Measurements of Intensity Fluctuations in a Tricritical Laser

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ABSTRACT

Steady-state intensity fluctuations of the laser with a saturable absorber are studied experimentally in a regime where the leading nonlinearity is of the fifth order in the field amplitude.

\[ \mathcal{E}(t) = \frac{1}{2} \left[ A - \bar{A} - C + \frac{1}{2} B(\alpha s - 1)|\mathcal{E}|^2 \right. \]
\[ - \frac{3}{8} \frac{B^2}{\bar{A}} (\alpha s^2 - 1)|\mathcal{E}|^4 \left. \right] \mathcal{E} + \sqrt{\bar{A}} Q(t), \]

(1)

where \( A, B, \) and \( C \) are the gain, self-saturation, and loss parameters of Scully and Lamb theory [5]. They can be expressed in terms of atomic transition dipole moment, decay and pump rates, and detuning of laser mode frequency from the center of atomic line. Parameters with a bar, \( \bar{A} \) and \( \bar{B} \), refer to the absorber. In general these parameters are complex [5,6]. For intensity fluctuations, however, only their real part is relevant. Consequently the parameters in Eq. (1) represent the real part of the generalized gain and self saturation parameters. \( Q(t) \) is a complex Gaussian white noise process with zero mean and unit intensity representing spontaneous emission fluctuations. \( \alpha = \bar{A}/A \) is the ratio of gain and absorption parameters and \( s = (\bar{B}/\bar{A})/(B/A) \) is the ratio of the saturation intensities for the laser and absorber atoms.

Equation (1) has been derived in the so-called rate equation approximation. This has been shown to be a good approximation for a laser where both the gain and absorbing media are strongly Doppler broadened. This is the case for our laser which consists of a HeNe gain cell and a pure Ne absorber cell. A perturbative approach was used to calculate induced polarization up to terms of fifth order in the field amplitude. In the ordinary laser only terms up to third order are required for the establishment of a steady-state. For the LSA terms up to fifth order are required in order to ensure the establishment of a steady-state because the third order term may become positive when \( \alpha s > 1 \). In this case the fifth order term, which is necessarily negative when
\( \alpha > 1 \), is needed to ensure a steady state. We are interested in this regime where

\[
s > 1 \quad \alpha s \geq 1. \tag{2}
\]

In view of the condition \( s > 1 \), the use of a perturbative approach for the absorber may not be valid. However, we continue to adopt this model because of its simplicity and because, at least qualitatively, this model predicts all the observed features of the LSA. In any case, Eq. (1) can still be considered as a phenomenological model to describe our system.

By a suitable scaling of the variables we arrive at the following equation of motion for the complex dimensionless field amplitude \( E(t) \)

\[
\dot{E}(t) = E[(a + b |E|^2 - |E|^4)] + q(t), \tag{3}
\]

where the pump parameter \( a \) and the saturation parameter \( b \) are given by

\[
a = (1 - \alpha - \mathcal{A}/\mathcal{A}) \left( \frac{8A^2}{3B(\alpha s^2 - 1)} \right)^{1/3},
\]

\[
b = (\alpha s - 1) \left( \frac{8A}{3B(\alpha s^2 - 1)} \right)^{1/3}, \tag{4}
\]

and \( q(t) \) is a complex Gaussian white noise process with zero mean and normalized strength 4. Equation (3) can be converted into a Fokker-Planck equation for the probability density for the electric field amplitude. The steady-state solution of this equation describing fluctuations of light intensity \( I = |E|^2 \) is

\[
P_s(I) = \text{const} \times \exp \left( \frac{1}{2} aI + \frac{1}{4} bI^2 - \frac{1}{6} I^3 \right). \tag{5}
\]

This equation predicts a variety of fluctuation properties for the LSA. In particular, Eq. (5) predicts bistability for \( b > 0 \) in the pump parameter range \(-b^2/4 < a < 0\) with a first order like phase transition at \( a = -3b^2/16\). For \( b < 0 \) only monostable behavior is predicted for all values of \( a \) with a second order like phase transition at \( a = 0 \). The point \( a = 0 = b \) is a special point at which the laser exhibits the so-called tricritical behavior [7].

An examination of equation (4) suggests that, for fixed values of \( s \) and \( \mathcal{A} \), by adjusting \( \alpha = \mathcal{A}/\mathcal{A} \) properly it is possible to access the point \( b = 0 \). The pump parameter \( a \) can then be varied by varying losses \( \mathcal{A} \) and one can study the fluctuation properties near the tricritical point. For \( b = 0 \) the leading nonlinearity is of the fifth order in the field amplitude. Equation (5) then leads to

\[
P_s(I) = \text{const} \times \exp \left( \frac{1}{2} aI - \frac{1}{6} I^3 \right), \tag{6}
\]

which describes fluctuation properties of the LSA near the tricritical point. From this equation we can calculate various moments of light intensity. Figure (1) shows the behavior of mean light intensity \( < I > \) as a function of \( a \) and relative variance \( \kappa_2 = < (\Delta I)^2 > / < I >^2 \) as a function of \( < I > \). For comparison we have also shown the corresponding quantities for the ordinary laser which obeys the steady-state intensity distribution

\[
P_s(I) = \text{const} \times \exp \left( \frac{1}{2} aI - \frac{1}{4} I^2 \right). \tag{7}
\]

Although the relative intensity fluctuations for the LSA may appear smaller than those for the ordinary laser, in absolute terms they are, in fact, dramatically enhanced. This can be seen by comparing the scaling of field amplitude in the LSA with the corresponding scaling for the ordinary laser [6]. For the LSA the scaling is \( \sim (B/A)^{1/6} \), whereas for the ordinary laser it is \( \sim (B/A)^{1/4} \), so that the intensity fluctuations in the LSA are larger by \( \sim (A/B)^{1/6} \sim 10 \) for a He:Ne laser with \( s \approx 9 \).

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Figure 1(a). Variation of the mean light intensity \( < I > \) as a function of pump parameter \( a \).

Figure 1(b). Relative variance of the light intensity \( \kappa_2 = < (\Delta I)^2 > / < I >^2 \) as a function of mean light intensity \( < I > \). The curves for the LSA are for \( b = 0 \).
In the experiment a He:Ne laser operating at 633 nm was used to investigate the fluctuation properties of the LSA. The laser cavity was 40 cm long and contained a pure Ne absorber tube in addition to a cold cathode He:Ne gain tube. The absorber tube was a hot cathode discharge tube and the ratio of the pressures in the two tubes was 10:1 which gave a value of approximately 9 for the ratio s. Because of pressure shifts of atomic lines the centers of atomic lines in the two tubes were not coincident. The laser was operated with its frequency close to the center of absorption line which was about 30 MHz wide. On the other hand, the gain line was relatively broad about 270 MHz wide. A high degree of stability was necessary in order to keep the laser operating in the appropriate regime. The whole laser assembly was enclosed in a temperature stabilized housing with acoustic insulation. The laser drift was found to be no more than 5 MHz in a period of several minutes. The light coming out from the laser was divided into two. One part was used to illuminate a photomultiplier tube (PMT) whose output was fed to a feedback loop. The voltage output from the PMT was compared with a reference voltage and any difference between the two was amplified several hundred times. The resultant voltage was applied to a piezoelectric transducer (PZT) which controlled laser light intensity by pushing a knife edge in and out of the laser beam. In this way the operating point of the laser was held constant to an accuracy of about 1%. The operating point of the laser was varied by changing the reference voltage.

\begin{equation}
< n > = \frac{\sum n N_n}{\sum N_n} = K < I >,
\end{equation}

\begin{equation}
< (\Delta n)^2 > = \frac{\sum (n - < n >)^2 N_n}{\sum N_n} = K^2 < (\Delta I)^2 >,
\end{equation}

where \( N_n \) is the number of counts stored in channel \( n \) and \( K \) is a scale factor that relates channel number to light intensity. From Eqs. (7) and (8) we find that relative variance \( \kappa_2 = < (\Delta I)^2 > / < I >^2 \) is independent of the scale factor \( K \). In order to compare the experimentally measured \( \kappa_2 \) with the theoretical predictions we need to determine the scale factor \( K \) that converts channel number to the dimensionless intensity used in the theoretical calculations. This was done by plotting measured \( \kappa_2 \) against \( \log_{10} < n > \) and comparing it with theoretical plots of \( \kappa_2 \) vs \( \log_{10} < I > \) for several different values of \( b \). The scale factor \( K \) then corresponds to a shift of the \( \log_{10} < I > \) axis. This procedure also determines \( b \). Once \( K \) and \( b \) are known mean intensities can be converted into pump parameter values. In order to find the point \( b = 0 \), a search was carried out by measuring \( \kappa_2 \) as a function of \( < I > \) for several different values of the pumping current in the absorber cell. This allowed \( b \) to be determined to an accuracy of about \( \pm 0.1 \). It is found, as expected, that for a fixed value of pumping current in the absorber cell, a single value of \( b \) fits data quite well. The results of this procedure for \( b = 0 \) are shown in Fig. 2. The dots represent the experimentally measured values. Statistical errors are smaller than the dot size. In practice, the errors due to electronic noise were found to be more important, especially at low excitation levels below threshold \( a < 0 \). For larger excitation its effect was not significant. For comparison we have also plotted \( \kappa_2 \) for the ordinary laser. The continuous curves have been computed from Eqs. (6) and (7). It can be seen that in the region of threshold the intensity fluctuations of the LSA are different from those of the ordinary laser. This difference is a direct reflection of the different saturation characteristics of the two lasers.

![Figure 2](image)

Figure 2. A comparison of the measured and theoretically predicted relative variance of light intensity for the LSA with \( b = 0 \) and the ordinary laser. Theoretical curves (solid lines) are derived from Eqs. (6) and (7).
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