Liquid-Supported Torsion Balance as Gradiometer

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We employ a liquid-supported torsion balance as a fixed-site long-term curvature variometer. The traditional torsion fiber is replaced by liquid support and electrostatic positioning. Thus the torsion constant is adjustable by varying the voltage applied to the torque electrodes, while the centering voltage remains constant. The sensitivity of this type of gradiometer will be discussed, along with critical parameters for success. Preliminary data will be presented.

The world's first gradiometers were the fiber-supported torsion balances (FSTB's) used by Baron Roland von Eötvös in the late nineteenth century. A wide variety of supported mass configurations were developed (the two basic ones are shown in Figure 1) and such torsion balance gradiometers were used in geophysical research through about 1940, at which time gravimeters replaced them. Torsion balances themselves were invented in the late eighteenth century and have been, and still are, used in a variety of applications for measuring very small forces. In our own work on the Einstein Equivalence Principle we have constructed,
FIGURE 1.
Baron Roland von Eötvös' FSTB Gradiometers (1896)
[Ref. 1, pp. 369-370]
ostensibly as an auxiliary instrument, a pair of torsion balance gradiometers to monitor longterm (T = 1 day) changes in what may be loosely called the "horizontal gravity gradients" (more precisely: in the curvatures of the gravity level surface). However, due to the often bizarre and always subtle problems associated with the use of fibers, and due to our desire to increase the sensitivity of the torsion balance, we have used a radically different design. FSTB's have an inherently limited sensitivity to mass-dependent forces as may be seen by noting that the supported weight (test masses and beam) is proportional to the cross-sectional area of the fiber, while the observed angle is proportional to the square of the area. This results in a maximum sensitivity as determined by the fiber material (yield strength versus elasticity) and by the resolution of the detection system.

To overcome this sensitivity limit, the supported object can be immersed in a liquid, to decrease the load on the fiber, as John Henry Poynting first suggested. However, to completely avoid the use of fibers, the recently-perfected liquid-supported torsion balance (LSTB) uses an electrode array to provide the centering and restoring torque usually provided by the fiber. Our LSTB curvature variometer is pictured in Figure 2. An advantage of this "electrostatic fiber" is that the voltage applied to the center electrode may be (and usually is) much larger than that applied to the torque electrodes, which allows the centering and torquing forces to be effectively independent. (The force between two electrodes is proportional to the square of the applied voltage, and for spherical electrodes the force is almost in inverse proportion to the gap between the upper and lower electrodes.) Furthermore, the torque voltage is readily adjustable so that we can, for
FIGURE 2.
Schematic of our LSTB Gradiometer (1986)
example, let the system thermally equilibrate with high torque voltage ("stiff fiber") and then simply lower the torsion constant to the correct value for critical damping (the optimum operating point). Similarly, we can quickly establish the true "zero" of the system so as to be able to measure the dc curvature values. A polyatomic inert gas (such as well-filtered N₂) used as the fill gas maximizes the dielectric strength of the electrode gaps and thus yields the maximum stiffness of the electrostatic fiber.

In a LSTB there are only a few undesirable mechanical oscillation modes, most of which have frequencies much higher than the torsional oscillation frequency (T = 10 min), and all of which are rapidly damped by the liquid. By contrast, FSTB's are plagued with a great number of high-Q modes which readily couple to the torsion mode. The bobbing mode of a LSTB (T = 1 sec) has never been observed to couple, while the off-centering mode (T = 20 min) is rarely excited and is highly damped. There are two tilting modes (T = 1 sec) one of which (or both, if they are degenerate) will of necessity couple to the torsion mode. This may be most readily seen by noting that a book (held shut with a rubber band) may be flipped into the air about any of three principal axes — and about two of those the rotation is stable. About the axis with the intermediate moment of inertia (or about the two axes of equal moments of inertia) the motion is unstable. Due to various practical constraints, in any real LSTB the moment of inertia about the torsional axis will always be smaller than the other two moments, and hence the torsional oscillation is always stable. Moreover, the coupling of the tilting mode(s) to the torsional mode only results in short episodes of small high-frequency "noise" (and the tilt modes are rarely excited).
We use a transmission-optics variant of the Gauss optical lever as our angle detection system. The infrared LED (noted in Figure 3) is a point source and the lens is a (borosilicate glass) rod with its axis vertical. The rays from the LED pass through the rod perpendicular to its axis and are focused into a vertical line. The focal plane (i.e., the line image) is made to coincide with the plane of the split photodiode. The two currents generated in the photodiode halves are converted to voltages in low-current-noise preamplifiers and their sum and difference formed. The four sums and four differences are combined in appropriate ways to generate radial translation (off-centering) and angular azimuth signals. The net gain is such that we have a measured sensitivity of about 30 mV/arcsec, using 4.5 mA in the LED.

The second derivatives of $U$, the scalar gravity potential, form a tensor, whose nine components are reduced by the constraints imposed by the curl of $g$ and the divergence of $g$ to five linearly independent components. Of these five we happen to be interested only in those two which correspond to masses near the instrument horizon, that is the two curvature values. In fact the azimuthal torque on a LST8 is given by

\[
N = \frac{1}{2} \sin(2\alpha) (I_x' x' - I_y' y') (U_{yy} - U_{xx})
\]

where $\alpha$ is the azimuth of the instrument relative to the axes of the principal radii of curvature, $U_{xx}$ and $U_{yy}$ are the two principal curvature values (so, e.g., $R_x = g_z/U_{xx}$), and the $I$'s are the two horizontal moments of inertia. Because of the symmetry of our float this reduces to (along the sensitive axes which are at $\pm 45^\circ$ to the line joining the test masses):
FIGURE 3.
Schematic of Optical Position-Sensing System

(LED's and Photodiodes attached to tank, lenses attached to float)
(2) \[ N = (mr^2)(U_{yy} - U_{xx}) \]

where \( m \) is the mass of each test mass (2.935 kg) and \( r \) is its radius (8.65 cm). For an external (point or spherical) mass \( M \) located on the sensitive axis (\( x \) or \( y \)) at a distance \( R \) from the axis of the torsion balance, we have for the torque

(3) \[ N = (mr^2)(2GM/R^3) \]

For our apparatus, with an external mass \( M = 150 \) kg at \( R = 2 \) m (corresponding to \( U = 2.5 \) EU, 1 EU = \( 10^{-9} \) sec\(^{-2} \)) we find a torque of about 0.55 mdyne-cm. Our critically-damped torsion constant is 1.09 dyne-cm/rad (at a voltage of about 55 V rms, electrode gap about 0.1 in.), so that the resultant angle is 0.50 mrad or 104 arcsec. This gives a sensitivity of over 400 arcsec/EU, or to put it another way the 2.5 EU signal is over 40 times the rms noise (from all sources) in our apparatus.

The noise sources may be conveniently characterized as internal or uncorrelated (i.e., uncorrelated with any external signal) and external or correlated. Tests have shown that the external noise sources have a very small effect.

The noise is observed to be insensitive to atmospheric pressure (the tank is sealed and fairly rigid). We have carefully avoided materials (or inclusions) of high magnetic susceptibility, and we keep all large magnets well away from the apparatus, so that the only significant noise of magnetic origin is due to eddy currents produced by rotating magnetic fields. It is quite possible to produce rotating fields of sufficient frequency and intensity to cause the float to rotate (at up
to 1 rpm), and we do so on occasion, but the ordinary 60 Hz rotating
magnetic fields produced by lab motors have no observable effect. (In
addition, we have obtained 62-mil high-permeability MIL-N14411C 80%-Ni
magnetic shields which we plan to install.) Ordinary floor vibrations
(people jumping at R = 2 m, e.g.) have no observable effect, but large
earthquakes (as in Figure 4) or pathological (badly unbalanced) machines
(as in Figure 5) which produce large horizontal accelerations or large
tilts of the floor do have an effect. Such noise is intermittent and
readily identifiable, and causes no real difficulty.

Internal noise sources cause more difficulties, none insurmount-
able. The torque and center voltages are ac to avoid the possibility of
static charge accumulation (though all exposed surfaces of the float are
conducting and grounded, and tests with dc voltages never showed any ef-
fects attributable to static charge). The center voltage is regulated
to ∆V/V = 10⁻² and the torque voltage to ∆V/V = 5x10⁻⁴. The (spurious)
change of angle, ∆θ, that arises from a shift in torque voltage is
10⁻³θ. The observed (gaussian) voltage noise in the optical detection
system amounts to 15 mV (pp) - i.e., 0.5 arcsec.

The dominant noise sources are due to convection currents in the
liquid, which may arise either from impurity concentration gradients or
from thermal gradients. Nothing is more crucial to the success of a
LSTB than the elimination of these two problems.

Water is the universal solvent, and we go to great lengths to pur-
ify the water. We use: predeionization, followed by 5µm filtration, UV
sterilization, 1 µm filtration, carbon filtration, deionization, and
0.22 µm filtration. All but the first stage are in a recirculating
system.
20 Sept 1985
Second Mexican Quake

6 Sept 1985, 03:47:29 GMT
43.2°N, 110.8°W; R = 4.6
(near Jackson, Wyoming)

FIGURE 4.
Effects of Earthquakes on LSTB
FIGURE 5.

Horizontal Floor Accelerations (2 μV/μm)
The resistivity of acceptable water is 10 MΩ-cm (though in practice we usually obtain 15 MΩ-cm). Experience shows that it is necessary to add a liquid non-ionic surfactant to the water to form a liquid surface monolayer which displaces the otherwise inevitable solid surface layer formed of residual (less than 1 ppm) impurities. We use Kodak Photoflo 2100 (not the premix Photoflo 200) or Triton X100, at about 500 ppm. In addition it is crucial to protect the aluminum from corrosive attack by the water. (Such attack is aggravated by dissolved oxygen, so we deoxygenate the water by bubbling filtered N₂ through it.) The main defense that we use against this otherwise inevitable corrosion is the application of a commercial chromate-conversion coating to the well-cleaned and unetched aluminum (as per MIL-C5541). We do not dissolve chromate ions in the water (which has also been shown to prevent corrosion effectively) as there is some indication that they react with our surfactants. Without this protective coating the LSTB fails ("locks up" in a weak surface gel) in about a week, but with the coating we can operate for eight months or more.

Water is used because it is cheap, safe, readily purified, and because it alone of all liquids has a maximum density point. Most thermal convection currents in a gravity field are driven by buoyancy and these buoyancy forces arise due to the temperature-dependent density of the liquid (similar to the principle of the hot-air balloon). At 3.98 °C the expansion coefficient of water is zero — so that small thermal gradients do not cause convection currents. There are five distinct sources of thermal gradient forces on a float. Two of these forces (which arise from the temperature-dependance of the surface-tension and the pressure) decenter the float and need not concern us here. The
other three forces (which are due to various types of convection currents) can torque the float. The dominant torquing convection current is the circumferential current in the water between the vertical walls of the float and tank. This is analogous to the "double-pane window" problem, where the idea is to minimize the heat flux. Here however we seek to minimize the convection (it can never go to zero unless the temperature gradient or the width of the gap is exactly zero). In the case of this circumferential current the maximum allowable $\Delta T$ (across the tank) for a specified $\Delta \Theta$ (at critical damping) scales as:

$$
\Delta T = (r_2 - r_1)^{-3}
$$

so that (all else being equal) we should minimize this gap.

We go to great lengths to insulate and control the temperature of our apparatus. A multilayer cubical enclosure (alternating insulation and reflective metal cans) three feet on a side surrounds the one-foot cylindrical tank. Around the innermost of these outer cans is wrapped flexible hose through which temperature-controlled water flows. Around this same can is wrapped a heater connected to a proportional-integral controller (gain 120 dB, time constant 11000 sec, stability better than 0.1 m$^\circ$C, resettability 1 m$^\circ$C). The temperature of this composite system is set to give 4.0 $\pm$ 0.1 $^\circ$C at the waterline of the tank. (A serendipitous effect of operation at 4 $^\circ$C is that the corrosive attack of water is slowed by over two orders of magnitude from its rate at 20 $^\circ$C.) The net effect is a stability of better than 0.5 m$^\circ$C on the tank over several days (Figure 6) and a sensitivity to room temperature fluctuations of less than 2 m$^\circ$C/$^\circ$C. We believe, in spite of all this, that
FIGURE 6.
Tank Temperature at the Waterline
(Start: 850915, 22:13)

FIGURE 7.

Data from Symmetric LSTB
our dominant noise term is temperature-induced convection currents.

In Figure 7 we present sample data from a run with an identical system in which the two masses were replaced with a disk (of the same total mass and moment of inertia about z). In such a device no torque should arise from curvatures and so any signal is "noise" — torques due to floor vibrations (the 10-min period) or torques due to convection currents. Note that in two weeks (16 Jan to 1 Feb 1986) the "zero" has drifted almost 0.2 V or 0.4 arcsec/day or 0.01 EU/day. Some sample data from the gradiometer system are presented in Figures 8 through 11. As can be seen, a claimed sensitivity of 0.1 EU would not be amiss. The drift rate in Figure 11 is less than 75 mV/day or 2.5 arcsec/day or 0.062 EU/day. (This drift was measured during a period we now have reason to suspect marked the beginning of the degradation of the water, after eight months of operation. Before this it seemed to be about 5 times less, consistent with the data of Figure 7.) It must also be noted that the response time is vastly slower than the various dynamic gradiometers now available. In fact the damping time is almost 20 min (so that the response to a step will be 95% complete after about three such times or one hour) which is about two orders of magnitude slower.

Future plans include the construction of three new machined float-and-tank sets (the current sets are modified spun-aluminum cook pots). These new sets will have a 0.5 cm gap between the float and tank which should reduce the effect of the convection currents by a factor of about 100, and thus floor vibrations and electronics, not convection, should be the dominant noise sources. In addition, the larger tungsten-alloy masses will be at a greater radius for a gain of a factor of 2.64 in sensitivity to curvatures. The first set is ready and will be given a
preliminary test as soon as the larger masses are ready.

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$U = 6.78 \text{ Eu}$

$30 \text{ min}$

$1.0 \text{ V}$

$33 \text{ sec}$

FIGURE 8.

Large Signal (17 Sept 1985)
FIGURE 9.
Small Signal (8 Sept 1985)

$u = 0.83 \text{ Eu}$

$0.1 \text{ V}$

$33 \text{ sec}$

$30. \text{ min}$
FIGURE 10.

Day (11 AM - 3 PM) and Night (1:30 AM - 5:30 AM)

(19 - 20 Aug 1985)
FIGURE 11.
Float Position and Fourier Transform
(Start 850819, 18:16)
1 Roland von Eötvös "Untersuchungen über Gravitation und Erdmagnetismus" Annalen der Physik und Chemie (Wiedmann) 59 (1896) 354-400.


6 O. V. Karagioz, V. V. Voronkov, V. P. Izmaylov "Effect of Swinging on Movement of a Torsion Pendulum" Determination of Gravity Constants and Measurement of Certain Fine Gravity Effects ed. Yuri D. Boul-


8 Nelson J. Newhard "Conversion Coatings for Aluminum: Chemical Properties and Applications" Metal Finishing 70.7 (July 1972) 49-53 and idem 70.8 (August 1972) 66-69. I am indebted to John Krogulski of Aluminum Amchem Co. for this reference and for many useful discussions.


