ON THE PROBLEMS OF INTERPRETING RAPIDLY OSCILLATING Ap STARS

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ABSTRACT

Two properties of the rapidly oscillating Ap stars discovered and studied by Kurtz pose immediate problems to the theorist. The first is that only very few modes are excited. The second is that the oscillation patterns, which are apparently axisymmetrical with respect to the magnetic axis, appear to rotate synchronously with the magnetic field. It will be explained why these properties are difficult to understand (at least for us). We conjecture that unobserved unstable modes are suppressed by the dominating oscillations, as is the case in some Cepheids, and that the modes do not actually rotate at exactly the same speed as the star. Instead, we suggest that the modes are excited to substantial amplitudes only when they are aligned appropriately with the magnetic field, so that phase between the magnetic field and the wave pattern is maintained.

INTRODUCTION

Kurtz (1982a,b) has argued very persuasively that the rapid oscillations he has observed in Ap stars are dominated by axisymmetrical nonradial pulsations which rotate with the large-scale magnetic field. Here we discuss two of the questions that come immediately to mind in any attempt to understand these oscillations: why are so few modes excited and why are the axes of pulsation aligned with the magnetic field? We do not answer either of these questions, but we do propose an hypothesis which may provide the germ of an eventual solution.

We adopt the picture of the magnetic rotator described by Kurtz. The star is pervaded by a magnetic field with a strong dipole component; that field is predominantly axisymmetrical, with an axis that is inclined to the axis of rotation. Therefore the angular velocity of the star is presumably uniform. Material diffusion takes place preferentially along the field, and causes abundance anomalies in and beneath the photosphere. In particular, one might naively believe that the outer layers of the envelope are likely to be hydrogen-rich near the magnetic poles, owing to the greater rate at which helium has settled in regions where the field is more nearly vertical.
What are the oscillations? Because they have such well-defined frequencies we follow Kurtz by assuming they are global p modes. They must have low degree \( \lambda \), for otherwise they would not have been detected, and they must be of high order \( n \), because their frequencies are so high. In that respect they are like the low-degree five-minute oscillations of the Sun, though they differ by having greater amplitudes. Their frequencies are given approximately by the asymptotic formula derived by Tassoul (1980) for high-order adiabatic modes:

\[
\nu \sim \nu_0 (n + \frac{1}{2} \lambda + n_e) - A[\lambda(\lambda+1)+\delta] \nu_0^{-1} + \ldots,
\]

where

\[
\nu_0 = (2 \int_0^R \frac{r}{c} \,dr)^{-1}
\]

(1.1)

(1.2)

\( r \) is a radial coordinate and \( R \) is the radius of the star; \( c \) is sound speed, \( n_e \) is an effective polytropic index characterizing the outer layers of the envelope, and \( A \) and \( \delta \) are more complicated integrals of the equilibrium model. For a prototype stellar model we adopt a mass \( M = 1.5 \, M_\odot \), and we take a luminosity \( L = 7.5 \, L_\odot \) and an effective temperature \( T_e = 8000 \, K \). Then we obtain \( \nu_0 = 90 \, \muHz \), and deduce immediately that for a mode with a frequency of about 10 \( \text{hr}^{-1} \), \( n = 30 \). We note, however, that for such large values of \( n \), the frequencies are relatively closely spaced. This leads one to enquire why it is that so very few modes are excited to observable amplitudes. This situation is quite different from the Sun, for example, where excitation is evidently broad-band: in the Sun the greatest power is generated in the frequency range 2.5 - 4.0 \( \text{mHz} \), and all \( p \) modes with frequencies in that range appear to be excited.

A second perplexing aspect of the oscillations is their apparent alignment with the magnetic field. Kurtz suggests that this is explained if the modes are standing waves that rotate at precisely the same speed as the star. His idea is consistent with his measurement of the rotational splitting, which yields a period equal to the rotation period deduced from variations in magnetic field strength and abundance anomalies. However, it is difficult to see why the standing waves should rotate with the star. It is to this problem that we first turn our attention.

**ON ROTATIONAL SPLITTING**

Since the star is presumed to be rotating rigidly, with angular velocity \( \Omega \), it is convenient to work in a coordinate frame rotating with the star and the large-scale magnetic field. In that frame there are Coriolis forces to perturb the dynamics of the oscillations. In particular, eastward and westward propagating waves are affected in opposite senses. Viewed from the rotating frame, the oscillating frequencies \( \nu_{n\lambda m} \) are modified by slow rotation according to (Cowling and Newing 1949, Ledoux 1951):

\[
\nu_{n\lambda m} = \nu_{n\lambda} - \frac{m}{2\pi} C_{n\lambda} \Omega,
\]

(2.1)

where \( \Omega = |\Omega| \), \( \nu_{n\lambda} \) is the cyclic eigenfrequency of the corresponding mode when \( \Omega = 0 \), and azimuthal order \( m \) is considered positive for prograde waves. Therefore the standing wave pattern precesses about the axis of rotation with
angular velocity \( -\Omega_n^\lambda \). The coefficient \( C_{n\lambda} \) is in some sense proportional to the component parallel to \( \Omega \) of the rotational part of the displacement eigenfunction \( \xi \). For high-order \( p \) modes \( \xi \) is nearly radial, and the fluid motion is locally almost rectilinear; hence \( C_{n\lambda} \) is small. Indeed, formally \( C_{n\lambda} \rightarrow 0 \) as \( n \rightarrow \infty \). But for finite \( n \), \( C_{n\lambda} \) is not zero. Therefore there must be some precession, and alignment with the magnetic field cannot persist indefinitely.

We have not computed \( C_{n\lambda} \) for Ap-star models, but we do know that for the Sun \( C_{n\lambda} \approx +0.01 \) for five-minute modes of low degree. Roughly speaking, both the Sun and Ap stars resemble polytropes of index 3, so we would expect similar values of \( C_{n\lambda} \). Thus, viewed from an inertial frame, the modes of the Ap stars would be expected to rotate at some 99% of the angular velocity of the star. The measurements of the rotational splitting of the eigenfrequencies are not accurate enough to distinguish that value from \( \Omega \).

In all cases Kurtz has succeeded in representing his observations in terms of axisymmetrical modes with \( \lambda = 1 \) or \( \lambda = 2 \), and in these cases when it was possible to infer the magnetic axis, it was found to coincide with the axis of symmetry of the modes. It is this coincidence that led Kurtz to suggest that, notwithstanding equation (2.1), the modes rotate at precisely the same speed as the star. In the next section we discuss why that might be difficult to understand. But we note in passing that of all nonradial modes only those with \( \lambda = 1 \) necessarily have an axis of symmetry. Therefore it is interesting that the quadrupole oscillations do not appear to have a strong genuine nonaxisymmetrical component.

THE DIRECT AND INDIRECT INFLUENCE OF AN OBLIQUE MAGNETIC FIELD ON THE PRECESSION OF THE MODES

In the light of Kurtz's discussion, one is led to ask whether it is possible for some agent to augment the rate of precession of the standing wave pattern by the 1% or so necessary to achieve synchronism with the star. Evidently, any such agent must itself necessarily be nonaxisymmetrical, and the most obvious candidate is the magnetic field.

The magnitude of magnetically induced frequency perturbations can be estimated roughly in the manner discussed by Ledoux and Walraven (1958). An internal dipole field of intensity \( B \) comparable with that observed at the surface of a typical rapidly oscillating Ap star makes only a small perturbation to the shape of the oscillation eigenfunction, and changes the frequency by an amount comparable in magnitude to \( C_{n\lambda} \). However, the dominant perturbation does not contribute to the precession of the wave pattern. Because, with respect to spherical polar coordinates \((r, \theta, \phi)\) about the rotation axis of the star, the shape of the oblique dipole field is invariant under the transformation of \( \phi \rightarrow -\phi \), and because the Lorentz force is quadratic in \( B \) and is thus independent of the sign of \( B \), the frequency perturbation is independent of the sign of \( m \). Thus it is only a coupling of the effects of \( B \) and \( \Omega \), via distortions of the eigenfunctions, that might permit some chance of synchronism being achieved. The additional precession would then arise from the modification to \( C_{n\lambda} \) produced by the magnetic distortion of the eigenfunction, and the east-west asymmetry in the effect on the Lorentz force produced by the rotational perturbation to the eigenfunction. Notice, however, that this
would involve at least a second-order perturbation about the nonrotating non-magnetic state, and to achieve the precession required would therefore demand that the magnetic field be much more intense in the interior of the star than it is at the surface.

To assess the role of the second-order perturbations, we have considered a very simple model. The star is represented by an infinite uniform isothermal circular cylinder of perfect gas, bounded by a rigid wall of radius \( a \). Gravity is ignored. The entire space is presumed to be permeated by a uniform magnetic field of intensity \( \varepsilon B_0 \) and, notwithstanding electromagnetic problems at infinity, the whole configuration rotates about the axis of the cylinder with angular velocity \( \Omega_0 \). The parameter \( \varepsilon \) is regarded as being small; \( B_0 \) and \( \Omega_0 \) are, in an appropriate sense, of order unity. The fluid is considered to be inviscid, with zero thermal and magnetic diffusivities, and the cylinder wall and its surroundings have infinite magnetic diffusivity. The eigenfrequencies \( \omega \) of acoustic oscillation were then computed by expanding the solution in powers of \( \varepsilon \) about the nonrotating nonmagnetic state. For modes with no variation in the axial direction, the result is

\[
\omega = \omega_{nm}(1 + \frac{1}{4} \varepsilon^2 V^2/c^2) + 2\varepsilon \Omega_0(a^2 \omega_{nm}^2/c^2 - m^2)^{-1}(1 + \frac{1}{2} \varepsilon^2 V^2/c^2) + \ldots ,
\]

(3.1)

where \( m \) is again the azimuthal order of the mode, and \( c \) and \( \varepsilon V \) are respectively the sound speed and the Alfvén speed in the undisturbed state. Also

\[
\omega_{nm}^2 = j_{nm}^2 c^2/a^2 + 4\varepsilon \Omega_0^2 ,
\]

(3.2)

where \( j_{nm} \) is the \( n \)th zero of the \( m \)th-order Bessel function of the first kind.

The term of order \( \varepsilon^3 \) that results from the leading-order coupling between the rotation and the magnetic field increases, rather than decreases, the retrograde precession of the standing wave pattern relative to the rotating star. Thus our simple model suggests that it is unlikely that the Lorentz force associated with a weak magnetic field (namely a field for which \( \varepsilon^2 V^2/c^2 \ll 1 \)) would synchronize the oscillations.

Although we have not proved that the wave patterns in Ap stars cannot be synchronized with the rotation by the magnetic field, we conjecture that this is the case. For though our model is highly idealized, it does appear to retain the essential features of the physics we wish to model. For high-order \( p \) modes, gravity and thermal stratification do not play an important role; indeed, to leading order in \( n^{-1} \) the equations governing the oscillations are approximated by those for a uniform isothermal sphere, if the acoustical radius (i.e. the sound travel-time from the centre of the star) is used as the radial coordinate. Since \( n \) is finite, and indeed small enough for \( \omega \) not to exceed Lamb’s acoustical cutoff frequency in the atmosphere, the approximation breaks down near the surface. Thus we have modelled the relatively inert outer evanescent layers of the star with a rigid wall. This is an essential part of the model, for if the boundary exerted a constant stress, the standing wave pattern would not precess relative to the rotating fluid. We doubt that the imposition of cylindrical geometry, or the assumption that the equilibrium magnetic field is perpendicular to the axis of rotation, have a profound influence on the aspect of the solution we are studying. We have found that to
order $\varepsilon^2$ the eigenfrequencies of oscillation of a rotating uniform isothermal sphere confined by a rigid boundary and permeated by an oblique uniform magnetic field are similar to those given by equations (3.1)-(3.2), especially for the sectoral modes which are the natural analogues of the cylindrical modes that we have studied in more detail. The assumption that the magnetic field is uniform is unlikely to have restricted the solution in a qualitative way.

We have also considered, less thoroughly, some indirect influences the magnetic field might exert on the oscillations. For example, we have permitted in the equilibrium state a dipole perturbation to the molecular weight, that we presume to have been brought about by preferential diffusion along the field lines. Such an inhomogeneity, like the magnetic field, cannot itself distinguish between east and west, and therefore cannot contribute to the precession in leading order. Since the inhomogeneity is likely to be confined to only the outer layers of the star, it is unlikely that the rotationally induced distortions of the eigenfunctions have a significant effect. We have considered the effects of nonadiabaticity, and whether the precession can be modified substantially by dipole variations in opacity, produced also by chemical inhomogeneities that result from diffusion. Once again, within the confines of our idealized model, we obtained negative results.

SELECTIVE EXCITATION

In view of the calculations described in the previous section, we abandon the idea that the wave pattern rotates at precisely the same rate as the magnetic field. Instead we suggest that the pattern does precess with respect to the rotating star, but that it exists only when its axis of symmetry is aligned with the magnetic axis. This we presume to occur as a result of the spatial variation of chemical elements resulting from differential diffusion, which impose horizontal variations on the equation of state and the opacity, both of which influence the growth rates of the oscillations. Our conjecture is that nonradial standing modes grow preferentially with a particular orientation, and after precession has destroyed the initial alignment the modes decay.

Evidently, if our conjecture is correct, the growth time of aligned modes must be short compared with the precession time; and the decay time of misaligned modes must also be short. We have not demonstrated that this is actually so, but we present calculations below which show that it is not out of the question. In particular, for radial modes of a nonmagnetic star we find growth times and decay times as short as a few hours for modes with periods near 10 minutes. This is the be compared with the time it takes a standing wave pattern to precess through an angle of, say, $\pi/4$, which is about 12 times the rotation period if $C_{n\ell} = 10^{-2}$.

Our preliminary calculations have ignored the direct influence of rotation and the magnetic field on the oscillation eigenfunctions. We note that the major contributions to the growth or decay of the modes come from the outer layers of the star. There the eigenfunctions are insensitive to the geometrical effects of $\lambda$; in the outer parts of the stellar envelope all low-degree $p$ modes with similar frequencies look alike. Therefore we have restricted attention to radial pulsations. Our idea was to model the magnetic poles and the magnetic equator by appropriate segments of two different
spherically symmetrical stellar models, and piece them together to estimate the growth rates of the nonradial modes.

As an example of the equatorial regions, we constructed a normal, chemically homogeneous stellar envelope, with hydrogen, helium and heavy element abundances \( X, Y, Z = 0.745, 0.235, 0.02 \). We adopted mass, luminosity and effective temperature: \( M = 1.5 M_\odot, L = 7.5 L_\odot, T_\text{e} = 8000 \text{ K} \). Mixing-length theory in the form quoted by Baker and Temesváry (1966) (cf. Gough and Weiss 1976) was used to model the convection zone, with the ratio \( \alpha \) of the mixing length to the pressure scale height chosen to be 2.0, in rough accordance with the solar calibration (Berthomieu et al., 1980, Lubow et al., 1980) The model extended down to a radius \( r \) of 0.2\( R_\odot \), where \( R = 1.7 R_\odot \) is the radius of the star. We adopt this model as our standard with which to compare others.

Linearized nonadiabatic radial pulsations were computed, treating the convective perturbations to the heat flux in the manner discussed by Gough (1977) and Baker and Gough (1979). Reynolds stresses were ignored. The vertical displacement eigenfunction was assumed to vanish at the base of the envelope (\( r = 0.2R_\odot \)). The growth rates of a selection of modes are plotted in Figure 1 as a function of cyclic frequency \( \nu = \omega/2\pi \); the points are joined by continuous lines.

A prominent feature in Figure 1 is the maximum in the growth rate at a pulsation period \( P \) of about ten minutes. This occurs when the first node of the displacement eigenfunction is near the base of the convection zone. The outermost wavelength of the oscillation is such that the mode derives maximum benefit from the excitation produced by helium ionization. As frequency increases, and wavelength decreases, spatial resonance is destroyed and the pulsations are abruptly stabilized. There is a second resonance at approximately twice the frequency of the first, where once again there is a spatial resonance of the eigenfunction with the ionization, but these pulsations are stable because dissipation in the interior now dominates.

It is difficult to compare this result with what one might expect to occur at the magnetic poles. One of the problems is that we do not know how the stratification of the equilibrium model at the poles compares with that at the equator. We have constructed two models in an attempt to get some idea of how growth rates are influenced. The first is composed purely of hydrogen in and above the convection zone, and has a normal homogeneous composition beneath. Otherwise it is the same as the standard. The second also has a helium-depleted convection zone, but was computed with \( \alpha = 0.5 \) to model some degree of suppression of convection by the magnetic field (though we did not change \( T_\text{e} \)). The growth rates of radial pulsations of these two models are included in Figure 1.

It is evident that a substantial modification to the growth rates is brought about by the variations in the equilibrium state we have considered. Therefore the growth rate of a nonradial mode is likely to be quite sensitive to its orientation. Our calculations suggest that the same mode could either grow or decay on a timescale of only a few hours, depending only on its orientation relative to the magnetic field. Therefore our conjecture that the observed alignment is purely a result of selective excitation seems at least potentially viable. However, there is an important discrepancy between our present models and observation which we must point out.
Fig. 1. Growth rates $\eta$ of a selection of radial pulsations of an Ap star model. The values are joined by straight lines. Continuous lines join the values for the standard model, which is a normal chemically homogeneous stellar envelope with $M = 1.5 M_\odot$, $L = 7.5 L_\odot$, $T_e = 8000$ K and $\alpha = 2.0$. Its fundamental radial period is 64 min. Dashed lines join the values for a model in which all the helium has been replaced by hydrogen in and above the convection zone, but which is otherwise the same as the standard model. Dotted lines join a model which also has a helium-depleted atmosphere and convection zone, but is computed with $\alpha = 0.5$. The growth rates of a model with $M = 1.5 M_\odot$, $L = 7.5 L_\odot$ and $T_e = 7500$ K are similar to those of our standard model when considered as a function of $\eta$; the frequencies of this model are roughly 63% of those of our standard.

Axisymmetrical modes have their greatest amplitude along the axis of symmetry. Therefore the growth rates of modes that are aligned with the axis of a dipole magnetic field would be the most strongly influenced by conditions near the magnetic axis. Our present calculations indicate that those conditions are stabilizing. This is brought about, as one would expect, by the depletion of helium from the region where it is partially ionized, which thus removes the dominant excitation mechanism. Therefore our first estimate is that the axes of pulsation and the magnetic field should not be aligned.

**DISCUSSION**

Our calculations are very rough, but we consider the results sufficiently interesting to warrant further study of the conjecture. Our principal conclusions are that nonradial standing $p$ modes do not rotate at precisely the same angular velocity as the star (we have not proved this, but we have presented evidence that it is likely in the case of rapidly oscillating Ap stars) and that the growth or decay of the modes is much faster than the rate at which they precess relative to the star. Moreover, the growth rates are very sensitive to the relative orientations of the pulsations and the magnetic field.

Our simple calculations have not demonstrated that aligned modes are preferred. That could be because there are many potentially important aspects of real Ap stars that we have not taken into account. Most obvious, perhaps, is that we have ignored the distortion of the eigenfunctions by the magnetic
field. Magnetic pressure and gas pressure are comparable at a pressure of about $10^5$ dyne cm$^{-2}$, which occurs at the top of the first helium ionization zone and is where much of the driving takes place. Therefore magnetic distortion could have a profound effect on the stability of the modes. The computation of the distortion appears to be difficult, but some headway has already been made by Biront et al. (1982). Our modelling of the helium distribution at the magnetic poles is oversimplified, and we have not taken into account the complicated distribution of the heavier elements that results from diffusion (cf. Vauclair and Vauclair, 1982). And what may be more important is the effect of magnetic suppression of convection on the stratification of the envelope; we have not attempted to model that in a realistic way.

The growth rates of the modes we have computed have a sharp maximum at $n = 15$. This is evident in Figure 1. For the stellar model considered, the corresponding period is about 11 min, which is comparable with the longest periods observed by Kurtz. It is not unlikely, therefore, that the modes are self-excited, and that the most rapidly growing mode suppresses the other unstable modes by nonlinear interactions, as is the case, for example, in some Cepheids. Though the amplitudes of the oscillations observed are very much lower than those of Cepheids, it is no less likely that nonlinear processes are operative: the limiting amplitudes must be determined by a nonlinear balance.

Finally we point out that the 2:1 frequency ratio found in HD83368 by Kurtz is also indicative of nonlinear interactions. It is likely that the higher frequency mode is driven by the lower frequency mode by resonant coupling, as Kurtz has suggested. This driving is helped by the maximum in the growth rate near twice the frequency of the most unstable mode (see Figure 1). In the case of our standard model, we find that $2\nu_{n,1} = \nu_{2(n+1),1}$ to within 0.5% when $n = 15$. (We estimated the nonradial frequencies from the radial frequencies using equation (1.1).) The precision to which the resonance is satisfied varies slowly with $n$, and it also changes as the equilibrium model changes. Indeed, raising the luminosity to 10 $L_\odot$ brings the resonance closer to $\nu_{2(n+1),2}$. Therefore we suggest that only in some stars is the resonance satisfied accurately enough for the high-frequency mode to be driven to an observable amplitude.

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