LATE STAGES OF CLOSE BINARY SYSTEMS — CLUES TO COMMON ENVELOPE EVOLUTION

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Those circumstances are outlined which theoretically lead to engulfment of one star by its companion, creating a common envelope binary. This evolutionary course is believed to lead to the formation of cataclysmic binaries, and other very compact systems containing degenerate components. We examine here the implications of these and other evolved binaries which, directly or indirectly, provide insight into the occurrence and nature of common envelope evolution: (1) The distribution of masses and orbital periods of Algol binaries shows clearly the deficiency or absence of systems in which the original primary had reached the giant branch prior to mass exchange. The absent systems would have been unstable to dynamical time scale mass exchange, and are therefore prime candidates for common envelope evolution. (2) Many, perhaps the majority, of the hot components in symbiotic stars are luminous, nuclear burning white dwarfs. It appears that these longer-period systems have somehow escaped or avoided common envelope evolution. (3) The apparent normality of the main sequence components in cataclysmic binaries, close binary planetary nebulae nuclei, and related
objects provides clear evidence that these components can neither have accreted nor lost a large fraction of their mass during common envelope evolution. (4) The distribution of the orbital parameters of barium stars shows evidence of tidal dissipation, even though the orbits are not fully circularized. Moreover, the companion masses appear to be correlated with the barium star masses themselves, and to be sufficiently small to necessitate placing the epoch of s-process enrichment in these systems earlier than the thermally-pulsing asymptotic giant branch phase. These circumstances suggest that mass transfer has occurred in these systems, but must have been quasi-conservative in character, in contrast with common envelope evolution.

1. **INTRODUCTION**

The past ten years have seen a growing realization of the central role played by common envelope evolution in the theory of close binary stellar evolution. In this context, the term common envelope refers to an extended envelope bound to a binary system within it, but, in contrast to classical W Ursae Majoris systems, neither synchronously rotating with the embedded binary, nor necessarily in hydrostatic equilibrium with it. In this sense, the concept was first fully developed by Paczynski (1976) and Ostriker (1975), although it can be traced to earlier work by Vauclair (1972), Refsdal, Roth, and Weigert (1974), and Sparks and Stecher (1974).

Common envelope evolution has been invoked in a number of contexts, most notably to explain the origin of cataclysmic variables, and to follow the future evolution of massive neutron star binaries, the two problems which motivated Paczynski and Ostriker, respectively, in their early work. In the broadest sense, there are two possible
circumstances which lead to common envelope formation. In one, a low-mass companion may be drawn into the envelope of a companion by secularly unstable tidal dissipation, if the sum of the moments of inertia of the two components exceeds one-third the orbital moment (Darwin 1879; Couselman 1973; Kopal 1978). In the other, the expanding envelope of one component engulfs its companion. This latter circumstance is generally associated with the occurrence of dynamical time scale mass transfer, since it involves the large-scale breakdown of hydrostatic equilibrium in the common envelope. Dynamical time scale mass transfer, in turn, occurs when the adiabatic response of a lobe-filling star will not permit that star to contract as rapidly as its Roche lobe — a condition fulfilled commonly by stars with virtually isentropic envelopes, i.e., stars with deep convective envelopes (see, e.g., Paczynski and Sienkiewicz 1972; Webbink 1979a), or by unevolved radiative stars which have been stripped rapidly of most of their mass (see Section IV below).

In the decade since common envelope evolution was outlined by Paczynski and Ostriker, a number of theoretical evolutionary calculations have attempted to follow its course in more detail: Alexander, Chau, and Henriksen (1976); Taam, Bodenheimer, and Ostriker (1978); Taam (1979); Meyer and Meyer-Hofmeister (1979); Delgado (1979, 1980); Tutukov and Yungel'son (1979); Livio, Salzman, and Shaviv (1979); Saltzman, Livio, and Shaviv (1980); Livio and Soker (1984a, b); Soker, Harpaz, and Livio (1984); and Bodenheimer and Taam (1984). This is, however, an extremely difficult problem, and consequently the only calculations presently feasible inevitably involve a large number of assumptions regarding various aspects of the
complex interaction between the common envelope and its embedded binary.

Direct observations of common envelope evolution are, for all practical purposes, nonexistent. We do not know of a single object to which we can point with confidence as an example of common envelope evolution, nor do such theoretical calculations as exist offer much insight into the telltale signs which might be used to identify one. However, we can identify several classes of evolved objects which are almost certainly products of common envelope evolution (such as cataclysmic variables and close binary planetary nebulae nuclei), others which have possibly passed through such a phase (symbiotic stars; barium stars), and still others which almost certainly have not undergone common envelope evolution (such as Algol systems).

In the following discussion, we will examine in turn Algol binaries, symbiotic stars, cataclysmic variables and close binary nuclei of planetary nebulae, and finally barium stars. Our objective will be to glean what insight these systems may offer into the details of common envelope evolution, and the circumstances under which it may or may not occur.

2. ALGOL BINARIES

The need to understand the evolutionary status of Algol-type binaries fostered the first detailed numerical studies of close binary evolution. The success of early investigators (Paczynski 1967; Kippenhahn, Kohl, and Weigert 1967; Plavec et al. 1968) in producing Algol-type systems, using the simple assumptions of conservation of total mass and orbital angular momentum within the context
of the classical Roche geometry, remains perhaps the
greatest success of close binary evolutionary theory.
Thus, while these systems do not themselves appear to be
products of common envelope evolution, they may prove
extremely valuable in defining empirically the boundaries
between common-envelope and quasi-conservative avenues of
evolution.

In Figure 1 are shown the present locations in the
mass-period diagram of the lobe-filling secondaries of 101
semi-detached systems tabulated by Giuricin, Mardirossian,
and Mezzetti (1983). Under the assumption of conservation
of total mass and orbital angular momentum, it is a simple
exercise to calculate their respective masses and orbital
periods at that point at which the two stars in each of
these semi-detached binaries had equal masses. The
results of this exercise are illustrated in Figure 2,
which shows that the present distribution of masses, mass
ratios, and angular momenta of Algol binaries are consis-
tent, within the uncertainties in observational data and
theoretical evolutionary tracks, with their all having
initiated mass transfer prior to the arrival of the origi-
nal primaries on the giant branch. Systems with longer
initial periods \(P \sim 10^4 - 100^4\), which most assuredly
exist (Popova, Tutukov, and Yungel'son 1982; Giuricin,
Mardirossian, and Mezzetti 1984), evidently do not survive
as recognizable Algol binaries. They have massive, deep
convective envelopes at the onset of mass transfer, and
are therefore expected to be unstable to dynamical time
scale mass transfer, a condition which we identified above
as conducive to common envelope evolution. The distribu-
tion of Algol binaries is therefore consistent with this
view, at least superficially.
FIGURE 1. The distribution of lobe-filling components of semi-detached binaries in the mass-period diagram. For reference purposes, the orbital periods of lobe-filling stellar models at various points in their evolution are also plotted as continuous curves.
FIGURE 2. The distribution of semi-detached binaries in the mass-period diagram at unit mass ratio, under the assumption of conservation of total mass and orbital angular momentum. Critical points in the evolution of lobe-filling stellar models are plotted as in Figure 1.
There are, however, some difficulties with so naive an analysis of the prior evolution of Algol binaries. As is clear from Figure 2, a number of systems fall below the lower period limit set by the zero-age main sequence when at their projected unit mass ratios, i.e., at their minimum periods in conservative mass transfer. These binaries have evidently lost a significant fraction of their initial angular momenta, because they do not presently have enough to have passed through the rapid mass reversal which occurs early in Algol-type evolution without becoming deep contact systems, and perhaps coalescing altogether.

This difficulty has been recognized since early in the history of close binary evolutionary studies (Ziolkowski 1969; Popov 1970), and a number of studies have examined the extent of mass and angular momentum losses quantitatively (e.g., Yungel'son 1972; Giuricin and Mardirossian 1981). Unfortunately, in each of these studies the investigators have been obliged to adopt an ad hoc recipe to specify the conditions and magnitudes of these loss rates, a recipe which has usually fixed the systemic mass loss rate as some fraction of the mass transfer rate between components, with the angular momentum loss per unit mass as either a fixed constant or a simple function of the Roche geometry.

There are clear inadequacies in these simple prescriptions, however. Numerous theoretical studies (Benson 1970; Yungel'son 1973; Webbink 1976; Kippenhahn and Meyer-Hofmeister 1977; and others) have shown that thermal time scale mass transfer during the early phases of Algol evolution should cause the accreting star to swell far beyond its thermal equilibrium, or main sequence, radius.
The faster the rate of mass transfer, the more pronounced is this bloating of the accreting star. The difficulty, then, with tying systemic mass and angular momentum loss rates to the mass transfer rate is that, unless the angular momentum content of mass lost from the system is very low (which it clearly cannot be if it is to explain the anomalously low specific angular momenta of present-day Algols), systemic losses tend to aggravate the thermal instability of the mass-losing star, and drive even faster mass transfer. The accelerated pace of mass transfer in turn aggravates the tendency of these binaries to evolve into deep contact. The obvious way to avoid this dilemma of creating contact binaries rather than Algol systems is to conclude that the bulk of the systemic mass and angular momentum losses now manifest in the distribution of Algol systems (Figure 2) occurred after the initial mass reversal, which must itself have been much more nearly conservative in character. That is, in the lowest approximation, such losses as occur do so nearly independently of the mass transfer process itself.

A promising explanation for this phenomenon lies with magnetically-coupled stellar winds, as discussed by Iben and Tutukov (1984). In this scenario, the interaction between convection and rotation leads to the generation of strong surface magnetic fields, which constrain the stellar wind to corotation out to many stellar radii. This wind dissipates the rotational angular momentum of the lobe-filling secondary stars on a rapid time scale, but that rotational momentum is continually restored from the orbit by tidal torques. Iben and Tutukov estimate that magnetic stellar winds are the principal mechanism for driving mass transfer among Algols with orbital periods
less than \(-4\) days, but cannot compete with nuclear evolution of the lobe-filling components in longer-period systems (where stellar winds can actually lead to increases in orbital periods). If this is indeed the case, the non-conservation of mass and angular momentum among short-period Algols does not affect the conclusion drawn above, that the division between quasi-conservative and common envelope evolution may be identified with the arrival of the primary components at the base of their giant branch evolution before they fill their Roche lobes. Figure 2 offers at least circumstantial support for this hypothesis, in that angular momentum losses appear much less severe on average among systems which have total masses in excess of \(7 M_\odot\) (Giuricin, Mardirossian, and Mezzetti 1983). These more massive systems are not expected to have long, nuclear time scale phases of semi-detached evolution as in lower-mass systems (Webbink 1979b).

3. SYMBIOTIC STARS

If very long period \((P > 100^d)\), low-mass Algol-type (i.e., post-mass reversal) systems were to exist, they would be relatively difficult to detect, not only because the long eclipse time scales mitigate against discovery, but also because the lobe-filling giants could well equal or exceed the light from their main sequence companions, making eclipses shallow and inconspicuous. Fortunately, there exists another class of intensely interacting binaries at long period, namely the symbiotic stars (Friedjung and Viotti 1982; Allen 1984; Kenyon 1985), which serve as a probe of this regime in the mass-period diagram. Those symbiotic stars with known or suspected orbital periods are listed in Table I.
### TABLE I  SYMBIOTIC STARS AND RELATED OBJECTS WITH KNOWN OR SUSPECTED ORBITAL PERIODS

<table>
<thead>
<tr>
<th>System</th>
<th>type</th>
<th>Spectrum</th>
<th>$P_{\text{orb}}$</th>
<th>note</th>
<th>Reference for $P_{\text{orb}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX CVn</td>
<td>n</td>
<td>M0 III + Al Vep</td>
<td>70.8?</td>
<td></td>
<td>Fried 1980</td>
</tr>
<tr>
<td>T CrB</td>
<td>a?</td>
<td>M3 III + sdBe</td>
<td>227.53</td>
<td>ecl?</td>
<td>Kenyon and García 1985</td>
</tr>
<tr>
<td>RS Oph</td>
<td>(a?)</td>
<td>M0 IIIp + OBeq</td>
<td>230.</td>
<td></td>
<td>García 1985</td>
</tr>
<tr>
<td>AX Mon</td>
<td></td>
<td>B1 I Veq + K0 III</td>
<td>232.5</td>
<td></td>
<td>Cowley 1964</td>
</tr>
<tr>
<td>V2756 Sgr</td>
<td></td>
<td>OBep + M2: (III)</td>
<td>243.?</td>
<td></td>
<td>Hoffleit 1970</td>
</tr>
<tr>
<td>SS Lep</td>
<td>(a)</td>
<td>A0 Veq + M1 III</td>
<td>260.0</td>
<td></td>
<td>Cowley 1967</td>
</tr>
<tr>
<td>RW Hya</td>
<td>n</td>
<td>M2 III + sdOe</td>
<td>372.45</td>
<td></td>
<td>Kenyon and Webbink 1984</td>
</tr>
<tr>
<td>EG And</td>
<td>(n)</td>
<td>M2 III + sdOe</td>
<td>470.?</td>
<td>ecl?</td>
<td>Smith 1980</td>
</tr>
<tr>
<td>AG Dra</td>
<td>n</td>
<td>K3 II + sdOe</td>
<td>554.</td>
<td></td>
<td>Meinunger 1979</td>
</tr>
<tr>
<td>V748 Cen</td>
<td></td>
<td>F2 Iep + M4 III</td>
<td>564.8</td>
<td>ecl</td>
<td>Van Genderen, et al. 1974</td>
</tr>
<tr>
<td>AR Pav</td>
<td>a</td>
<td>Of + M3 III</td>
<td>604.6</td>
<td>ecl</td>
<td>Andrews 1974</td>
</tr>
<tr>
<td>CL Sco</td>
<td>a</td>
<td>K3 (II)? + sdBe</td>
<td>624.7?</td>
<td></td>
<td>Kenyon and Webbink 1984</td>
</tr>
<tr>
<td>SY Mus</td>
<td>n</td>
<td>M2 III + sdOe</td>
<td>627.0</td>
<td>ecl</td>
<td>Kenyon and Bateson 1984</td>
</tr>
<tr>
<td>AX Per</td>
<td>(a)</td>
<td>M3 III + sdOe</td>
<td>681.6</td>
<td></td>
<td>Kenyon 1982</td>
</tr>
<tr>
<td>HV 13055</td>
<td></td>
<td>Beq (+ K5 III?)</td>
<td>682.?</td>
<td>in LMC</td>
<td>Hodge and Wright 1970</td>
</tr>
<tr>
<td>HK Sco</td>
<td></td>
<td>OBep + M1 (III)</td>
<td>700.?</td>
<td></td>
<td>Swope 1941</td>
</tr>
<tr>
<td>Z And</td>
<td>a?</td>
<td>M2 II + sdOe</td>
<td>756.85</td>
<td></td>
<td>Kenyon and Webbink 1984</td>
</tr>
<tr>
<td>BF Cyg</td>
<td>n</td>
<td>Beq + M4 III</td>
<td>757.3</td>
<td>ecl?</td>
<td>Pucinkas 1970</td>
</tr>
<tr>
<td>BL Tel</td>
<td></td>
<td>F8 Ib + M0 III</td>
<td>778.6</td>
<td>ecl</td>
<td>Van Genderen 1983</td>
</tr>
<tr>
<td>AG Peg</td>
<td>n</td>
<td>WN6 + M1.7 III</td>
<td>816.5</td>
<td></td>
<td>Fernie 1985</td>
</tr>
<tr>
<td>V2601 Sgr</td>
<td></td>
<td>M5 (III) + sdOe</td>
<td>850.?</td>
<td></td>
<td>Hoffleit 1968</td>
</tr>
<tr>
<td>CI Cyg</td>
<td>a</td>
<td>M4 II + sdOe</td>
<td>855.25</td>
<td>ecl</td>
<td>Whitney 1953</td>
</tr>
<tr>
<td>HD 128220</td>
<td></td>
<td>G0 III + sdO</td>
<td>870.?</td>
<td></td>
<td>Wallerstein and Woolf 1966</td>
</tr>
<tr>
<td>V1329 Cyg</td>
<td>(n)</td>
<td>WN5 + M4 II</td>
<td>950.07</td>
<td></td>
<td>Chochol, et al. 1980</td>
</tr>
<tr>
<td>CH Cyg</td>
<td>?</td>
<td>M6 III + A0 Veq</td>
<td>5750.?</td>
<td></td>
<td>Yamashita and Maehara 1979</td>
</tr>
</tbody>
</table>

By virtue of the same arguments which led one to expect a "zone of avoidance" among Algol-type systems (the region above, and to the left of, the dashed line in Figure 2), this author was led to the conclusion that the symbiotic binaries, with their longer orbital periods, are probably pre-mass-transfer rather than post-mass-transfer objects. The intense activity and high-excitation spectra characteristic of symbiotic stars could then be understood in terms of accretion processes onto main sequence stars in the beginning phases of the dynamical time scale mass transfer expected to prevail in systems with lobe-filling convective primaries.
A survey of 19 symbiotic and symbiotic-like objects by Kenyon and Webbink (1984) revealed a number of systems which did indeed appear to conform to this model. Those included in Table I are indicated by an "a" in column 2. However, at least as large a number of objects were found to contain hot stellar companions, presumably nuclear-burning degenerate dwarfs. These systems are indicated by an "n" in column 2 of Table I. They are probably fueled by accretion from the stellar winds of their late-type giant companions, as suggested by Tutukov and Yungelson (1976), since no evidence could be found for accretion disks in these symbiotics. In this case, it is possible that the component stars of very long period systems could have evolved independently, without ever encountering tidal mass exchange. However, those systems with orbital periods less than $-10^4$ days (see Figure 1) are almost certainly products of mass transfer in a binary system.

How then are we to understand the existence of degenerate components in symbiotic binaries, which appear to have escaped the ravages of common envelope evolution? Several possibilities suggest themselves: (1) Binaries with initial primaries of $-3 - 10 \, M_\odot$ reaching mass transfer in the Hertzsprung gap are expected to leave white dwarf remnants in this period range (Webbink 1979b; Iben and Tutukov 1985). Since the mass-losing stars have radiative envelopes in this case, mass exchange should proceed at least approximately conservatively, except for losses due to the stellar winds of the individual binary components. This explanation would suggest that these symbiotics are rather more massive systems than either of the two following hypotheses. (2) Tidal mass exchange can be stabilized or avoided altogether if the initial mass
ratio is not too far from unity, and the stellar wind from
the original primary can remove one-third or more of that
star's mass prior to its filling its Roche lobe. Thus the
first-evolving star was already the less massive component
by a substantial margin before mass transfer ever occur-
red. (3) The symbiotic binaries containing degenerate hot
components could actually be products of common envelope
evolution. To be a viable model for systems as long in
orbital period as symbiotic stars, this scenario requires
that the mass ratio of the initial system be rather moder-
ate at the onset of mass transfer \( q \geq 0.5 \), that the
ratio of envelope mass of the lobe-filling star to mass of
the companion star be small (and thus that the envelope
mass be a small fraction of the total mass of the pri-
mary), and finally that common envelope evolution be very
efficient at using orbital energy to unbind the envelope
of the primary.

Which, if any, of these explanations is correct is a
question which does not now appear answerable. The possi-
bilities could be narrowed substantially if the component
masses were known for several of these systems, but sym-
biotic stars are notoriously difficult objects for spec-
troscopic orbit determinations (see, for example,
Thackeray and Hutchings 1974; Hutchings, Cowley and Redmon
1975). Indeed, only eight of the orbital periods listed
in Table I are originally spectroscopic in origin (TX CVn,
T CrB, RS Oph, SS Lep, EG And, AG Peg, HD 128220, and CH
Cyg), and half of these are very questionable. Neverthe-
less, there is circumstantial evidence that the symbiotic
stars containing degenerate components are of low total
mass, therefore favoring the second and third possibili-
ties enumerated above: EG And and AG Dra are notable as
high-velocity systems (Eggen 1964), and the symbiotic stars generally have kinematics (Wallerstein 1981) and space distributions (Allen 1980; Duerbeck 1984) characteristic of an old disk population. Further evidence supporting one or both of these scenarios will be drawn from the discussion of barium stars in Section V, below.

4. CATACLISMIC VARIABLES AND CLOSE BINARY NUCLEI OF PLANETARY NEBULAE

As products of common envelope evolution, cataclysmic variables should in principle provide considerable insight into that phenomenon. Unfortunately, it is not possible, given the present state of knowledge, to reconstruct the prior evolution of individual systems, and their statistics are strongly influenced by the effects of secular evolution (see, e.g., Ritter 1983; Patterson 1984) and by observational selection effects (Ritter and Burkert 1985). In contrast, the handful of close detached white dwarf-red dwarf binaries now known (see Ritter 1984 for a tabulation) should be free from the secular effects of mass transfer, and close binary planetary nebulae nuclei from orbital evolution altogether. Nevertheless, it is a daunting prospect to draw more than qualitative conclusions about the nature of common envelope evolution from the properties of these systems as we now understand them. For the present, we wish to make only the following point: The properties of the non-degenerate components of these systems strongly indicate that they cannot have accreted or lost a large fraction of their mass during the common envelope phase.

What is the justification for this statement? Let us consider first the case in which the present non-degener-
ate star was initially more massive than it is now. If that initial mass had been as much as 70 to 80 percent that of the initial primary, or more, we would expect central hydrogen depletions by a factor of two or more. Thus, while the donor stars in cataclysmic binaries appear to lie on or near a "main sequence" (Ritter 1983; Patterson 1984), the appropriate main sequence may be hydrogen-deficient, a consequence of nuclear evolution in the present secondary during the earlier phase of its lifetime when the primary was growing the degenerate core seen now as a white dwarf. Stars on these hydrogen-deficient main sequences may be considerably more luminous than those with normal compositions, as seen in Figure 3. This result is well-known for stars in radiative equilibrium (cf. Eddington 1926), but the effect is even more marked for stars with deep convective envelopes. Stars of the same degree of hydrogen depletion, but inhomogeneous composition profiles, would be more luminous yet.

In very few cataclysmic binaries do we have direct spectroscopic estimates of the masses of the secondary components. In most cases, their masses have been deduced from the binary periods, which, aside from a very weak dependence on mass ratio, in effect fix the mean densities of the lobe-filling components. It is clear from Figure 3, however, that the mean density of a star is not a strong function of the hydrogen depletion (at least for homogeneous models), except for stars of very low mass ($M \lesssim 0.2 M_\odot$), among which highly depleted models become significantly denser. In all cases, were the secondaries in cataclysmic binaries significantly hydrogen-depleted, we would expect them to be anomalously luminous and early in spectral type, at a given orbital period. Not only
FIGURE 3. The theoretical Hertzsprung-Russell diagram of main sequences of solar metallicity, but varying degrees of hydrogen depletion.
does this not appear to be the case observationally (Ritter 1983), but in some cases the secondary actually appears too late in spectral type for its mass (in U Gem, for example — see Wade 1981). Only in long-period systems \((P \gtrsim 7^h)\), which are sufficiently wide to permit nuclear evolution since reaching the cataclysmic state (Taam, Flannery, and Faulkner 1980), do we find evidence of chemically-evolved secondaries. Interpreted at face value, the mass-luminosity diagram for secondaries in cataclysmic binaries (Figure 5d in Ritter 1983) shows no evidence whatever of discernable hydrogen depletion \((\Delta X/X = 0.0 \pm 0.05 \text{ (s.d.)})\) in these stars. A similar inference may be drawn from the sharpness of the 80-minute short-period cutoff to the orbital period distribution of cataclysmic binaries (Rappaport and Joss 1984).

Of course, the argument just outlined really only demands that, in the absence of a mechanism for mixing the entire star (see, for example, Webbink 1977), the primordial mass ratio must have been quite small \((q_0 \lesssim 0.3)\). This already greatly restricts the possible degree of mass loss from the secondaries during common envelope evolution, but a much stronger constraint may be found, in fact, among close binary planetary nebulae nuclei. Their importance stems from the fact that, with the planetary nebula ejecta still visible around the systems, we can be certain that these binaries emerged from the common envelope phase so recently that the secondary component cannot have undergone any significant thermal relaxation since that phase. That is, the thermal, or Kelvin-Helmholtz, time scale for the secondary is several orders of magnitude greater than the age of the planetary nebula. Indeed, according to all available estimates, the entire
mass transfer and common envelope phase is itself so brief $(\Delta t \lesssim 10^4 \text{ yr})$ that the same conclusion may be drawn in its regard. Thus, the entropy profile in the present secondary’s interior must be that which prevailed at the start of mass transfer. The only possible exception to this statement is if the adiabatic compression of the secondary’s core during an early accretion phase drives the nuclear energy generation rate up to values several orders of magnitude higher than the equilibrium rate, thereby significantly increasing the specific entropy of the core (see Webbink 1977). But in no case do we expect the specific entropy of the interior of the secondary to have decreased significantly.

In the structure of any chemically unevolved star (which we have argued above is the case for the secondaries in cataclysmic variables), the inner one-half to two-thirds of the star will be approximately isentropic (see Figure 2.2.2 in Webbink 1985). This condition merely reflects the fact that the scale-heights characterizing the gradients of thermodynamic variables in a star are generally of the order of the depth below the surface in its outer layers, but become asymptotically infinite at its center (where all radial gradients vanish). In stars with radiatively stable envelopes, the specific entropy rises rapidly in the outer envelope. Fully convective main sequence stars, on the other hand, have virtually isentropic interiors, with a drop in specific entropy only in a very thin, superadiabatic surface layer.

If we now consider what happens when mass is stripped rapidly from a star in a binary system, it is clear that, as more and more mass is removed from the star, the remnant star approaches more and more closely an isen-
FIGURE 4. The specific entropy of zero-age main sequence stars at their centers, and (for cooler stars) at the bases of their convective envelopes.

tropic state, characterized by the same adiabat as the center of the original star. The actual values of specific entropy at the centers of model main sequence stars of solar composition are plotted in Figure 4, where it is clearly seen that central entropy is a monotonically increasing function of mass. Because of the efficiency of convection, where it occurs, this central entropy is, for all practical purposes, a minimum for the entire star. It follows that the remnant of rapid mass loss can never be more compact than an isentropic star whose specific entropy is that at the center of the original star.
The consequences of adiabatic mass loss are illustrated in Figure 5. Plotted in this figure are the zero-age radii of main sequence stars of solar composition. Also shown are the radii of isentropic stars having the same

![Graph showing mass-radius relationship](image)

**FIGURE 5.** The mass-radius diagram for homogeneous stars. The heavy solid line is the zero-age main sequence. The lighter solid line is an estimate of the minimum radius of any more massive star undergoing adiabatic mass loss. The dash-dotted line represents the radii of fully isentropic stars characterized by the same central entropy as the zero-age main sequence. Rapid mass transfer remnants must lie on, or above, a line of slope -1/3 originating on the dash-dotted line at their initial masses, as illustrated by the arrows. Remnants lying above the upper dashed line correspond to the convective cores of their progenitor stars, and are fully isentropic. Those lying below the lower dashed line (and above the main sequence) are remnants of fully convective progenitors, and are also isentropic.
mass, composition, and central specific entropy. These radii have been estimated here by approximating these isentropic stars by \( n = 3/2 \) polytropes, which should be good approximations in this mass range, where the effects of radiation pressure and non-ideal corrections to the equation of state are small. According to the preceding argument, then, the radii of rapid mass remnants from stars of a given mass must lie above a line of slope \( \log R/d \log M = -1/3 \) intersecting the lower curve at the initial mass of that star. Thus, in the case illustrated in Figure 5, a mass transfer remnant of mass \( 0.80 \, M_\odot \) (log \( M = -0.10 \)) cannot possibly have a radius smaller than the zero-age main sequence radius of an \( 0.80 \, M_\odot \) star if its initial mass exceeded \( 1.74 \, M_\odot \) (log \( M = +0.24 \)). In fact, a realistic estimate of the lower limit to the radii of mass transfer remnants of a given mass can be gleaned from the properties of stripped polytropes of index \( n = 3 \), but adiabatic exponent \( \gamma = 5/3 \) (Hjellming and Webbink 1985), and is illustrated in Figure 5. We note, finally, that if a star is stripped down to a mass as small as its convective core, then the isentropic approximation for its remnant should be very accurate indeed, as it will be for stars which were fully convective originally. The boundaries of the region so defined are shown in Figure 5.

What, then, do the close binary nuclei of planetary nebula have to tell us about possible mass loss during common envelope evolution? About ten such systems are known, and they are listed in Table II. The periods quoted for the central stars of IC 418 and LT-5 are uncertain due to aliasing of the available radial velocities, and that of NGC 6543 is very doubtful, as photoelectric photometry has revealed no evidence of short time
scale variability (Patterson 1979). Of the remaining
systems, all except NGC 6826 have photometric periods, but
we will concentrate here on Abell 41 (MT Ser), DS 1 (LSS
2018), Abell 63 (UU Sge), and Abell 46 (V477 Lyr), since
these short-period systems provide the strongest con-
straints on mass loss during common envelope evolution.
The cool components of three of these four systems are
plotted on the mass-radius diagram in Figure 6.

In the case of MT Ser, the central star of Abell 41,
neither a photometric nor a spectroscopic solution exists.
Nevertheless, the requirement that the cool component
should fit within its Roche lobe at this very short orbi-
tal period (163 minutes) already poses a severe upper
limit on its radius. This upper limit to \( R_c \), the radius
of the secondary is plotted in Figure 6 for two extreme
masses for the companion pre-white dwarf: \( M_h = 0.2 \, M_\odot \)
(upper boundary to the shaded region), corresponding to
the smallest degenerate core which can be formed in a
binary system within ~ \( 10^{10} \) years; and \( M_h = 1.4 \, M_\odot \) (lower
boundary to the shaded region), corresponding to the
Chandrasekhar limit. If the present mass of the secondary
exceeds 0.10 \( M_\odot \), it cannot therefore have lost more than
41% of its initial mass during common envelope evolution.
FIGURE 6. The mass-radius diagram for the cool components of close binary planetary nebulae nuclei. The zero-age main sequence and minimum radii for mass transfer remnants are plotted as in Figure 5. Geometrical constraints require the secondary in UU Sge to lie within the long strip outlined; theoretical constraints further limit it to the portion bounded by the heavy solid line; and the colors during totality limit it to the small, irregularly shaped, heavily shaded segment. The secondary of LSS 2018 must lie on the short, heavy solid line indicated. That of MT Ser must lie below the lightly-shaded strip, and above the zero-age main sequence, as shown.

LSS 2018, the central star of the newly-discovered planetary nebula DS 1, is a double-lined spectroscopic binary with an ellipsoidal light curve (Drilling 1985).
Although the orbital inclination of this system is unknown, Drilling argues that it must lie between $i = 53^\circ$, below which he is unable to reproduce the amplitude of the ellipsoidal variation, and $i = 73^\circ$, above which eclipses would be observable. He estimates the fractional radii of the cool and hot components as $r_c = 0.18$ and $r_h = 0.12$, respectively. The limits on orbital inclination, plus the radial velocity amplitudes of both components (with that of the secondary corrected for proximity effects) then place the cool component slightly above the main sequence in Figure 6. The displacement with respect to the main sequence then limits potential mass loss during the common envelope phase to no more than 26% to 39% of the initial mass of the secondary, corresponding to the upper and lower limits, respectively, to its present mass.

V477 Lyr, the central star of Abell 46, undergoes partial eclipses. No formal light curve solution has been published, but a rough analysis (after Irwin 1962) of the primary eclipse light curve published by Grauer and Bond (1981) yields fractional radii $r_h = 0.11$ and $r_c = 0.23$ for the hot and cool components, respectively, with an orbital inclination $i = 79^\circ$. No radial velocity study exists for this binary, but its photometric parameters are, apart from orbital inclination, almost identical to those of UU Sge. It has therefore not been plotted in Figure 6.

UU Sge, the central star of Abell 63, also lack a spectroscopic orbit, but photometric solutions have been published by Bond, Liller, and Mannery (1978), and by Budding and Kopal (1980; later refined by Budding 1981). We adopt here the solution of Bond, Liller, and Mannery, because it is based on a superior light curve, even though the eclipse model is somewhat naive. They find $r_h = 0.13$,
$r_c = 0.24$, and $i = 85^\circ$. The fractional radius of the cool component, plus Kepler's third law, restricts the location of the secondary to the elongated box indicated in Figure 6. The upper and lower bounds to this box correspond to assumed companion masses of 1.4 $M_\odot$ and 0.2 $M_\odot$, respectively, the same mass limits invoked above for MT Ser. At low masses ($M_c \sim 0.27 M_\odot$), the allowed radii of the cool component are bounded by the requirement that it fit within its Roche lobe.

The position of the secondary of UU Sge in the mass-radius diagram is further constrained because we have an estimate of its intrinsic color during totality. Corrected for its close visual companion, the depth of primary eclipse in $B$ and $V$ is 4.3 magnitudes (Miller, Krzeminski, and Friedhorsky 1976) and 2.8 magnitudes (Budding 1981), respectively, yielding $(B-V)_o = +1.0$ for the secondary, assuming $(B-V)_o = -0.3$ for the hot primary. This corresponds to a spectral type of K3V for the secondary, and, in effect, fixes the specific entropy at its surface. By demanding that the specific entropy at the surface of a mass transfer remnant equal that at the surface of a K3V star (or, more appropriately, that at the base of the surface convection zone of a K3V star), the secondary in UU Sge is further constrained, if it has suffered a net loss of mass during common envelope evolution, to lie within the heavily-shaded region in its allowed strip in Figure 6. A nominal uncertainty of ±0.1 in $(B-V)_o$ has been assumed in delineating these regions. The upper limit to the fraction of its initial mass which the secondary may have lost, as constrained by its present colors, then varies from $\lesssim 7\%$ for $(B-V)_o = +0.9$ to $\lesssim 46\%$ for $(B-V)_o = +1.1$. \[ \text{LATE STAGES OF CLOSE BINARY SYSTEMS} \]
In none of the cases examined here can the secondary have lost a very large fraction of its mass during common envelope evolution. Indeed, physically there is no compelling reason to suppose that they may have suffered a net loss of mass at all. Even in these short-period systems, the present secondaries evidently underfill their Roche lobes, as there is no evidence of ongoing mass transfer activity, and yet they have not had time to contract significantly since common envelope ejection. The mass constituting the original secondaries must always have remained tidally stable, and it seems more reasonable, physically, to attribute the modest degree to which individual stars may exceed their main sequence (thermal equilibrium) radii to the accretion of matter from the common envelope.

What constraints can then be placed on possible mass accretion by the secondary during the common envelope phase? The argument here is essentially a theoretical one. In answering this question, it is important to note that any accreted envelope which is retained will, at least at the moment when the last vestiges of the common envelope are shed by the binary, inevitably consist of very high entropy material, in comparison with the original surface layers of the accreting star. This follows both from the fact that the common envelope itself is of very high entropy, by virtue of its initial very low density (compared with the mean density of the embedded secondary) and because it is heated by dissipation of the orbital energy of the embedded binary, and from the fact that the material first accreted by the secondary will be heated by accretion shocks to even higher temperature and entropy. Because this accreted material is buried on a
dynamical time scale, it can cool efficiently only if it is convectively unstable. The Schwarzschild criterion for convection requires that the specific entropy of the envelope must decrease outwards for convection to occur. Thus, until it is exposed by planetary nebula ejection, the accreted envelope is prevented from falling in specific entropy much below the high value characteristic of the common envelope.

It is possible to make a very rough estimate of the amount of mass which has been retained, as follows: Let $M_e$ and $R$ be the mass and radius, respectively, of the accreted envelope, and likewise $M$ and $r$ be the mass and radius of the underlying star. Then it is easy to show that, for $M_e \ll M$, the mass of an envelope of polytropic index $n$ is related to its radius by the relation

$$M_e = 4\pi R^3 \left( \frac{GM}{(n + 1)K} \right)^n F(R/r) \ ,$$

where

$$F(R/r) = \left( \frac{R}{r} \right)^{3-n} \int_1^{R/r} x^{-4} (x - 1)^n \, dx \ ,$$

and

$$F(R/r) \leq \frac{1}{n + 1} \left( \frac{R}{r} - 1 \right)^{n+1} \ ,$$

where $K$ is the polytropic constant of the envelope. To the extent that the thermal timescale of the accreted envelope is long compared to the interval since common envelope ejection, $K$ will be unchanged in the interim. The polytropic constant characterizing the common envelope is of order

$$K \sim \frac{GM}{R_o} \left( \frac{M_{ce}}{R^3} \right)^{-1/n} \ ,$$
where $M_{ce}$ is the mass of the common envelope, $M_\odot$ the total mass of the system (including the common envelope), and $R_\odot$ the radius of the common envelope. The fraction of the common envelope retained by the secondary is

$$\frac{M_e}{M_{ce}} \leq 2 \times 10^{-5} \left(\frac{R_\odot}{r}\right)^{5/2}.$$ 

Therefore, even in fairly long period, lobe-filling binaries, the accreted envelope mass must be much smaller than the underlying stellar mass. In fact, for the short-period systems discussed above, any accreted mass could well be so small that its thermal time scale,

$$\tau \approx \frac{3}{7} \frac{G M M_e}{R L} \left(\frac{R_\odot}{r} - 1\right),$$

is smaller than the age of the planetary nebula, contrary to our assumptions above. Nevertheless, this condition in itself requires that the envelope mass be order

$$M_e \leq \frac{\tau_{PN} R L}{G M},$$

where $\tau_{PN}$ is the age of the planetary nebula and $L$ the luminosity of the secondary. The secondaries of the nuclei discussed above all have masses $M \leq M_\odot$, radii $R \leq R_\odot$, and appear to be intrinsically quite cold ($T \leq T_\odot$); the corresponding limit to the accreted envelope mass is $\leq 10^{-3} M_\odot$. It is clear, then, that any potential increase in mass of either star during common envelope evolution must have been negligibly small. (This conclusion is guaranteed for the degenerate star because its nuclear burning — i.e., core growth — time scale is at least a
factor of $10^3$ longer than the duration of the common envelope phase.)

It should come as no surprise that the envelope retained by either star must be very small indeed. The picture which emerges is that shock-heating of the first-accreted matter by the secondary tends to build an initial envelope with an inverted entropy profile — that is, one which is itself convectively unstable. When the envelopes of the two stars reach contact and merge, in a common envelope, the matter accreted by the secondary is prevented from cooling below the specific entropy of the common envelope, because to do so would cut off convection, the only cooling process which can compete on the time scale of dynamical evolution of the binary. Thus, the envelope structure surrounding the secondary is very similar to that around the degenerate primary — a red giant, but one with a non-degenerate core in this case. It is well-known (Refsdal and Weigert 1970; Paczynski 1970a,b) that such structures do not begin to shrink from giant dimensions, as they must do in this case to achieve tidal stability, until their envelope masses are extremely small.

This picture of the structure of the accreted envelope also explains why the cool components of close binary planetary nebulae nuclei and their evolutionary descendants appear so remarkably undisturbed by the rigors of common envelope evolution. It is because the accretion of a very high entropy envelope in effect throws a thermal "blanket" over the surface of the star. The imposition of a sharp entropy increase at the surface of the original secondary creates an isothermal boundary between that star and its accreted envelope, cutting off all heat transport across it. For examples of this behavior, see Sarna.
1984; Prialnik and Livio 1985; also Webbink 1977). Since this duration of the common envelope phase is in any case much shorter than the thermal time scale of the secondary, the entire episode can be regarded as one of adiabatic compression and decompression of the secondary. With the ejection of the common envelope, it is returned to its initial state of hydrostatic and thermal equilibrium. If this were not the case, we should expect to see, observationally, secondary components in planetary nebulae binaries which were very overluminous and oversized for their masses — in all likelihood filling their Roche lobes. There is no clear evidence to support this position. Quite to the contrary, in none of the four systems discussed in detail above do the secondaries have radii much (if at all) exceeding those of main sequence stars of the same mass; and in the case of UU Sge, we know that the intrinsic luminosity of the secondary is near that of a main sequence star.

5. BARIUM STARS

Barium stars are Population I G and K giants characterized by an enhancement of s-process elements relative to iron (Bidelman and Keenan 1951). They appear to span a large range in absolute magnitude ($M_V = +2.2$ to $M_V = -3.7$: Scalo 1976), but, with few exceptions, are too low in luminosity to have reached a double-shell-burning state where thermal pulses can bring s-process material to the surface of a star (Becker and Iben 1980; Renzini and Voli 1981). This dilemma would seem to have been broken by the discovery that barium stars occur only in binary systems (McClure, Fletcher, and Nemec 1980; McClure 1983, 1984a, 1985). Their Population II counterparts, the CH stars, appear as
well to have a very high incidence of binarity, in comparison with normal giants (McClure 1984b, 1985). If the companions to the barium stars are white dwarfs — and this has been demonstrated only for one classical barium star (ζ Cap: Böhm-Vitense 1980), and three mild barium stars (56 Peg: Schindler et al. 1982; ζ Cyg: Dominy and Lambert 1983; ξι Cet: Böhm-Vitense and Johnson 1985) — then those companion stars may have been the sites of the s-process nucleosynthesis, with mass transfer responsible for depositing these products on the star now visible (McClure, Fletcher, and Nemec 1980; Smith, Sneden, and Pilachowski 1980). Alternatively, the companion stars could have provoked mixing within the barium stars themselves (Smith, Sneden, and Pilachowski 1980).

Griffin and Griffin (1980), Culver (1981), and McClure (1983) have published orbital solutions for a total of seven classical barium stars. They range in orbital period from 80.54 to 2300 d, and all have small mass functions, f(M) ≤ 0.037 M☉. In addition, velocity variations have been detected for all but three of thirteen other barium stars monitored by McClure (1983, 1985); these systems are apparently all of long period (P ≥ 1600 d). The orbits of all of these systems are sufficiently short-period, nevertheless, to fall within the domain of close binary evolution (see Figure 1), and it would therefore appear inevitable that mass exchange between components should play some role in their evolution — although whether it has done so already may not be so clear.

The two questions to be addressed within the context of the present study are thus: (1) Are barium stars the
common envelope evolution?

In addressing the first of these questions, let us turn first to the distribution of the orbital eccentricities of barium stars. There is a well-known correlation between orbital period and mean eccentricity among binary systems (see, e.g., Bouigue 1974), and for systems as long in period as the barium stars, one would expect a moderately high mean eccentricity. In fact, the mean for the seven known orbits is $\bar{e} = 0.17 \pm 0.05$, and the eccentricity is clearly nonzero for all but two stars (HD 31487 and HD 46407). This fact in itself is somewhat surprising if these are indeed post-mass-transfer objects. Significantly nonzero eccentricities are very rare among semidetached binaries: of the 101 systems tabulated by Giuricin, Mardirossian, and Mezzetti (1983) and included in Figures 1 and 2, the mean orbital eccentricity of the 74 with spectroscopic orbits (Batten, Fletcher, and Mann 1978; Pédoussaut, Ginestet, and Carquillat 1984) is $\bar{e} = 0.05 \pm 0.01$. What nonzero eccentricities appear among these systems can almost always be attributed to distortion of their velocity curves by gas streams. Similarly, none of the close binary planetary nebula nuclei shows any photometric evidence (asymmetric phasing or unequal durations of primary and secondary eclipse) of an eccentric orbit. Thus, mass transfer or common envelope evolution seems, empirically, to damp quite strongly any orbital eccentricity. This is, of course, not at all unexpected on the basis of theoretical studies of tidal dissipation (e.g., Zahn 1977), or of the dynamics of mass transfer streams (Piotrowski 1964; Kruszewski 1966).

Nevertheless, the orbital eccentricity distribution of barium stars does show clear evidence of dissipation.
To illustrate this point, and as a basis for comparison, a sample of "normal" giants was selected from the work of Griffin (1983, 1985). His orbits are comparable in time basis and quality with those of McClure. This sample, totaling 23 systems, was chosen on the basis of similarity of spectral type (G5 to K1), luminosity class (II to III), and orbital period (80 to 2300 days) with the range spanned by the barium stars. Giants of known spectral peculiarity were omitted, although, of course, this does not preclude the inclusion of some systems whose peculiarity is not yet recognized. For these "normal" giants, the mean eccentricity is $\bar{e} = 0.24 \pm 0.04$. The significance of the difference in orbital eccentricities between normal giants and barium stars is perhaps more apparent in Figure 7, where their cumulative distributions are compared.

FIGURE 7. The fraction of barium stars and comparable normal giant binaries with orbital eccentricities
Indeed, if we compare mean values of the square of the eccentricity, a quantity which equals the fractional excess of orbital energy of a binary over its minimum energy state (a circular orbit), the significance of this difference is quantitatively apparent: $e^2 = 0.043 \pm 0.016$ for barium stars, $e^2 = 0.091 \pm 0.020$ for the comparable sample of normal giants. This difference points clearly to tidal dissipation among the barium stars, from which one may infer that one of the binary components must, in the past, have filled or nearly filled its Roche lobe.

![Graph](image)

**FIGURE 8.** The fraction of barium stars and comparable normal giant binaries with spectroscopic mass functions less than or equal to a given fraction of their respective mean values. The smooth solid curve is the distribution expected for a population of binaries with a single value of $m_2^3/(m_1 + m_2)^2$ seen at random inclinations.
Clear evidence that the component in question must have been the companion to the barium star, and that these binaries are therefore post-mass-transfer systems, derives from an examination of the magnitudes and distribution of their mass functions. As noted above, these mass functions are, in the mean, small. But they possess one other remarkable property — their distribution is entirely consistent with a single value for \( m_2^3/(m_1 + m_2)^2 \) = 0.042 ± 0.008 \( M_\odot \), with a random distribution of orbital inclinations, \( i \). This distribution is illustrated in Figure 8, where it is clear that it departs radically from that of our normal giant sample, a sample which presumably contains a random assortment of companion stars at random orbital inclinations. It therefore appears that, at the very least, the masses of the companions to the barium stars must be highly correlated with those of the barium stars themselves, and it is possible that both are virtually single-valued.

Current mass estimates for the barium stars place them in the range 1.0 to 1.5 \( M_\odot \) (Eggen 1972) or 1.7 to 3 \( M_\odot \) (Culver and Ianna 1976, 1980; Culver, Ianna, and Franz 1977). The corresponding values for their companion masses (see Table III) are eminently reasonable for white

<table>
<thead>
<tr>
<th>( \frac{M_{\text{Ba}}}{M_\odot} )</th>
<th>( \frac{M_{\text{comp}}}{M_\odot} )</th>
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<tbody>
<tr>
<td>1.00</td>
<td>0.45 ± 0.04</td>
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<tr>
<td>1.50</td>
<td>0.57 ± 0.05</td>
</tr>
<tr>
<td>2.00</td>
<td>0.67 ± 0.05</td>
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<tr>
<td>2.50</td>
<td>0.77 ± 0.06</td>
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<tr>
<td>3.00</td>
<td>0.86 ± 0.07</td>
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dwarfs. This fact, plus the mere existence of a correlation between components masses, and the fact that the orbital periods of barium stars are too short to permit either component to complete its evolution without filling its Roche lobe, all point towards mass transfer as fundamental to the barium star phenomenon.

If we accept the mass transfer hypothesis, does it follow that barium stars are products of common envelope evolution? Iben and Tutukov (1985) note that the orbits of the shortest-period systems are too small to accommodate thermally-pulsing double-shell-burning giants, a circumstance which would indeed point to common envelope evolution. There exist a number of problems with mass transfer scenarios in general, however, and with this one in particular, as enumerated by Trimble (1984). The most serious problem for mass transfer hypotheses generally is the apparent absence of a comparable population of chemically peculiar main sequence precursors to barium stars. Lambert (1985) hints, however, that this conclusion may derive more from the absence of evidence bearing on chemical peculiarity than from evidence of absence of the same. As for thermally-pulsing giants, the pattern of heavy element enhancement in barium stars appears curiously at odds with the hypothesis that such stars are the source of that anomaly. The $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ reaction responsible for s-processing in these giants is self-limited by the $^{25}\text{Mg}(n,\gamma)^{26}\text{Mg}$ reaction, and appears incapable (Truran 1980, 1981) of producing the ratio of enhancement of heavy elements (barium and beyond) to that of intermediate-mass elements (iron to barium) observed in the barium stars (Danziger 1965; Tomkin and Lambert 1979, 1983; Smith 1984).
Interest has therefore shifted to the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction as the neutron source (e.g., Tomkin and Lambert 1979; Truran 1980; Smith 1984). This reaction, in turn, raises the question of $s$-process nucleosynthesis prior to the thermally-pulsing, double-shell-source phase, perhaps during the helium core flash (see, for example, the discussion by Lambert 1985). Although the manner in which such a scenario might work remains obscure, we should note that the companion masses deduced above are too small for the stars in question to have reached a thermally-pulsing stage (see Truran and Iben 1978). Mass transfer would appear to have cut off their evolution during thick helium shell burning. In this case, the difficulty in fitting such a star within the dimensions of present barium star systems is removed (see Figure 1), as is the necessity, then, of appealing to common envelope evolution.

In the context of the discussion in Section IV, it would be easier to understand the retention within the binary of the $s$-process-enhanced envelope of the present white dwarf if mass transfer were quasi-conservative (thermal or nuclear time scale). In that case, mass transfer onto the present barium star need not have been so rapid as to preclude thermal relaxation of the accreted envelope, at least to the extent needed to keep that star within its Roche lobe. If this interpretation is correct, therefore, the barium stars may offer further evidence, beyond that discussed above in Section III, that, under suitable circumstances, lobe-filling giants with deep convective envelopes can somehow avoid dynamical time scale mass transfer and common envelope evolution.
6. **CONCLUSION**

The picture which emerges from the preceding discussion is not altogether a coherent one. On the one hand, the deficiency of Algol systems which must have first reached mass transfer with the primary on the giant branch suggests that systems whose primaries develop deep convective envelopes prior to mass exchange probably suffer extensive mass and angular momentum loss, presumably through common envelope evolution. This would scarcely by surprising, since we expect on theoretical grounds (Paczynski 1965; Paczynski, Ziolkowski, and Zytkow 1969) that such systems are unstable to dynamical time scale mass transfer. On the other hand, the existence of long-period, low-mass, post-mass-transfer systems among symbiotic stars and possibly barium stars suggests that this may not always be the case. This difference requires some explanation, but it may be related to the stabilizing effects, among systems with much more evolved giants, of mass loss in a non-magnetic stellar wind (e.g., Eggleton 1985) and of their relatively large ratios of degenerate core mass to envelope mass (Hjellming and Webbink 1985).

We have seen that the properties of the cool components of cataclysmic variables and close binary planetary nebulae nuclei reflect remarkably little disturbance of these components, either through mass loss or mass accretion, during common envelope evolution. This conclusion holds the promise of greatly simplifying detailed calculations of common envelope evolution. On the other hand, the apparent normality of the cool components of close binary planetary nebulae nuclei mitigates against forming them from objects of sub-stellar masses (see Eggleton
I wish to thank the scientific organizing committee for this opportunity to expound upon these views. I also want to thank Drs. James W. Truran, Mario Livio, Peter Eggleton, and Erika Böhm-Vitense for very helpful discussions regarding several aspects of the objects and processes discussed here. This research was supported by National Science Foundation grant AST 83-17916 to the University of Illinois and by a Visiting Fellowship to the Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards.

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