Quantum Effects in Atomic Interferometry

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Abstract
Mechanical forces generated by the interaction between an atomic beam and a transverse optical field can be used to make sensitive measurements of certain system variables. We present the results of a numerical simulation which illustrate that the solution of the generalised Bloch equations correctly predicts the structure of experimentally observed deflection profiles, allowing the transition between diffractive and diffusive regimes to be understood. We show that in the diffractive regime it is possible to use the atomic probe to make a sensitive Quantum Nondemolition measurement of the field intensity of a standing wave mode.

1 Introduction
The study of the forces on atoms by light has been the subject of increasing theoretical and experimental research [1, 2, 3]. An experiment basic to our understanding of these forces has been realized by Gould et al. [4] In this experiment they measured the momentum distribution of a highly collimated monoenergetic beam of sodium atoms scattered by a plane standing light wave. As an atom passes through the light field momentum exchanges occur between the light field and the atom in multiples of the photon momentum. The dipole force on the atom results from the momentum exchanged by absorption and stimulated emission of photon pairs. Thus the projection of this momentum along the laser axis is quantized in integral multiples of $2\hbar k$.

In the absence of spontaneous emission, that is for sufficiently detuned atoms, the output in the far field of the scattered atoms consists of several sharply defined peaks. This distribution may be viewed as atomic diffraction of the atoms due to scattering of the atoms from a phase grating formed by the light intensity varying with spatial period $\lambda/2$. As one tunes closer to resonance, the spontaneous emission becomes important. The recoil imparted to the atom by a spontaneously emitted photon occurs in a random direction so that its momentum component in the direction of the standing wave can range from $-\hbar k$ to $+\hbar k$ [5]. Thus spontaneous emission causes
the diffractive peaks to be smeared out. This can be viewed as a loss of coherence between parts of the atom wave which scatter from the in-phase parts of the grating formed by the light field. Whereas previous experiments by Pritchard and coworkers were restricted to the diffractive region [6, 7], the recent experiment has been able to probe the transition from the diffractive to the diffusive regime. A general treatment capable of treating the transition region has been formulated by Tanguy et al. [8] who derived a set of generalized Bloch equations valid in the Raman-Nath regime. To date computational complexities in solving these equations have prevented direct comparison with experiment.

This paper is organised as follows. In the second section, we present a solution of the generalized Bloch equations which enables us to compare the theory with the experimental results of Gould et al. in all regions. The method developed is computationally efficient, and allows the action of the optical field to be characterized by a relatively small set of numbers which may then be used to determine its effects on an initial atomic state with arbitrary angular distribution of spontaneous emission.

In the third section, we use the mechanical atom-photon interaction to propose a Quantum Nondemolition measurement scheme in the diffractive regime which allows us to monitor the standing field in a high quality cavity. We show that in this scheme, the atomic position measurements at some distance from the interaction region carry information about the intensity of the field, and that if the cavity does not relax in the time between adjacent atoms, the state of the field is contracted into an ideal photon number.

2 Generalized Bloch Equations in the Diffractive & Diffusive regimes

Consider a beam of two-level atoms in their ground state which crosses a transverse optical standing wave. We wish to determine the deflection of the atoms as they traverse the light field. We denote the detuning between the atomic and optical frequencies by \( \delta \omega \) and let \( F(x,t) = \beta_0 \cos kx \cos \omega t \) denote the standing-wave electric field encountered in the frame of the moving atoms. This leads to a Rabi frequency \( \Omega = \mu E_0 / h \) where \( \mu \) is the atomic dipole moment of the transition. The spontaneous emission of the atoms is described by \( \gamma^{-1} \), the radiative lifetime of the excited state.

We specify the total atomic state by the density matrix \( \rho \), and define \( \xi = kx \) as a dimensionless position variable. The equations of motion of the density matrix are called the generalized optical Bloch equations [8], and in the Raman-Nath regime have the form

\[
\frac{\partial \rho}{\partial t} = \mathcal{L}(\xi, \xi') \rho
\]
where $\mathcal{L}(\xi, \xi')$ is the matrix

$$
\begin{pmatrix}
-\Gamma' & i\cos\xi' & -i\cos\xi & 0 \\
 i\cos\xi' & -(\frac{\Gamma}{2} + 2i\Delta) & 0 & -i\cos\xi \\
-i\cos\xi & 0 & -(\frac{\Gamma}{2} - 2i\Delta) & i\cos\xi' \\
\Gamma\chi(\xi - \xi') & -i\cos\xi & i\cos\xi' & 0
\end{pmatrix}
$$

(2)

Here $\tau = \Omega t$, $\Delta = \delta\omega/(2\Omega)$ and $\Gamma' = \gamma/\Omega$. The term $\Gamma\chi$ describes the transfer of atoms from the excited state to the ground state by spontaneous emission. The experimental results of Gould et al. are for a monoenergetic, highly collimated atomic sodium beam crossing a circularly polarized standing wave laser field for which

$$
\chi(u) = \int d^2n \phi(n) \exp(-i\mathbf{k} \cdot \mathbf{u})
$$

where

$$
\phi(n) = \frac{3}{16\pi} (1 + \cos^2 \theta)
$$

(3)

is the distribution for spontaneous emission at angle $\theta$. The generalized Bloch equations form an initial-value problem which can be solved using a method similar to that employed by Tanguy et al. [8] for finding the propagator of the Wigner function in the intermediate regime between the diffractive and diffusive regimes. This generates coefficients which completely characterize the action of the light field on the atoms, to which arbitrary initial state and arbitrary spontaneous emission pattern may be applied using a sequence of convolutions. Note that atoms which leave the field in the excited state will spontaneously emit on their way to the detector, so allowance must be made for the change in momentum due to recoil.

The experimental parameters are shown in Table I for three scans, where $t$ is the time taken for the atoms to travel between the $1/e^2$ intensity points of the light field, and $N$ represents the average number of spontaneous emissions during the interaction time. In the simulations, there are three adjustable variables $\Delta$, $\Gamma$ and $\tau$ which correspond to the normalized detuning, damping and interaction time respectively. The Rabi frequency was set to take account of the effective interaction time. The input beam was taken to have a full-width at half maximum of $1.2\hbar k$.

In Fig. 1, Fig. 2, and Fig. 3, the simulation results (dashed line) are superimposed upon the experimental results (solid line). Both simulation and experimental results have been normalized to give an area of unity under the graphs.

Agreement is quite good in all cases, except for the height of the central peak which represents those atoms which are undeflected by the standing wave. In each case the experimental result for this peak exceeds the theoretical prediction. This may indicate that the atoms were not all prepared in the correct initial state so that some were unaffected by the light field.
Comparison of experimental data (solid line) and simulation results (dashed line)

Fig 1. Scan (a) of Table 1, $\bar{N} = 4.5$.  
Fig 2. Scan (b) of Table 1, $\bar{N} = 1.2$.  

![Graphs showing probability distributions for different scans.]

<table>
<thead>
<tr>
<th>Scan</th>
<th>$\delta \omega$</th>
<th>$\Omega$</th>
<th>$\bar{N}$</th>
<th>$\tau$</th>
<th>$\Delta$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>2.36$\gamma$</td>
<td>4.5</td>
<td>13.93</td>
<td>0</td>
<td>0.424</td>
</tr>
<tr>
<td>(b)</td>
<td>4.0$\gamma$</td>
<td>3.34$\gamma$</td>
<td>1.2</td>
<td>19.33</td>
<td>0.611</td>
<td>0.365</td>
</tr>
<tr>
<td>(c)</td>
<td>8.0$\gamma$</td>
<td>3.10$\gamma$</td>
<td>0.4</td>
<td>18.26</td>
<td>1.293</td>
<td>0.323</td>
</tr>
</tbody>
</table>

Fig 3. Scan (c) of Table 1, $\bar{N} = 0.4$.  

Table 1. Experimental and simulation parameters.  In all scans, $\gamma = 2\pi(10$ MHz$)$ and $t = 4.71/\gamma$.

3 Quantum Nondemolition Measurement of the Photon Number

In the diffractive regime $\Omega \ll \delta \omega$ both spontaneous emission and inversion of the atomic internal state are negligible. In this region the photon number of the field
is not disturbed by the interaction. This model also describes the interaction of slow Rydberg atoms with a microcavity field [9]. We denote the boson annihilation operator for the field as \( a \), the single photon Rabi frequency by \( \gamma \), and consider again the Raman-Nath regime. The effective interaction Hamiltonian is

\[
V_i = \hbar \frac{\gamma^2}{\omega} a a^\dagger \cos^2 \xi
\]

in which the photon number is not altered by the evolution of the atom in the cavity. The strength of the perturbation is proportional to the intensity of the field. The probability of the atom exiting with momentum \( p \) after interaction time \( t \) is given by

\[
Q(p,t) = Q(p,0) \ast \sum_r \left[ \sum_n J_r^2 \left( \frac{|\gamma|^2 n t}{2\hbar \omega} \right) P(n) \right] \delta(p - 2\pi r \hbar)
\]

where \( \ast \) denotes momentum convolution, \( \delta \) and \( J \) are the Dirac delta and Bessel functions respectively, and \( P(n) \) describes the photon statistics of the cavity. For simplicity, we have taken the Rabi frequency to be independent of time. The term \( \delta \) represents the probability that a momentum of \( 2\pi r \hbar \) will be transferred to the atom in the cavity. This is strongly dependent on the field photon statistics, so that the back-projection of each atomic position measurement reduces the density operator corresponding to the field state. If the cavity lifetime is long compared to the time between atomic injections, Eq. (5) can be inverted and sufficient repeated measurements will eventually completely determine the photon statistics. However, if the cavity is not significantly coupled to any external reservoir, continual probing of the cavity will eventually result in the complete collapse of the field state to that of an exact number state corresponding to the absolute energy of the radiation within the normalisation volume. We illustrate this effect by simulating repeated atomic position measurements and examining the residual density of states. For simplicity we consider a monokinetic atomic beam in which the longitudinal velocity spread is small compared to the mean velocity. Based on an initial choice of field statistics, a particular output momentum \( p_0 \) for an atom exiting the cavity is chosen. The diagonal elements of the field density matrix \( P(n) \) are then altered by the back-projection of the measurement \( P(n|p_0) = M P(n|p_0) P(n) \) where \( M \) is the normalisation constant. The next momentum \( p_1 \) is then selected with this probability weighting for the statistics of the field, and the process is repeated. As the measurement proceeds the lack of knowledge or entropy about the state of the field \( \sum_n P(n) \ln P(n) \) is reduced. Figure 1 illustrates a simulation of 5 probe atoms with a non-relaxing field initially described by a coherent state with mean 10 photons. The atomic interaction parameters were \( (|\gamma|^2 t)(2\Delta) = 50 \). Each such simulation collapses the field to a single photon number which then does not change. The proportion of times that each number is selected is completely determined by the initial photon statistics. The final entropy can be used as an indicator of the quality of measurement.

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Fig 4. Simulation of the collapse of the field density of states. The probability scale on the right corresponds to all six bar graphs, while projected on to the back wall is the entropy of the field state with scale denoted by the entropy vertical axis on the left.

References


