the auxiliary surface and the plane defined by \( \hat{\mathbf{g}} \) and the line connecting \( P \) with the mass center, because for given angles \( \theta \) and \( \phi \), the surface points are localized in radial direction applying the Newton-Raphson iteration scheme. Further details of the search algorithm for the locus of the intersection point \( D \) are omitted here for the sake of clarity. A complete description of the iteration method is given by Haas (1993). It should only be noted that a comparison of numerically determined potential gradients with analytical gradients on the shadow side show that a relative accuracy better than \( 10^{-3} \) % is achieved after only few iterations. The numerically determined potential gradients are now used instead of the analytical gradients of the classical Roche model to calculate the local gravity darkening in an analogous way as described by WD (1971).

4.2. Parameter optimization

On the basis of the modified Roche potential, synthetic light curves can now be calculated as a function of the usual parameter set complemented with the radiation pressure parameters. This so-called direct problem is solved in a more or less
straightforward way. However, we are mainly interested in the inverse problem, namely in deriving the characteristic amount of radiation pressure together with all other unknown quantities from observations. This implies that one has to adjust \( b_1 \) and \( b_2 \) as part of the sample of free parameters, and to optimize the theoretical light curve in a way to fit best the observed one. The original WD approach uses differential corrections for this purpose, which essentially means a linearized least squares method. This is a feasible and proved technique, if the number of simultaneous free parameters is not too large, and convergence problems due to parameter correlations can be alleviated, e.g. by applying the method of multiple subsets (see Wilson & Biermann 1976).

4.2.1. Simplex method

Additional free parameters normally affect the numerical stability and add to the well-known difficulties connected with the convergence behavior and uniqueness of photometric solutions. Kallrath & Linnell (1987) had first combined the WD approach with an alternate parameter optimization procedure, the simplex algorithm, which has various advantages compared with the differential corrections method. The most important ones are:

1. that it cannot diverge,
2. that it does not require the specification of partial derivatives \( \partial l / \partial p_i \) of the light function \( l \) with respect to the free parameters \( p_i \), and
3. its numerical stability is sufficiently good even in the case of a large number of simultaneous free and partially correlated parameters \( p_i \).

For example, third light can cause convergence problems due to its correlation with the luminosity of the primary component. A systematic comparison and discussion of the application of the differential corrections method and the simplex algorithm — both combined with the WD approach — was, e.g. given by Drechsel et al. (1989) for the third light-affected systems LY Aur and AH Cep. Once the parameter set was not too remote from the global minimum in the multi-dimensional parameter space, both optimization procedures yielded essentially identical results; when using differential corrections, this requires the knowledge of a start parameter set, which is already sufficiently close to the final solution, otherwise the numerical procedure either diverges or ends up in a local minimum different from and less deep than the global one. In principle, such difficulties can be avoided by a proper handling of the simplex algorithm, which can systematically search for the best solution in a broad section of the parameter space without encountering numerical problems arising from the increased number of free parameters. It was therefore preferred to apply this non-linear parameter optimization method instead of differential corrections for the solution of light curves based on an improved binary model including radiation pressure effects.

The simplex is a polyhedron with \((m+1)\) corner points (so-called vertices) in the \(m\)-dimensional parameter space, where \(m\) denotes the number of free parameters. Each vertex defines a set of adjustable parameters, which are used together with the fixed parameters to generate a synthetic light curve. The standard deviation of the observations with respect to this calculated light curve is taken as a criterion to decide upon the relative fit quality of the \((m+1)\) simplex corner points. At each iteration step, the worst vertex is discarded and replaced by a better one, which is constructed with predefined geometrical operations (reflection, contraction, expansion, or shrinkage of the simplex; see Nelder & Mead 1965); if no better corner point can be found, the whole simplex will be shifted in the direction of the best vertex. This way the simplex approaches the final minimum in the parameter space and contracts while changing its volume and shape from step to step. The procedure cannot diverge, and the fit quality improves monotonously with the number of iterations. For a full description of this direct search algorithm, which does not require the calculation of any partial derivatives, the reader is referred to Kallrath & Linnell (1987) and Lorenz (1988), who also provides experiences and instructions for its practical use.

4.2.2. Implementation of radiation pressure

The implementation of radiation pressure effects in the light curve analysis code presupposes the replacement of the classical Roche potential by the modified potential field described in Sect. 2.

For this purpose, a program was developed which calculates the grid points and local potential gradients of the surfaces of the binary components for arbitrary values of \( b_1 \) and \( b_2 \). This subroutine replaces the SURFAS module in the original WD code. The new radiation pressure-modified version is restricted to detached and semi-detached systems, if \( b_1 \) or \( b_2 > 0 \). Since the new routine consumes about four times as much CPU time than the original one, an adapted version of the original SURFAS subroutine is employed in any radiation pressure-free case and for over-contact systems.

To allow for an adjustment of the \( \delta \) parameters in addition to the usual sample of free parameters, the simplex parameter optimization procedure as described by Kallrath & Linnell was changed accordingly. The subroutines concerned with the parameter adjustment as well as various I/O routines were rearranged. Further modifications were made in the CSTRAIN subroutine, which guarantees that the adjustable quantities remain within user-defined and physically relevant limits during the automatic iteration procedure. Besides the values of \( \delta_1 \) and \( \delta_2 \), the ratio \( \delta_1 / \delta_2 \) can be constrained to a physically reasonable range. CSTRAIN also prevents that the values \( \Omega_1(\delta_1, \delta_2) \) and \( \Omega_2(\delta_1, \delta_2) \) of the modified potential ever fall below the inner critical value \( \Omega_{\text{crit}} \); if \( \Omega(L_2) \) or \( \Omega(L_1) < \Omega_{\text{crit}} \), CSTRAIN keeps \( \Omega_1 \geq \Omega(L_1) \) and \( \Omega_2 \geq \Omega(L_2) \). Further details of the modifications of the source code can be found in Haas (1993).

The CPU time consumption of the program is about four times bigger than in the radiation pressure-free case. This is mainly due to the calls of the subroutine, which calculates the
actual and reference fluxes incident at each surface element by an integration over the visible parts of the surface of the irradiating star. The CPU requirements therefore depend on the grid fineness. Typically a few hundred surface points are adequate for the grid of the irradiated component, while a somewhat coarser grid is sufficient for the representation of the irradiating star. For our first applications, 154 elements were adopted for the irradiating component, which is probably close to the lower limit, but still yields reasonably accurate values of the incident flux. A typical run of 200 simplex iterations with 13 free parameters applied to a set of three simultaneously analyzed light curves consisting of about 70 normal points each requires about 15 CPU minutes on a DEC3000 model 300 ALPHA machine under the VMS operating system.

5. Application to IU Aurigae and AB Crucis

The new numerical light curve solution code was first applied to the late O-type eclipsing binaries IU Aur and AB Cru.

IU Aur was chosen as a first test case for two reasons: it is a well studied system, for which reasonably good light curves exist; also a reliable set of initial system parameters was available as a starting point for the radiation pressure code; on the other hand IU Aur is an important system, because it is one of very few early-type triple systems, for which orbital elements and absolute dimensions can be derived.

AB Cru belongs to the small group of binaries earlier than O9 (the review of Hilditch & Bell (1987) lists only three such systems; in the General Catalogue of Variable Stars (Kholopov 1985) only eight systems of such early type are found). The application of the light curve solution program to new photometric observations allowed for the first precise determination of the system parameters of AB Cru. Combined with our spectroscopic results, absolute masses, radii, and luminosities could be derived, which are of particular importance for the stellar evolution theory of massive and hot OB stars.

As an illustration of the radiation pressure-modified code, selected light curve solutions for both binaries are presented in Sects. 5.1 and 5.2. Detailed results of these studies can be found in two separate papers: Drechsel et al. (1994) for IU Aur, and Lorenz et al. (1994) for AB Cru, respectively.

5.1. IU Aurigae

IU Aur (HD 35652) is a semi-detached eclipsing binary of type O9.5-B0 Vp in a common orbit with a massive third component of about $17 - 18M_\odot$, which is probably not a single star, but a binary itself — as suggested by its relatively low luminosity. The triple system nature is proven by a well defined light time effect, by a third light contribution of about 20%, by the temporal variation of the eclipse minimum depth due to the precessional motion of the binary orbit as well as by direct spectroscopic evidence of the third component. All of these effects have been discussed in detail by Drechsel et al. (1994), Lorenz et al. (1993), Mayer & Drechsel (1987), and references therein. Here we only present the solution of the UBV light curves obtained at the mean epoch 1973.1 (see Mayer (1976) for a description of the measurements), which had served as a first test case for our new program.

The individual UBV measurements were transformed into 46 normal points in each color. Typically 3-5 points in the centers of the eclipse minima and a few points close to quadrature phases were weighted higher than average due to the special importance of these phase ranges. A suitable set of start parameters was available from the earlier investigation of Mayer & Drechsel (1987), who had applied the original WD method to these data.

The mass ratio $q$ plays a decisive role for the photometric solution: frequently numerical solutions turn out to be non-unique in the sense that different parameter combinations can yield equally good and nearly indistinguishable fits. Only an independent determination of $q$ makes it possible to sort out the global minimum in the parameter space, and helps to discern between the physically correct solution and other artifacts. In this case, a spectroscopic mass ratio was available, which was derived from recent Coudé/CCD measurements of IU Aur (Drechsel et al. 1994). Nevertheless, in the photometric solution $q$ was a free parameter; the adjusted value is very nearly the same as the spectroscopic one, which supports the reliability of the new code.

The gravity darkening exponents and bolometric albedos were fixed at their theoretical values for radiative envelopes, and theoretical limb darkening coefficients were assumed. The effective temperature of the primary was chosen according to its spectral type. The binary configuration was not restricted to the semi-detached case; the final solution could also have converged at a detached or contact configuration. A compilation of all fixed parameters is found in Table 1.

The UBV light curves were simultaneously analyzed, and a total of 13 free parameters had to be adjusted: inclination $i$, mass ratio $q$, surface potentials $\Omega_1$ and $\Omega_2$, effective secondary temperature $T_2$, primary luminosity $L_1(U, B, V)$, third light $L_3(U, B, V)$, and radiation pressure parameters $\xi_1$ and $\xi_2$. In view of the important implications of radiative effects, special attention was paid to the determination of the $\delta$ parameters. Hence we started the iterations with a variety of trial values for $\delta_1$ and $\delta_2$ of up to 5%, including the radiation pressure-free case of $\delta_1 = \delta_2 = 0$. Due to the later spectral type of the secondary, $\delta_2$ tended to very small values, but $\delta_1$ always converged at non-zero level. Further details about the adjustment procedure may be looked up in the paper of Drechsel et al. (1994), which deals with the solution of IU Aur light curves from four different epochs between 1964 and 1984. Absolute dimensions could be derived from recently attained spectroscopic observations. The main aim of this first application of the radiation pressure code was the precise determination of the system parameters as well as a verification of the time variability of the inclination. The temporal variation of the orbital inclination together with the spectroscopic information made it possible to investigate dynamical properties of the three body system and to extract an estimate of the third body mass.
As an example for the achieved light curve solutions we show here the representation of the UBV curves measured around the mean epoch 1973.1 in Fig. 8. The light curves are given in intensity units and are normalized to 1 at their maximum values at quadrature phase. The weighted standard deviation $\sigma$ for the set of UBV measurements amounts to about 0.004. No systematic effects are apparent, and all three light curves are represented equally well.

Especially the eclipse minima are fitted with literally no deviations. A minor distortion of the observed curves between phases 0.8 and 0.9 is ascribed to a possible depression due to circumstellar extinction in a stream of matter leaving the Roche lobe filling secondary star (see Drechsel et al. 1994). Such effects cannot be easily accounted for by conventional light curve modelling methods. Since the distortion is of minor nature and limited to a relatively narrow phase interval, its impact on the photometric solution is certainly not important.

Table 1 lists the fixed and adjusted system parameters of IU Aur as derived from the 1973 light curves. The adjusted parameters come out practically identical in the solutions for the four different epochs 1964.3, 1973.1, 1974.9, and 1984.4, except for the time-variable inclination. Though the radiation pressure parameters of this moderately hot B0 system merely exceed the 1% level, the adjustment procedure always converged at small, but definitely non-zero values, and this first application of the radiation pressure code yielded an improved theoretical representation of the observed light curves with unprecedented fit quality.

Figure 9 shows the residuals between the best fitting $V$ light curve and the corresponding normal points. The arithmetic mean of all residuals is very close to zero ($-8.7 \times 10^{-4}$ intensity units), and indicated by a horizontal line. For the $U$ and $B$ curves, the mean residual values are even smaller ($-3.7 \times 10^{-4}$ and $-4.2 \times 10^{-4}$, respectively). The weighted standard deviations for all three curves are very nearly the same (about 0.004 intensity units). The dashed and dash-dotted lines give the $1\sigma$ and $3\sigma$ belts around the mean residual value.

A graphic description of the system structure and configuration is given in Fig. 10, showing a 3D-simulated aspect of the semi-detached binary for the actually derived system parameters, viewed under an arbitrary angle of 60° relative to the orbital plane.

5.2. AB Crucis

AB Cru (HD 106871) is one of very few relatively "clean" O-type systems with only minor light and velocity curve distortions due to proximity effects. Hence it appeared as a promising
Table 1. Photometric solution of the IU Aur UBV light curves of epoch 1973.

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th>Adjusted parameters</th>
<th>Roche radii $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>effective temperature $T_1$</td>
<td>32,000 K</td>
<td>$T_2$ 28,160 K (-120)</td>
</tr>
<tr>
<td>gravity darkening $g_1$</td>
<td>1.0</td>
<td>$i$ 84.0 (-8.0)</td>
</tr>
<tr>
<td>exponents $^b$</td>
<td></td>
<td>$g_2$ 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mass ratio $(M_2/M_1)$</td>
</tr>
<tr>
<td>bolometric albedos $^b$</td>
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<td>surface potentials $^c$</td>
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<td>$\Omega_1$ 3.480 (+0.011)</td>
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<tr>
<td>$A_2$</td>
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<tr>
<td>limb darkening</td>
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<td>critical potential $^c$</td>
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<tr>
<td>coefficients $^e$</td>
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<td>luminosity $^d$</td>
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<td>$x_1(U)$</td>
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<td>$L_1(B)$ 0.586 (+0.004)</td>
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<td>$L_1(V)$ 0.580 (+0.004)</td>
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<td>0.20</td>
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<td>$x_2(U)$</td>
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<td>$L_2(B)$ 0.213 (+0.007)</td>
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<td>$L_2(V)$ 0.219 (+0.005)</td>
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<tr>
<td>$x_2(V)$</td>
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<td>radiation pressure $^f$</td>
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<td>parameters $^f$</td>
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</table>

$^a$ fractional Roche radii in units of separation of mass centers;

$^b$ theoretical value for radiative envelope;

$^c$ dimensionless equipotentials in the notation of Kopal (1959);

$^d$ percentage values given as $L_1/(L_1 + L_2)$;

$^e$ theoretical values according to Eaton (1978);

$^f$ percentage values given as $L_2/(L_1 + L_2 + L_3)$.

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![Fig. 9. Residuals between the best fitting V light curve of IU Aur as shown in Fig. 8 and normal points of epoch 1973.1. No systematic trend is evident, and the mean residual value is very small ($-8.7 \cdot 10^{-5}$); 1σ and 3σ belts are shown](image-url)
case for the determination of reliable fundamental parameters of early-type stars. The early spectral type (O8 Vn according to Garrison et al. 1977) and its close semi-detached configuration obviously suggest that radiation pressure effects might be relevant. The first photoelectric light curves of the system since its discovery by Oosterhoff (1933) together with our ECHELLEC spectra obtained at ESO-La Silla in the years 1992 and 1993 could be used to derive absolute dimensions; the detailed analysis is published elsewhere (Lorenz et al. 1994). Here we present the photoelectric solution in order to demonstrate the feasibility of our new analysis code for a system with relatively strong irradiation effects.

A description of the UBV photometric observations is given by Lorenz et al. (1994). The normal points are shown as dots in Fig. 11. Five points around the primary minimum and four points in the secondary minimum were weighted between three and five times higher than the rest of the light curves to emphasize the importance of these phase ranges for the solution.

A spectroscopic mass ratio was available from our recent phase-resolved measurements with the ECHELLEC instrument of the ESO 1.52m telescope, which are also described in the Lorenz et al. paper. The primary effective temperature was fixed according to the spectral type classification. Gravity darkening exponents and bolometric albedos for both components were assumed as 1.0 for radiative envelopes. Theoretical limb darkening coefficients were taken from Wade & Rucinski (1985).

There is no indication for third light, neither by a physical third body in the system, nor by any field star or nebulousness. Since the third light contributions \(l_3(U, B, V)\) always tended towards zero when adjusted as free parameters, these quantities were fixed at zero for all further runs. The remaining set of 10 free parameters was adjusted using the radiation pressure code combined with the simplex optimization algorithm. As in the case of IU Aur, no restriction for the system configuration was presumed. After a few trial runs for each individual light curve, the preliminary system parameters were taken as start parameter set for the simultaneous fit of the UBV sample of light curves. These first trial solutions were started with several combinations of \(\delta_1\) and \(\delta_2\) values, including the radiation pressure-free case \(\delta_1 = \delta_2 = 0\). In all cases, the iterations converged at non-zero \(\delta\) values, lending support to our physical and numerical treatment of the radiative effects.

The final solution achieved after a few thousand iterations in several runs carried out with a variety of different start simplices and increments is given in Fig. 11. The general agreement between observed and calculated light curves is very satisfactory. The fit quality of the three curves is similarly good: the relatively poorest agreement with a weighted standard deviation of \(\sigma = 8.50 \times 10^{-3}\) is found for the U curve, which has the highest intrinsic scattering, while the corresponding values for the B and V curves are \(\sigma = 6.80 \times 10^{-3}\) and \(6.94 \times 10^{-3}\), respectively. The mean residual values are very small: \(5.8 \times 10^{-4}\) for U, \(-3.3 \times 10^{-4}\) for B, and \(-5.2 \times 10^{-4}\) for V. As an example, the V residuals are shown in Fig. 12 with 1\(\sigma\) and 3\(\sigma\) belts. The largest residuals are obviously due to some irregularities in the observed curves, but generally no systematic trend is evident, except for a similar depression of the observed light around orbital phase 0.9, which cannot be accounted for by the synthetic curves. Since the system configuration and evolutionary state with a Roche lobe filling secondary is comparable to IU Aur, it is tempting to explain this effect also by mass exchange. Further support for this assumption comes from the orbital period change discussed by Lorenz et al. (1994).

Table 2 is a compilation of the fixed and adjusted system parameters of AB Cru. The Roche radii \(r_{\text{pole}}, r_{\text{point}}, r_{\text{side}}\), and \(r_{\text{back}}\) (in units of the mass center separation) correspond to the adjusted mass ratio and surface potentials of both components. The fact that the photometric mass ratio \(q\) converges very close to the spectroscopic value can be considered as an important clue for the uniqueness of the solution. The radiation pressure parameters have adjusted values of about 4 \% and 1.5 \% for the primary and secondary components, and are considerably larger than those of IU Aur, apparently due to the earlier spectral type of AB Cru. The surface potential \(\Omega_2\) of the secondary converges at a value, which is only 0.14 \% above the inner critical limit for the mass ratio and radiation pressure parameters of the solution, though it was not constrained to match the semi-detached value.

The 3D simulation of the AB Cru system shown in Fig. 13 illustrates the remarkable similarity of the configuration compared with IU Aur. The main difference is the relative size of the secondary, which is smaller in AB Cru (mass ratio 0.37 compared to 0.69 for IU Aur). In absolute units, however, the AB Cru components are of course larger. The absolute dimensions of both systems have been discussed in the above cited papers.

6. Discussion and conclusions

The radiative interaction between the components of close binaries can have significant influence on the stellar shapes and system configuration. Such effects are probably not important for binaries of wide separation and for the majority of inter-

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th>Adjusted parameters</th>
<th>Roche radii $^a$</th>
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<tr>
<td>effective temperature $T_1$</td>
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</table>

$^a$ fractional Roche radii in units of separation of mass centers;
$^b$ theoretical value for radiative envelope;
$^c$ dimensionless equipotentials in the notation of Kopal (1959);
$^d$ theoretical values according to Wade & Rucinski (1985);
$^e$ percentage values given as $L_1/(L_1 + L_2)$;
mediate to late-type objects. However, the ratio of radiative to gravitational forces rapidly increases with $T_\text{eff}$. For hot and luminous stars the inclusion of radiation pressure terms in the total effective potential therefore leads to considerable deviations from the conventionally assumed Roche geometry. Depending on the strength of the radiation field and the relative efficiency of the radiation pressure action in the irradiated photospheres, several effects relevant for the geometrical shape of the stars and the binary configurations are apparent:

- The overall geometry of the equipotential surfaces in the vicinity of the mass centers and in the circumstellar region is changed;
- the positions of the Lagrangean saddle points $L_1$, $L_2$ and $L_3$ are shifted;
- the existence of a critical inner contact surface is no longer warranted; above a certain critical limit of the radiation pressure parameters one of the components may switch from an inner to an outer contact configuration;
- radiation pressure counterbalances the tendency to take up inner contact to a certain degree in the course of the binary evolution, but on the other hand can trigger outer contact configurations, if one or both $\delta$ values should gain large values, so that the potential of $L_1$ becomes smaller than that of an outer Lagrangean point — a situation, which is never encountered in the frame of the classical, radiation pressure-free Roche model;
- in such outer contact systems with strong radiation fields mass loss through an outer Lagrangean point will be enhanced and obviously influences evolutionary processes; especially systems with very luminous components like Wolf-Rayet or X-ray binaries can be affected;
- due to the altered shape of the stars and modified system configurations eclipse light curves exhibit radiation pressure-dependent effects; if the radiative deformation is taken into account, somewhat different orbital elements and stellar parameters will result;
- finally, the hydrodynamics of mass transfer and circumstellar mass flows are governed by a modified potential field.

We have developed a numerical representation of the equipotential field of binaries with non-negligible radiation effects by means of a parametrized radiative force. A correspondingly modified Roche potential was taken as basis for an improved binary model, which accounts for the essential consequences of the mutual irradiation for the geometrical structure of the binary. We have demonstrated the systematic effects on
the shape of the stars and on the system configurations as the radiation pressure parameters are increased.

The numerical calculation of eclipse light curves also fully accounts for these effects. The radiation pressure parameters were included in the sample of adjustable quantities to yield more realistic photometric solutions for close early-type binaries. First applications of the new solution code to the OB systems IU Aur and AB Cru are very promising and have shown that improved solutions are possible even in the case of very small radiation pressure parameters (cf. IU Aur).

There are a couple of early-type systems with strong interaction effects, for which no satisfactory light curve solutions could be achieved so far by any of the conventional methods. The verification of the existence of outer contact configurations could possibly lead to an improved representation of the observed light curves. At the same time a more conclusive explanation would be at hand for the systematically larger mass loss rates of many early-type binaries compared with single stars of comparable spectral type (de Loore 1981; Garmann et al. 1981). Also large rates of period decrease in combination with expanding circumstellar envelopes like in the well known case of SV Cen (Drechsel et al. 1982; Herczig & Drechsel 1985; Drechsel & Lorenz 1993) could easily be explained by means of mass loss through an outer Lagrangean point triggered by radiative processes.

In the future we will apply the new analysis program to hot O-type systems with possibly important radiative effects in order to derive more realistic solutions and improved absolute dimensions, which will be crucial for a comparison with the latest revisions of the stellar evolution theory for massive stars.

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