MODE INTERACTION IN U TrA

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Abstract

Photoelectric observations of the double-mode Cepheid U TrA previously reported by Oosterhoff (1957a) and Jansen (1962) have been reanalyzed using Fourier techniques. The two reported periods have been confirmed and it has been shown that no further statistically significant periodicity is present. The two observed modes are subject to a strong mode interaction, but it is not yet clear whether this will have any implications for the pulsational masses derived for double-mode Cepheids.

I. INTRODUCTION

The double-mode or beat Cepheids form a group of eight variables with remarkably uniform properties, particularly with regard to the ratio of the shorter to the longer periods (see Table 1). If the two periods are identified with radial fundamental and first overtone oscillations, linear pulsation theory may be used to yield a mass and a radius for each double-mode Cepheid, based on period observations alone (Petersen 1973; King, Hansen, Ross and Cox 1975). The resulting masses are in the range 1.2 $M_\odot$ to 2.0 $M_\odot$, approximately half the values suggested by evolutionary theory for stars crossing the instability strip at their luminosity (Rodgers 1970; Fricke, Stobie and Strittmatter 1971, 1972; Petersen 1973; King et al. 1975). This "Cepheid mass problem" is much more acute for these double-mode Cepheids than for single-mode pulsators, for which the mass discrepancy may now be largely removed by one or more of a number of plausible hypotheses (Iben and Tuggle 1972a,b; King et al. 1975; and references therein).

The determination of the period of the secondary pulsation for beat Cepheids has followed one of two approaches (Table 1). In general the more sophisticated autocorrelation analysis has been applied to Cepheids with fewer observations, while those with more measures have been analyzed by assigning beat period phase values, $\psi$, to various epochs, either from the shape of the light curve (Oosterhoff 1957b; Jansen 1962) or by searching for the highest maxima (Leotta-Janin 1967).

The best observed of the variables in Table 1 is U TrA for which there exist 603 blue and visual measures made in two series (but with the same equipment) over the years 1953–59. The 1959 data in particular are in long runs of up to 30 observations per night and sometimes made on consecutive nights (see Jansen 1962, Fig. 1). It was in examining this figure that the author became aware that additional periodicity information may be derived from the existing observations. The long night runs make it possible to distinguish epochs when the mean primary light variation is crossed in the sense of rising superposed light variation (e.g., the night of
Table 1
Period Analyses of Double-Mode Cepheids

<table>
<thead>
<tr>
<th>Star</th>
<th>Reference</th>
<th>No. of measures</th>
<th>Analysis</th>
<th>$P_{\text{prim}}$ (d)</th>
<th>$P_{\text{secon}}$ (d)</th>
<th>Ratio</th>
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<td>TV Cas</td>
<td>Oosterhoff (1957b)</td>
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<td>2.13930</td>
<td>1.5183</td>
<td>0.710</td>
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<td>0.711</td>
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<td>Oosterhoff (1964)</td>
<td>165 p.g.</td>
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<td>2.1993</td>
<td>0.703</td>
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<td>BK Cen</td>
<td>Leotta-Janin (1967)</td>
<td>49 p.e. (+ p.g.)</td>
<td>$\psi$ method</td>
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<td>2.2366</td>
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<td>Gusev (1967)</td>
<td>117 p.g.</td>
<td>$\psi$ method</td>
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<td>1.343</td>
<td>0.709</td>
</tr>
<tr>
<td>VX Pup</td>
<td>Stobie (1970)</td>
<td>17 p.e.</td>
<td>--</td>
<td>3.01172</td>
<td>2.136</td>
<td>0.709</td>
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<td>AX Vel</td>
<td>Stobie and Hawarden (1972)</td>
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<td>auto-correlation</td>
<td>2.59285</td>
<td>3.6731</td>
<td>0.706</td>
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<tr>
<td>Y Car</td>
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<td>auto-correlation</td>
<td>3.639760</td>
<td>2.559</td>
<td>0.703</td>
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JD2436807) and in the sense of falling superposed variation (e.g., JD2436753). Inspection of these epochs indicates that the superposed variation contains a period of just over 1 day, which may be refined to $1^{d}067\pm0.002$ by taking epoch differences in order of increasing time interval. This is not an independent period but is related to the two previously reported periods being at the sum of their frequencies. It is thus a cross-coupling mode due to nonlinear interaction between the two oscillations.

In view of this result, and in view of the importance of understanding the mass discrepancy for the double-mode Cepheids, we have made a Fourier analysis of the data for U TrA. Our conclusion is that the light variation is completely explained by the two previously reported modes, but that there is considerable mode interaction.

II. FOURIER ANALYSIS

We may analyze the light variations of a star with two periods, $P_1$ and $P_2$ (days), and corresponding frequencies $f_1$ and $f_2$ (day$^{-1}$), in terms of the Fourier expansion, of order $n$:

$$L = <L> \left\{ 1 + \sum_{i,j=0,n}^{} (a_{ij} \sin \theta_{ij} + b_{ij} \cos \theta_{ij}) \right\}$$

(1a)
\[ L = \langle L \rangle \left\{ 1 + \sum \! A_{ij} \cos(\theta_{ij} - \phi_{ij}) \right\} \]  

(1b)

where

\[ \theta_{ij} = 2\pi(i f_1 + j f_2) t \]  

(2)

\[ a_{ij} = A_{ij} \sin \phi_{ij} \]  

\[ b_{ij} = A_{ij} \cos \phi_{ij} \]  

(3)

and \( <L> \), the average luminosity, is defined by the above equations. The terms with \( i j = 10, 20, \ldots, n0 \), describe the primary variation and its harmonics, those with \( i j = 01, 02, \ldots, 0n \), describe the secondary variation and its harmonics, while all other terms describe the cross-coupling interaction between the two modes.

We have least-squares fitted the observations of U TrA to such Fourier expansions at several orders. The quality of the fit may be described by the standard deviation, \( \sigma \), of the residuals between the observed light values and the series values. The least-squares fit is most conveniently made with the Fourier expansion expressed in the form (1a), but our results are tabulated in terms of mode amplitudes, \( A_{ij} \), and phases, \( \phi_{ij} \), for the equivalent form (1b).

Equation (1) with \( j = 0 \) becomes a simple Fourier series representing a single frequency pulsation. We have used least-squares fitting to the series in this form to check the reported periods for U TrA, and also to search for further periodicities in the residuals after the two-mode Fourier fits. In this method the run of standard deviation for the Fourier series fit, \( \sigma \), with test frequency, \( f \), provides the frequency analysis spectrum. It may be shown that for an adequate distribution of the observations with phase of the variation, the familiar periodogram method of frequency analysis is equivalent to the first order Fourier search by least-squares fitting; the Fourier technique has the advantage, however, that it may be generalized to higher order if desired. This was done in the case of the initial period check in U TrA where fitting to the third order Fourier series was used. The \( \sigma-f \) plot will have a minimum both at the frequency associated with the periodicity being sought, and also at alias frequencies reflecting the epochs at which the observations were obtained (Wehlau and Leung 1964). The two principal alias structures surround the true frequency at separations of 1 cycle per day (reflecting the night-to-night pattern of observing) and of 1/T cycles per day, where T (days) is the total time span of the observations.

III. RESULTS FOR U TrA

(a) Observational Data

Two series of photoelectric observations exist for U TrA. Oosterhoff (1957a) made 101 measures in the blue and visual during the years 1953-56, while Jansen (1962) has reported an additional 502 measures made in 1959. The observations of Oosterhoff were reduced to the photometric S-system of the Cape of 1953. Unfortunately it is not clear to which system the (relative) magnitudes of Jansen were reduced, although they appear to have been made with the same instrumental arrangements. This being the case we have followed Jansen in deriving simple transformations without color terms. We find that both the earlier and the later observations have the same mean luminosity in the fifth order Fourier solution (see §III c below) if
\[ \Delta V = SPv - 7.78 \]
\[ \Delta B = SPg - 8.08 \]

which are almost identical with the relationships found by Jansen.

b) Period Check

The primary and secondary periods reported for U TrA (Jansen 1962) were first checked before a full Fourier analysis was attempted. The V observations were used for this purpose.

For each period the observations were pre-whitened by removing the mean variation associated with the other period. Stobie (1970) has described an iterative method for separating two mean light curves, which was employed here. The mean variation for each period was found by averaging all the observations in each of 20 phase bins (intervals of \( \Delta \phi = 0.05 \)) corresponding to that period, and making a quadratic fit, at any phase \( \phi \), to the nearest three bin averages. Separation (all bin averages repeating to 0.0005) was achieved in four iterations.

After pre-whitening a third order Fourier analysis (as described in §II) was applied to determine the periods. The results are shown in Fig. 1. In both cases the principal minimum is surrounded by aliases at constant frequency intervals of \( \Delta f = 0.0009 \, \text{d}^{-1} \). This reflects the total time span of the observations (~1100 d) since virtually all were obtained in 1956 (83 observations) and 1959 (501 observations). We were unable to improve Oosterhoff's (1957a) value for the primary period of U TrA, but have measured the secondary period from Fig. 1 obtaining a value only slightly different from Jansen's (1962). We adopt the elements and phase formulae

Fig. 1. Primary and secondary period determination for U TrA. In each case the V observations have been pre-whitened by removal of the mean variation at the other period, before being least-squares fitted to a third order Fourier series at each of the test frequencies shown. \( \sigma \) is the standard deviation of the residuals (in L/<L>) after the Fourier fit. The alias structure reflects the 3 year observing interval of the observations.
\[
JD_{\text{max} 1} = 2436750.96 + 2.568438 \text{ E} \\
\pm 2 \quad \pm 17 \\
\phi_1 = 0.3893417 \quad (\text{JD} - 2430000) \\
\pm 26
\]
\[
JD_{\text{max} 2} = 2436751.09 + 1.82485 \text{ E} \\
\pm 2 \quad \pm 4 \\
\phi_2 = 0.54799 \quad (\text{JD} - 2430000) \\
\pm 1
\]  
(5)  
(6)

where the epochs of maximum light have been calculated from Fig. 2 below.

(c) Double-mode Fourier Analysis

Third, fourth and fifth order double mode Fourier analyses were made for both the V and the B observations of U TrA using the periods just derived and the method described in §II. Table 2 gives the amplitudes and phases [see equation (1b)] for the several modes, together with the standard deviations of the remaining residuals after each Fourier fit. The latter are somewhat larger for the B than for the V solutions but only in proportion to the respective amplitudes of the light variation. The \( \sigma = 0.0017 \) value obtained for the V residuals after the fifth order fit may be compared with a value of \( \sigma = 0.0053 \) which is the best attainable with the two periodicities alone, i.e., neglecting the cross-coupling terms.

The amplitudes and phases for the fifth order fits are accompanied by error estimates; these are standard deviations calculated on the assumption that the actual stellar variation may be completely represented by a Fourier fit of that order, and that the uncertainty of the amplitudes and phases is thus due entirely to observational errors. One may obtain some feeling for how well this assumption is justified by comparing the amplitudes and phases for the several modes as the order of the Fourier fit is increased. It is our conclusion that the data for the fifth order solution are meaningful, and that the total uncertainties, due to both observational error and the neglect of still higher order modes, are about twice the standard deviations shown. It will be seen from Table 2 that the cross-coupling modes are quite substantial, having amplitudes which are a sizable fraction of the modes associated with the secondary period and its harmonics.

In Fig. 2 we have plotted the V light variation at both the primary and the secondary periods. In each case the observations were whitened by removal of all the fifth order Fourier fit modes associated with the other period and with the cross-coupling. The remaining scatter of points thus represents the residuals after the fifth order fit. It will be seen that the Fourier analysis, including the mode interaction, provides a very clean picture indeed of the light variation. Figure 2 was used to define the maximum light epochs for the primary and secondary variation elements which were given in equations (5) and (6).

(d) Periodicity Search for Residuals

We made a first order Fourier periodicity search for the V residuals after the third and fifth order analyses using the method described in §II. Since the frequency range searched (0.3 - 2.0 d\(^{-1}\)) was large compared with the alias spacing of 0.0009 d\(^{-1}\) it would have been prohibitively expensive to search the data as they stood. Accordingly we adopted the device of Fitch (1967) by grouping the residuals
### Table 2: Amplitudes and Phases for the Double-mode Fourier Fits to the Observations of U TrA

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<th>Mode</th>
<th>A&lt;sub&gt;i&lt;/sub&gt;</th>
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<th>3</th>
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#### Footnotes
- **0.032** (3rd order)
- **0.023** (4th order)
- **0.017** (5th order)

**Standard deviations of residuals after Fourier fits:**
- **0.049** (3rd order)
- **0.032** (4th order)
- **0.023** (5th order)
Fig. 2. Primary and secondary \( V \) light variations for \( U \) \( \text{TrA} \). In each case the observations have been whitened by removal of the variation associated with the other period and the cross-coupling as determined by the fifth order double-mode Fourier analysis. The remaining scatter reflects the residuals after the fifth order fit. Dots indicate 1953-1956 observations; full curves the mean nightly runs of the 1959 observations.

into six sets each spanning a 40 day interval, carrying out the search separately in each set, and then combining the results to obtain a total \( \sigma \) for all the residuals. In this way the alias structure is broadened to a separation of \(~0.025\ \text{d}^{-1}\), and a test frequency separation of \(0.005\ \text{d}^{-1}\) was entirely satisfactory for the periodicity search.

The "periodograms" for the residuals after the two double-mode Fourier fits are shown in Fig. 3, which provides convincing evidence that the third order double-mode solution is far from adequate in describing the \( V \) light variations, while the fifth order solution is entirely adequate. In particular, there is no remaining periodicity of any significance which is not accounted for by the modes.
Fig. 3. First order Fourier periodicity search on the V residuals after the third order (top) and fifth order (bottom) double-mode Fourier fits for U TrA, grouped into 40 day intervals. \( \sigma \) is the standard deviation of the residuals (in \( L/\langle L \rangle \)) in the Fourier search. The most prominent fourth order modes and their 1 cycle per day aliases \( (\text{A}) \) are indicated in the third order residual spectrum. It will be seen that no appreciable periodicity remains after the fifth order fit.

of the primary and secondary variations and their interactions to fifth order. The most prominent of the periodicities remaining after the third order fit may be identified with the strongest 4th order terms and their 1 cycle per day aliases (indicated on Fig. 3).

**IV. DISCUSSION**

If we regard the sum \( \Sigma A_{ij}^2 \) for any group of modes as a measure of the pulsational energy associated with that group, then the fifth order Fourier solutions we have obtained indicate that the energies associated with the primary pulsation, \( E_1 \), the secondary pulsation, \( E_2 \), and the cross coupling, \( E_x \), are in the ratio:

\[
E_1 : E_2 : E_x = 1.00 : 0.14 : 0.17 \quad (V \text{ observations})
\]

\[
E_1 : E_2 : E_x = 1.00 : 0.15 : 0.18 \quad (B \text{ observations}) \quad (7)
\]

Clearly the nonlinear cross-coupling is a quite significant factor in the pulsational behavior of U TrA. It is appropriate to ask whether this could have any significant effect on the periodicities. This is particularly important because the "pulsation" masses and radii for double-mode Cepheids are derived from the fundamental and first overtone periods using period fitting formulae that have been based on linear pulsation calculations. Furthermore the derived masses and radii are extremely sensitive to the periods; a change in the periods of as little as one percent will affect the derived mass by as much as 20 percent.

Unfortunately it does not appear possible to give a satisfactory answer to this question at present. Nonlinear calculations for single-mode pulsators (e.g., King, Cox, Eilers and Davey 1973; Stellingwerf 1975a) indicate that the nonlinear periods agree with the linear results to better than 3 percent. There is, however, only one nonlinear calculation which has succeeded in producing a double-mode pulsation --
Stellingwerf's (1975a) aperiodic limit cycle model of a 0.578 M₀, 63 L₀, 6200 K, population II, RR Lyrae star. Stellingwerf (1975b) has considered the implications of this model for the double-mode Cepheid problem. He performed an autocorrelation periodicity check on an 8 day segment of the data from his model calculations in the same manner as if they had been observational data from a pulsating star. He confirmed that his "observed" periods and the linear and nonlinear calculation periods all agreed within 0.5 percent for both the fundamental and the first overtone. Stellingwerf also derived mean fundamental and first overtone light curves for his aperiodic solution and compared them with the periodic limit cycle light curves computed for these modes separately. He noted two effects that could be indicative of the type of mode interaction we have found in U TrA: (i) the first overtone amplitude is reduced in the mixed mode case, and (ii) at phases where the two maxima coincide the peaks are sharper than the sum of the separate variations. It would be of great interest to apply the type of double-mode Fourier analysis we have used to Stellingwerf's smoothed calculation data to see if the calculations are subject to the same degree of mode interaction as the double-mode Cepheids themselves. If they are, then we must accept that these effects produce little change in the periods, and the double-mode Cepheid mass problem remains unsolved. If the interaction in the calculated models is considerably less than observed, however, the possibility arises that sufficient period changes may yet be necessary to bring the evolutionary and pulsation masses into agreement.

There are at present two suggestions for the physical state of the double-mode Cepheids (Stellingwerf 1975b). They may be stars at the cool (<5850 K) edge of the instability strip which have a stable mixed-mode pulsation, or they may be currently mode switching at Teff ≈ 6300 K. If the latter is the case, then the e-folding time for the growth of one mode at the expense of the other has been estimated by Stellingwerf to be ~80 yrs. We would thus expect a change of ~30 percent in the relative energies associated with the two modes in the period since the observations here under discussion were made. Such a change should be readily detectable in view of the precise interpretation we have found for the old observations. A new and similarly extensive set of observations of U TrA is thus highly desirable.

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