PRELIMINARY STUDY OF GPS ORBIT DETERMINATION ACCURACY
ACHIEVEABLE FROM WORLDWIDE TRACKING DATA

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ABSTRACT

Studies of the GPS satellite orbit determination problem which have been published previously have assumed that the tracking data would be provided by pseudo-range measurements from about 4 ground stations. Substantial allowances were included in the error budget for drifts in the satellite and ground station clocks, and for uncertainty in the water vapor contribution to the tropospheric propagation correction. We have started an investigation recently to see how much improvement in the orbit accuracy can be expected if high-accuracy tracking data from a substantially larger number of ground stations is available. For observations from 20 ground stations, the results indicate that 20 cm or better accuracy can be achieved for the horizontal coordinates of the GPS satellites. With this accuracy, the contribution to the error budget for determining 1000 km baselines by GPS geodetic receivers would be only about 1 cm.

INTRODUCTION

A preliminary study of the accuracy with which geodetic baselines can be determined using signals from the Global Positioning System (GPS) satellites was very encouraging (Bosssler et al. 1980), and we have since carried out a much more detailed study (Larden and Bender, 1982a). It was assumed that the GPS receivers would measure the phase of the reconstructed carrier signals which are generated in some receivers by using the modulation code (Ward, 1982) and in other instruments without requiring knowledge of the code (MacDoren et al., 1982; Counselman and Steinbrecher, 1982). Baselines up to about 100 km long were considered, and uncertainties in the horizontal

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coordinates for the GPS satellites were taken to be 2 meters. The measurement accuracy was assumed to be 1 cm for the range difference from two ground stations to a given satellite, with most of the error due to the expected uncertainty in the tropospheric correction even if water vapor radiometers are used at both ground stations to determine the integrated water vapor content along the line of sight. A "modified worst case" type of analysis was employed because of the expected substantial correlation of the tropospheric correction errors over the measurement times.

With the above assumptions and analysis method, the expected baseline uncertainties were found to be less than 1 cm for horizontal components and 2 cm for the vertical for short baselines in most parts of the world, even with only the initial 6 GPS satellites in operation. However, for baselines substantially longer than 100 km, the largest part of the uncertainty would come from the assumed GPS satellite position uncertainty of 2 meters for the horizontal coordinates. This was one reason why we decided to investigate the extent to which determination of the GPS satellite orbits could be improved if accurate reconstructed carrier phase measurements were made at a number of worldwide tracking sites whose coordinates were known from other types of measurements (Larden and Bender, 1981). The other reason was the probable major importance of receiving GPS signals aboard altimeter satellites such as the TOPEX (Topographic Experiment) satellite and the ERS (Earth Resources Satellites) in order to determine their orbits accurately, provided that the GPS satellite locations could be determined accurately (Bender and Larden, 1982). The initial results of our study of the GPS orbit determination problem are given in the remainder of this paper.

GPS SATELLITE ORBIT DETERMINATION

The objective of our study is to establish the accuracy achievable for determining the GPS satellite orbits when all 18 of the planned satellites are in operation. If the GPS signals are as useful as expected for geodetic applications and for determining the orbits of other satellites, then the tracking of the GPS satellites from as many as 20 fixed ground stations seems likely. Such a future network might be a combination and extension of the present TRÄNET 2 and MEDOC networks, with each station equipped with a high-accuracy GPS receiver and a water vapor radiometer.

Since the goals of the NASA Crustal Dynamics Project and of related projects in other countries include determining intercontinental baselines with 3 cm accuracy, we assume 3 cm uncertainty in each coordinate for the GPS tracking stations. It appears quite feasible to achieve this accuracy with both LAGEOS ranging and VLBI measurements using highly mobile stations. The additional cost of such measurements appears to be reasonable, in view of the expected worldwide benefits, provided that the main part of the Crustal Dynamics Project or some similar program is being carried out. Repeat measurements would be needed only every few years for most stations, provided that the relative plate motions are being monitored and that the expected low rates of intraplate motion are found in most areas.

The main problem in determining the expected accuracy of the GPS orbits is that the most serious limitation is likely to come from time-dependent
systematic errors whose time variations are not known and cannot be modeled. In order to make some appropriate allowance for such errors, we assume that measurements are made at each ground station only once every half hour. For orbital arcs of 2 and 4 hours, this means that either 5 or 9 sets of measurements are included in the simulations. The measurements actually would be made much more frequently, and the part of the error which is truly uncorrelated from measurement to measurement would nearly average out. Thus the assumed random measurement errors really are intended to represent the effects of the time-varying systematic errors.

In our simulations we assume that reconstructed carrier phase measurements are made at nearly the same time from each ground station to all GPS satellites which are above 20° elevation angle. The time spread for the different measurements with respect to the nominal measurement time is assumed to be short enough so that only a single clock correction needs to be considered for each satellite and each ground station. We take the difference in range from a particular satellite to a given pair of ground stations as the observable, so that corrections to the satellite clocks do not appear explicitly in the analysis. Corrections to the clock differences between 19 of the ground station clocks and one reference clock are solved for at each nominal measurement time. The only other parameters solved for are the six orbit elements for each satellite.

The measurement noise was taken to be 1.4 cm for the difference in range to a given satellite between a reference station and each of the other stations for which the elevation angle was greater than 20°. The station with the highest elevation angle for the satellite usually was chosen as the reference station. Other error sources allowed for were a 5% uncertainty in the solar radiation pressure force acting radially away from the sun, a $5 \times 10^{-8}$ fractional uncertainty in $GM$, where $M$ is the earth's mass, and the GEM-10 scaled standard deviations for each of the gravitational field harmonic coefficients through degree and order 4. An additional thrust uncertainty equal in magnitude to the 5% uncertainty in the solar radiation pressure force usually was included, with its direction making equal angles with the along-track, cross-track, and radial directions.

The 20 ground station locations chosen for use in the simulations are listed in Table 1. The first 3 are currently GPS monitor station sites, and the next 6 are TRANET sites. The following 3, Bermuda, Reunion, and Unalaska are close enough to the TRANET sites in Herdon, Virginia, in Mahe, Seychelles Islands, and in Anchorage, Alaska so that these sites probably would give comparable accuracy. The following 2 in Djibouti and Pretoria are MEDOC sites, and the remaining six are not presently connected with Transit or GPS satellite tracking. Auckland and Geraldton are included instead of the TRANET site in Adelaide, Australia, in order to give better geometrical coverage. Dakar, New Delhi, Quito, and Rio Gallegos represent general areas where coverage would be desirable.
Table 1.
Assumed GPS tracking station locations

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<td>1.</td>
<td>Guam</td>
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<td>2.</td>
<td>Hawaii</td>
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<tr>
<td>3.</td>
<td>Vandenberg AFB, Calif.</td>
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<td>4.</td>
<td>Grasse, France</td>
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<td>5.</td>
<td>Kushiro, Japan</td>
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<td>6.</td>
<td>McMurdo Sound, Antarctic</td>
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<td>7.</td>
<td>Pretoria, South Africa</td>
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<td>8.</td>
<td>Sao Paulo, Brazil</td>
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<tr>
<td>9.</td>
<td>Thule, Greenland</td>
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<td>10.</td>
<td>Bermuda Islands</td>
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<td>11.</td>
<td>Reunion Island</td>
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<td>12.</td>
<td>Unalaska, Aleutian Islands</td>
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<td>13.</td>
<td>Djibouti, Republic of Djibouti</td>
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<td>14.</td>
<td>Papeete, Tahiti</td>
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<td>15.</td>
<td>Auckland, New Zealand</td>
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<td>16.</td>
<td>Dakar, Senegal</td>
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<td>17.</td>
<td>Geraldton, Australia</td>
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<td>18.</td>
<td>New Delhi, India</td>
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<td>19.</td>
<td>Quito, Equador</td>
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<td>20.</td>
<td>Rio Gallegos, Argentina</td>
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RESULTS AND CONCLUSIONS

The results obtained from the main simulations are given in Table 2. The uncertainties in the GPS satellite horizontal coordinates are the values corresponding to the middle of the arcs. It can be seen that the dominant errors are those due to the unmodeled parameters. Of these, the major part comes from the assumed 3 cm uncertainty in each component of the ground station locations. The uncertainties in GM and in the other low degree gravity field coefficients give a total of only roughly 3 cm uncertainty in each of the horizontal coordinates in the 4 hr case and about half as much in the 2 hr case. The uncertainties due to solar radiation pressure and to the additional assumed thrusts also are small. The contributions from the random errors were scaled up by a factor $(k/4)^{1/2}$, where $k$ is the number of measurement times. This corresponds to normalizing the results to the rather arbitrarily chosen case of 4 observation times, so that correlated errors don't tend to average out as much.

From the results given, it appears that uncertainties of less than 20 cm for the horizontal coordinates of the GPS satellites probably can be achieved using the 4 hr arcs, provided that a sufficient number of ground stations equipped with the right kinds of receivers and water vapor radiometers provide regular tracking data. The uncertainties obtained for the 2 hr arcs were about 35% larger. The model assumed for radiation pressure force uncertainties was not realistic enough to give good estimates of the effect of this error source, since the actual error in the satellite acceleration will not be confined to the two directions included in the model and will vary appreciably over the 4 hr arc. However, the magnitudes of the corresponding orbit errors were small enough so that the radiation pressure effects do not appear to be serious limitations on the accuracy for arcs of this length.

In view of the results obtained, it appears that GPS receivers can be used to obtain accurate positions over baselines up to at least 1000 km in length. For this distance, the contributions of our horizontal coordinate uncertainties of less than 20 cm for the GPS satellites to the baseline...
Table 2.
Uncertainties in GPS satellite horizontal coordinates

<table>
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<tr>
<th>Contributions to total uncertainty</th>
<th>Position Uncertainties (cm)</th>
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<tr>
<td></td>
<td>2 hour case</td>
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<tr>
<td></td>
<td>Cross track</td>
</tr>
<tr>
<td>Modified random noise,* RMS</td>
<td>10.5</td>
</tr>
<tr>
<td>Unmodeled effects, RMS</td>
<td>18.0</td>
</tr>
<tr>
<td>RSS total</td>
<td>20.8</td>
</tr>
<tr>
<td>Worst individual uncertainty for any satellite</td>
<td>25.3</td>
</tr>
</tbody>
</table>

*Random noise contributions were multiplied by a factor \((k/4)^{1/2}\), where \(k\) is the number of measurement times.

**The thrust uncertainties were not included in this case.

uncertainty can be shown to be only about 1 cm in each coordinate (Larden and Bender, 1982b). Thus it is likely to be useful for a number of countries to participate in accurate tracking of the GPS satellites, so that the distances over which GPS receivers can be used for high-accuracy geophysical and geodetic measurements can be considerably extended. The use of improved GPS satellite positions for determining the orbits of the TOPEX and ERS satellites and other altimeter satellites also is likely to be of major importance (Bender and Larden, 1982; Larden and Bender, 1982b).

ACKNOWLEDGMENTS

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REFERENCES


