Partial Coherent Scattering in the Wings of Lyman $\alpha$

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Abstract

We have used the unified theory of Vidal, Cooper and Smith (1970) to obtain a frequency-dependent Stark broadening parameter. This frequency dependence appears in the coherence fraction of scattering in the transfer equation (Cooper, 1978), while previous work has assumed a frequency-independent coherence fraction. This new formalism is applied to calculations of the H I $\alpha$ wing, leading to reductions in intensity of 2-3 times, and a steeper profile near line center.
I. Introduction

Partial frequency redistribution (PRD) has been shown to be a significant effect in the transfer of radiation in important optically thick resonance lines like the Ca II H, K, Mg II h, k and H I λλ lines in the Sun (Shine et al., 1975). Previous work has assumed that all mechanisms of line broadening produce a Lorentzian line shape, which leads to a frequency-independent ratio between the fraction of radiation scattered coherently in the atom's frame and the radiation which is redistributed in frequency. In this case a simple form for the noncoherence fraction has been derived by Omont, Smith and Cooper (OSC) (1972). This result is valid only in the impact approximation of broadening theory, which for hydrogen holds only in the extreme line core.

Recent work by Cooper (1978) shows that for the non-impact regime, the results of OSC carry over if one uses a frequency-dependent Stark broadening parameter in place of the old impact expression. The form of \( \Gamma_{ST}^{(u)}(\Delta \nu) \) may be obtained from the unified theory of Stark broadening by electrons given by Vidal, Cooper, and Smith (VCS) (1970). Ion stark broadening is quasi-static and will, therefore, not redistribute the radiation or contribute to the noncoherence fraction. In section II we show how the transfer equation is modified by these effects, in section III we give an expression for \( \Gamma_{ST}^{(u)}(\Delta \nu) \), and in section IV we apply these results to the formation of the wings of \( \lambda \lambda \) in the Sun.

II. Redistribution in the Scattering Integral

To evaluate the source function of a spectral line, one must know the redistribution function \( R(\nu, \nu') \) which describes the probability of absorption at frequency \( \nu \) followed by emission at frequency \( \nu' \). The theory of OSC shows that the appropriate expression is

\[
R(\nu, \nu') = (1-\gamma) R^{II}(\nu, \nu') + \gamma R^{III}(\nu, \nu') \quad \text{where} \quad \gamma = \frac{\Gamma_E}{\Gamma_E + \Gamma_R + \Gamma_I}
\]

(1)

with \( \gamma \) the frequency-independent noncoherence fraction. \( R^{II}(\nu, \nu') \) is the redistribution function corresponding to coherent scattering in the atom's frame, and \( R^{III} \) refers to noncoherent scattering (usually approximated by complete redistribution). Here \( \Gamma_R \) is the natural damping width, \( \Gamma_I \) is the width due to inelastic collisions, and \( \Gamma_E \) is the width due to elastic collisions which redistribute photons in the upper state. For the problem of hydrogen in the solar atmosphere, \( \Gamma_E = \Gamma_{ST} + \Gamma_{RES} \) where \( \Gamma_{ST} \) is the width due to electron Stark broadening and \( \Gamma_{RES} \) is the width due to collisions with neutral hydrogen. For typical values of \( n_e \) and \( T \) in the chromosphere, \( \Gamma_E \) is only a few percent of \( \Gamma_R \), \( \Gamma_{ST} \) is greater than \( \Gamma_{RES} \) above the temperature minimum, and \( \Gamma_I \) is negligible.

Expression (1) is derived for the impact regime, which is valid for frequency differences \( \Delta \omega < 1/\tau_I \) (\( \tau_I \) is the duration of a typical collision). For electron Stark broadening of hydrogen, the validity condition is \( \Delta \omega \lesssim \omega_p \) (the plasma frequency). For \( \lambda \lambda \) in the chromosphere this corresponds to \( \Delta \lambda \lesssim 1.4 \times 10^{-3} \AA \), so that the impact approximation is essentially invalid.
for an analysis of the La profile, especially outside the Doppler core where
PRD effects become very important (Δλ > 0.1 Å). Cooper (1978) has recently
extended the OSC theory to the non-impact regime. In this case all the
broadening is Lorentzian except ΓST. We may summarize Cooper’s results by
stating that Eq. (1) remains valid provided that ΓST + ΓST(u)(Δν), where
ΓST(u)(Δν) must be derived from a theory of Stark broadening or from experi-
ment. Since ΓST(u)(Δν) generally is less than ΓST and decreases with increasing Δν,
we see that the frequency-dependent noncoherence fraction, γ(Δν), will cause
the coherent scattering term to be even more dominant than in the previous
case. Thus for a given model atmosphere, the non-impact theory will produce
a different line shape than the OSC analysis.

III. The Stark Broadening Parameter

The unified theory of VCS derives ΓST(u)(Δν). There are three regions
of interest: the impact regime Δω ≤ Δωp, the quasi-static or Holtsmark
limit Δω ≈ Δωc (the Weisskopf frequency), and the transition regime between
these two limits. To use the results of VCS we define (using their notation)
the following:

ΔωR ≡ Δω
p
Δw , with Δω = \frac{2πcΔλ}{\lambda^2} and Δw
p = \sqrt{2} \omega_p = \left(\frac{8\pi ne^2}{m_e}\right)^{1/2} = 7.98 \times 10^4 n_e^{1/2}

C = \frac{\hbar \Delta\omega_p}{2kT} and ω_c = \frac{\Delta\omega_p}{C}.

For La and in cgs units we have numerically that

\Delta\omega_R = 1.6 \times 10^8 \Delta\lambda (Å) \bar{n}_e^{-1/2}, C = 9.13 \times 10^{-7} n_e^{1/2} T^{-1} and \Delta\omega_R |_{\omega_c} = C^{-1}.

The asymptotic expressions for ΓST(u)(Δν) are, in the impact regime

ΓST(1)(ΔωR) = ΓST = 4.6 \times 10^{-4} n_e T^{-1/2} [6.81 - ln(9 \times 10^{-10} n_e T^{-2})] for ΔωR ≤ 1, (2)

and in the Holtsmark limit

ΓST(3)(ΔωR) = ΓST \frac{π^{3/2}}{4} \left\{ (CΔω_R)^{1/2} [B - ln(4C^2)] \right\}^{-1} \text{ for } Δω_R ≥ C^{-1}. (3)

Here B is a parameter describing a strong collision cut-off whose value is
still in dispute. We adopt the VCS value of B = 0.27.

The exact results for the transition regime between these two are
difficult to calculate but are compiled for certain cases in tables by
Vidal, Cooper and Smith (1973). One finds from these tables or examining
graphs like Fig. 1 that a reasonable approximation in this regime is a linear
dependence in log (ΔωR). This dependence is in fact exact in the Lewis
theory, but with the wrong slope. We therefore adopt this dependence and
require that it agree with \( \Gamma_{ST}^{(1)} \) and \( \Gamma_{ST}^{(3)} \) in the two known limits. This yields the expression

\[
\Gamma_{ST}^{(2)}(\Delta \omega_R) = \Gamma_{ST} \left[ 1 + \left( \frac{\pi^{3/2}}{4[B - \ln(4c^2)]} - 1 \right) \frac{\log(\Delta \omega_R)}{\log(c^{-1})} \right] 1 < \Delta \omega_R < c^{-1}.
\] (4)

Equations (2) - (4) provide the full expression \( \Gamma_{ST}^{(u)}(\Delta \omega_R) \) which must be used in place of \( \Gamma_{ST} \) in Eq. (1). For \( n_e = 5 \times 10^{10} \) and \( T = 6500 \) K the values of these parameters and the frequency dependence of \( \Gamma_{ST}^{(u)} \) are tabulated in Table I. Note that in this typical case \( \Gamma_{ST}^{(u)}(\Delta \omega_R) \) is one-third of \( \Gamma_{ST} \) at \( \Delta \lambda = 2 \) Å and for the extreme wings of \( \Delta \lambda \), \( \Gamma_{ST}^{(u)}(\Delta \omega_R) \approx \Gamma_{RES} \) (we have assumed \( n_H I = 4 \times 10^{11} \) for \( \Gamma_{RES} \)). Actually, \( \Gamma^{(2)} \) is an overestimate of \( \Gamma^{(u)} \) while \( \Gamma^{(3)} \) gives a better fit (underestimate) in the regime \( \Gamma^{(3)} < \Gamma^{(2)} \) as seen in Fig. 1. This effect is exaggerated in Fig. 1 because for the chromosphere \( \omega_c \) is much larger. Thus one could change the limits of validity for Eqs. (3) and (4) to reflect this, but there is no simple expression for these new limits.

\[
\begin{array}{cccccccc}
\hline
\Delta \lambda (\AA) & 1.4 \times 10^{-3} & 0.5 & 1 & 2 & 5 & 10 & 44.5 & 100 \\
\hline
\Delta \omega_R & 1 & 358 & 715 & 1431 & 3578 & 7155 & 3.185 \times 10^4 \ast & 7.155 \times 10^4 \\
\Gamma_{ST}^{(u)}(\Delta \omega_R)/\Gamma_{ST} & 1 & .47 & .41 & .35 & .27 & .20 & .071 & .047 \\
\hline
\ast \Delta \omega_R = c^{-1}.
\end{array}
\]

**Table I**

IV. Results

To test the effects of the correct frequency dependence of \( \gamma(\Delta \nu) \) in a realistic case, we have calculated the wings of \( \Delta \lambda \) for the Vernazza et al. (1973) solar model. A two-level atom partial coherent scattering code based on the work of Ayres (1975) was used first with \( \Gamma_{ST} \) (case A) and then with \( \Gamma_{ST}^{(u)}(\Delta \omega_R) \) (case B) in Eq. (1). Preliminary work has shown that the wing profiles obtained with this simple code have the same shape but generally lower intensities than profiles from a multi-level PRD transfer code. Differential comparison of cases A and B should therefore be valid. In Fig. 2 are plotted the red wings of \( \Delta \lambda \) for each of the two cases.

The effect of increasing the coherence of the scattering is to lower the intensities everywhere for case B. This difference is largest for \( \Delta \lambda \approx 6 \) Å, where case A has 2-3 times the intensity of case B. At 0.4 Å the difference is 1.5 times while at 16 Å it is 1.7 times. Thus the profile in case B is steeper than in case A for the first 10 Å or so, and shallower beyond that. It begins to converge back to case A beyond 25 Å because this part of the wing is formed sufficiently deep in the atmosphere that \( \Gamma_{RES} \) is the dominant
term in $\Gamma_E$, particularly in case B. The increased noncoherence due to $\Gamma_{RES}$ in the extreme wing is also the reason that the intensity goes through a minimum around 20 Å. These portions of the profile are formed above the temperature minimum, at $T \approx 5300$ K.

Use of the proper frequency-dependent treatment of $\gamma(\Delta \nu)$ is primarily important for semi-empirical modeling of the solar atmosphere based on the wings of La. The temperatures inferred from case B will be higher than for case A, given an observed profile to fit. The details of the temperature structure are also affected. In either case the actual noncoherent fraction $\gamma(\Delta \nu)$ is only 1% or less in the region of line formation; significantly less than the 7% used by Vernazza et al. (1973) or Milkey and Mihalas (1973). It is important for models based on the La line to use the correct value of $\gamma(\Delta \nu)$.

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References

Fig. 1 (from Fig. 7 of Vidal, Cooper, Smith, 1970)

The frequency-dependent ratio of $\Gamma_{st}^{(u)}$ to the impact value $\Gamma_{st}$ for $n_e = 1.3 \times 10^{13}$ and $T = 1850^\circ$.
Fig. 2 H Lyman-α wings for a solar model