New Laboratory Test of the Equivalence Principle

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A test of the principle of equivalence using a large fluid (surrogate) fiber Éötvös apparatus is presently being undertaken at the Joint Institute for Laboratory Astrophysics. Preliminary measurements using a 0.25 m diameter fluid system were sufficiently encouraging that we have embarked on the construction of a five times larger (1.27 m diameter) system employing approximately 500 kg of lead and 500 kg of copper as the test masses. The first experimental results on the equivalence of gravitational and inertial mass with this new large apparatus are expected in 1983 or 1984.

Key words: equivalence principle; general relativity; gravitation.

1. Introduction

The ratio of the inertial mass of an object to its passive gravitational mass is postulated to be independent of the material in metric theories of gravitation such as general relativity [1]. In fact, in the last century, the equivalence of inertial mass ($m_i$) and gravitational mass ($M$) for two materials (A and B) as measured by the parameter $\eta$, where

$$\eta(A, B) = \frac{1}{2} \left[ \frac{M}{m_A} - \frac{M}{m_B} \right],$$

was known to $2 \times 10^{-5}$ [2], and was measured to be less than $3 \times 10^{-5}$ by 1922 [3]. Two more recent measurements have lowered the upper limit on $\eta$ to $3 \times 10^{-11}$ and $0.9 \times 10^{-12}$ respectively [4, 5]. Nevertheless, a fundamental motivation exists for checking this hypothesis to the highest possible accuracy: the principle of equivalence is basic to theories of gravity.

At the Joint Institute for Laboratory Astrophysics (JILA), we have developed a new type of torsion-pendulum apparatus for the purpose of improving the accuracy of the Éötvös experiment, which tests the equivalence of gravitational and inertial mass. All of the experiments mentioned above were done with a traditional torsion balance; but, despite a significant amount of effort, none of these experiments reached the sensitivity that would be expected if they were limited by the Brownian motion of the torsion balance. In all cases, other sources of noise dominated the fundamental thermal noise. The torsion fiber itself is a source of noise, while the classical torsion balance is extremely sensitive to seismic noise. Also, the sensitivity of a torsion balance is limited since the fiber both supports and torques the masses. Any attempt to increase the sensitivity of the balance by increasing the size of the masses also requires an increase in the diameter of the fiber, and this then places more severe requirements on the position detector since the torsional stiffness of a fiber increases as the fourth power of its diameter.

The approach we have taken at JILA to deal with these limitations of the traditional fiber involves the utilization of a new type of (surrogate) fiber, in which the functions of support and torque are separated. We do this by using a fluid (water at 3.98°C, its maximum density point) to provide the support function of a fiber, and an electrode array to provide the centering and restoring functions. A 0.25 m diameter float using 3 kg test masses of copper and tungsten was successfully tested and used by Keiser and Faller [6]. While their result ($\eta = 0.2 \pm 4.0 \times 10^{-11}$) was an order of magnitude worse than what one would expect if the motion of the float were predominantly determined by Brownian motion at periods of 24 h, a power spectrum analysis showed that down to frequencies of 10 cycles/day the observed motion was in fact limited by the Brownian motion of the float.

2. Description and Analysis of the Apparatus

We are now constructing a 1.27 m diameter float, shown schematically in Fig. 1. Note there are seven balls in a hexagonal pattern on the lid of the float (see also Fig. 2), and three balls in a line on the underside of the lid of the float container. These latter three are electrically insulated from the container. The central ball functions as centering electrodes—an ac potential of about 1 kV is applied across them to center the float. The outer balls function as torque electrodes. (The float can be zeroed in any of six positions, differing by 60°.) Varying

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the ac potential applied to the outer balls serves to vary the torsion constant. As seen in Fig. 1, we are using 24 cylindrical masses; 12 of copper and 12 of lead. The total mass of each 12-piece test mass is about 490 kg. A breakdown of equivalence (i.e., a non-zero $\eta$) would result in a slight difference in the acceleration of the two sides of the float toward the sun, and this would evidence itself as a 24-hour-period variation in the position of the float.

The sensitivity of this type of apparatus can be expected to improve with size since most of the noise terms couple in through area (surface) effects, and therefore their relative importance decreases as one scales up the apparatus. Further, as one increases the size, $D$, of the apparatus, the fundamental thermal-noise-limited sensitivity improves, scaling as $D^{-52}$. To see this, note that the noise torque in a bandwidth $\Delta f$ due to Brownian motion is given by:

$$\tau_{\text{noise}} = (2k_bT b \Delta f)^{1/2}$$

(2)

where $k_b$ is Boltzmann’s constant, $T$ is the absolute temperature, and $b$ is the damping coefficient in the equation of motion for the float with an external torque, namely:

$$I \ddot{\Theta} + b \dot{\Theta} + K \Theta = I_{\text{external}}.$$  

(3)

Here $I$ is the moment of inertia of the float, and $K$ is the torsion constant. The amplitude of the 24-hour-period signal torque is given by:

$$\tau_{\text{signal}} = \eta M a D \cos \delta,$$  

(4)

where $M$ is the mass of one of the two test masses, $a$ is the acceleration toward the sun (0.62 cm/sec$^2$), $D$ is the effective moment arm of the test masses, and $\delta$ is the declination of the sun. The coefficient of $\Theta$ in Eq. (3) will depend on some geometric factor times the viscosity $\nu$ of the fluid. Straightforward dimensional analysis (remembering that the dimensions of $\nu$ are $M/LT$) shows that $b \propto \nu D^{5/3}$ ($D$ being a characteristic linear size of the apparatus). Equating $\tau_{\text{signal}}$ and $\tau_{\text{noise}}$ (i.e., S/N = 1), noting that $M \propto D^3$, and solving for $\eta$, we see that

$$\eta_{\text{min}} \propto \frac{1}{D^{52}} \left( \frac{2k_bT}{a} \frac{\nu \Delta f}{\cos \delta} \right)^{1/2}. $$

(5)

That is, the $k_bT$ limit on $\eta$ improves as $D^{-52}$, or the sensitivity of a given apparatus improves as the five-halves power of its size. For our new apparatus, the calculated 1$\sigma$, one-day Brownian motion limit on the equivalence of gravitational and inertial mass is $7 \times 10^{-14}$.

3. Discussion

In both the Princeton [4] and the Russian experiments [5], seismic disturbances turned out to be a primary noise limit for their systems; and to this type of noise, the fluid approach seems to be remarkably insensitive. We have yet to detect any effect attributable to seismic disturbances on the position of fluid pendulums. One explanation for this can be found by noting that at least for low frequencies, time-varying accelerations do not—as they do with a normal fiber—stretch the system. Also, the fact that the zero of the system is established by a macroscopic electrode array rather than a microscopic fiber under stress probably accounts to some extent for the observed insensitivity of the zero to vibrations (and for that matter, changes in temperature).

Of the various noise sources identified in the earlier experiment of Keiser and Faller [6], only the effect of the residual mass-quadrupole moments can be expected to be substantially worse in this new larger apparatus. This effect can be reduced by measuring the quadrupole moments of the float and systematically tuning them out by mass tuning (as was done on the small float); or one can remove the effect of residual mass-quadrupole moments on the data by directly measuring the changes in the gradients of the external gravitational field and then correcting the float’s response for these changing gradients. In the case of the large float, we expect to employ both methods.

A number of practical problems have been encountered as a result of scaling up the smaller system to the large one, which is presently under construction, and which is probably near the practical size limit for scaling. As one example, the float as received from the fabricator was not as round on top as we had hoped. However, by using a hydraulic jack to deform the top just past the elastic limit, we were able to improve its as-received roundness of ±0.035 cm to ±0.010 cm, resulting in a fractional out-of-roundness of $\Delta R/R \approx 1.6 \times 10^{-4}$. Also, due to the vastly increased size of the apparatus, price and ease of fabrication became much more critical factors in choosing the materials (e.g., not only were gold and silver out of the question, but even tungsten was). It appears that lead and copper are the most practical choices. Initially we had planned to use Pb and Zn, however, the Zn castings were not as nonmagnetic as a sample had led us to expect, and they were returned to the manufacturer. We have subsequently switched to Cu. The paramagnetism problem associated with CuO is avoided by using OFHC (oxygen-free, high conductivity) copper, and by gold-plating the masses to prevent any oxidation with time.

At least a part of the non-$k_bT$ performance at long periods of the Keiser-Faller experiment has now been traced to their use of an external auto-collarimeter to monitor the float’s angular position. This introduced not only the stability of the auto-collarimeter, but also the stability of the stand on which it was located as well as the mechanical integrity of the intermediate linkages between the auto-collarimeter stand and the container lid whose electrode array determined the "zero" of the fiber. A new sensing system (see Fig. 2) which we have devised for this experiment promises to have a much higher mechanical integrity in that the sensing optics are a part of the same container lid on which the zero-determining electrode array is located.

![Figure 2. Schematic of new angle-measuring system.](image-url)
the LED-photodiode pair, causing the line image to move on the split photodiode. This results in a change in the output of the two halves of this device, giving rise, when amplified, to a rotation-dependent voltage as shown on the right half of Fig. 2. With two such detection systems—one on each side—and proper summing of their outputs, the detector is insensitive to sideways motion of the float without the sacrifice of any of its sensitivity to rotation.

4. Conclusion

This new large apparatus is now in the final stages of construction. We are awaiting delivery of the (encompassing) magnetic shield before installing this apparatus in the "Spectroscopy Lab" in the sub-basement of JILA to begin a prolonged (possibly a year-long) shake-down of the full apparatus. We plan to put a 0.2 to 0.5-cm layer of oil on the surface of the water, which we have experimentally determined lowers the evaporation rate by a factor of about 1000. This should help avoid local evaporative cooling of the surface of the water (which could lead to thermal gradients) and prevent the subsequent recondensation of water on the float (which could produce undesired quadrupole mass moments). We also plan to make the initial tests without cooling; and, by establishing a positive upward temperature gradient, we will try to achieve sufficient temperature stability to inhibit convection. Cooling, while straightforward and serving to reduce the linear expansion coefficient, α, water—and therefore the tendency to convect [7]—by more than an order of magnitude (at 20 °C, α ≈ 7 \times 10^{-5}/°C, while at 4 °C, α ≈ 1.5 \times 10^{-7}/°C), extracts a certain price in experimental inconvenience. This is particularly true in a system of this size, and we would rather not have to pay this price unless it proves absolutely necessary. However, even without cooling, the scale of this experiment is such that the first improved results on the equivalence principle using this large float should not be expected until 1983 or 1984.

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References
