Improved Kennedy-Thorndike Experiment - a preliminary report

Dieter Hils and J. L. Hall
Joint Institute for Laboratory Astrophysics
National Institute of Standards and Technology and University of Colorado
Boulder, CO 80309-0440

The possibility of using lasers to improve the accuracy of the classical experiments (1-3) of Special Relativity (SR) was originally suggested by Javan and Townes and in fact they were the first to make a more precise Michelson-Morley experiment (4,1). The full potential of modern laser frequency metrology for length measurements was however not exploited until the more recent precision Michelson-Morley experiment (MM) of Brillet and Hall (5) which achieved a fractional frequency uncertainty of ±2.5×10^{-15} in showing the isotropy of space. They noted also the technical difficulties which would have to be overcome to achieve similar large improvements in a laser version of the Kennedy-Thorndike (KT) experiment (2), which compares the transformations of time and length in a moving frame. The present measurements yield a sensitivity of ~2×10^{-13} for a term with the expected 24 sidereal hr term, corresponding to a ~300-fold higher accuracy than the original KT experiment.

Following Roberson (6), Mansouri and Sexl (7-9) have developed a useful framework for explaining what an experiment measures and how it relates to other experiments. They consider two coordinate systems in relative motion, and write the most general transformation between Σ (the preferred frame) and S (the moving frame). They introduce multiplicative kinematical parameters which might be determined by theory but, more importantly, can be determined by experiment. Because of isotropy in Σ, these transformation parameters are even functions, dependent only on (v/c)^2. Apart from synchronization, the transformation between the preferred frame Σ and a moving frame S is specified by these parameters. The velocity of a light ray in S in general depends on the direction of propagation. Special Relativity makes the unambiguous statement Σ = -1/2, β=1/2, δ=0, corresponding to c(0) = c for all frames.

Modern astrophysical measurements (10) of anisotropy in the microwave background (µW) seem to define a universal standard of rest which reasonably could be taken to be the preferred frame Σ. In the following we will assume that the relevant velocity v is given by the observed motion of earth with respect to the µW frame. The signature of the effect we search for is its dependence on the sidereal modulation of v due the earth's rotation.

Present knowledge of the parameters Σ, β, δ (which also quantifies the degree of agreement of Einstein's SR and observation) comes principally from the second order MM and KT optical experiments and from optical and Mossbauer rotor experiments which determine the time dilation parameter a. (For a more detailed discussion of the many excellent experiments we refer to Refs. 7-9, 11.) The most accurate time dilation experiments (12-14) imply a = -(1/2) ± 1×10^{-7} and the most accurate MM experiment (5) determines β = (1/2) ± 5×10^{-9}. The original KT experiment (2) leads to a-β = -1±2×10^{-2} which thus introduces the single greatest uncertainty in the transformation equations. For these reasons, the importance of improving the KT experiment has been stressed repeatedly (9, 15, 16).

In the KT experiment a differential comparison was made between a standard of time defined by a mercury lamp and a standard of length in the form of an unequal arm Michelson interferometer. Our laser experiment utilizes instead two He-Ne

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lasers, one locked to a molecular absorption line in I$_2$ (R(127) 11-5), while the other is locked to a very stable Fabry-Perot reference cavity. Their frequencies are compared by optical heterodyne detection. Let $\nu^S_2$ denote the I$_2$-stabilized laser frequency. From Eqs. (6.15-18) of Ref. 8, we determine (17) the cavity locked laser frequency $\nu^S_1$ in the moving frame S. The heterodyne beat of the two optical frequencies is

$$\frac{\nu^S_{\text{beat}}}{\nu_C} = 1 + (\beta - \alpha - 1) \frac{(\nu/c)^2}{(\delta - \beta + 1/2)} \left( \frac{(\nu/c)^2 \sin^2 \theta}{\nu^S_2/\nu_C} \right),$$

(1)

where $\nu_C = pc/(2L)$, $p$ is an integer, and $L$ is the length of the Fabry-Perot interferometer as measured in S. The precise MM result of Brillet and Hall (5) implies $\delta - \beta + 1/2 = 0.55 \times 10^{-9}$ and so to this accuracy

$$\frac{\nu^S_{\text{beat}}}{\nu_C} = 1 + (\beta - \alpha - 1) (\nu/c)^2 - \frac{\nu^S_2}{\nu_C}.$$  

(2)

Working in the preferred frame S one can show that $\nu^S_{\text{beat}}/\nu_C = [1 + \alpha (\nu/c)^2] \cdot (\nu^S_{\text{beat}}/\nu_C)$. The result is to be expected due to the time dilation effect.

We next determine the velocity $v$ of our laboratory with respect to the $\mu$W frame. Working in the earth equatorial frame we find for the main sidereal component:

$$(\nu/c)^2 = (u/c)^2 + 2 (u/c)(\Omega R_E/c) \cos \phi_L \cos \delta_\mu \sin [\Omega (t-t_0) + \Phi].$$

(3)

In this equation $u = 377 \pm 14$ km/s is the velocity of earth with respect to the $\mu$W frame (10,18), $\Omega = (2\pi/365$ days) with $R_E$ being the sidereal period and $R_E$ the earth radius, $\phi_L = 40^\circ$ is the latitude of Boulder, $\delta_\mu = -6.4 \pm 1^\circ$ is the observed declination of the $\mu$W velocity vector (10,18) and $\Phi$ is the phase at the start of the analysis epoch. From Eqs. (2) & (3) we see that determination of the factor $(\beta - \alpha - 1)$ critically depends on our ability to measure the 24 hr sidereal variation of the fractional beat frequency.

The principle of our experiment is discussed with reference to Fig. 1. A He-Ne laser ($\lambda = 6328 \text{ } \lambda$) is locked (19) to a highly stable, isolated Fabry-Perot interferometer, thereby satisfying optical standing-wave boundary conditions. The servo system then transforms length variations of the cavity (of accidental or cosmic origin) into laser frequency variations. These can be sensitively detected by optically heterodyning some of the laser power with an optical frequency reference provided by an I$_2$-stabilized laser. The beat frequency (~160 MHz) if the I$_2$-stabilized laser is locked to the "d" component of the R(127) 11-5 transition is counted for 40 s and stored with negligible dead-time. We usually acquire 4320 beat frequency readings (~2 days) in memory before we store the data on disk and restart the measurement.

Our fundamental standard of length is the Fabry-Perot interferometer. It uses Zerodur "gyro" quality mirrors optically-contacted to the ends of a Zerodur spacer (length 30 cm, diameter 15 cm). The radii of curvature are $R = 575 \text{ } \text{cm}$ and $R = \infty$; transmission is T = 30 ppm. The cavity fringe width = 72 kHz fwhm giving a finesse $F = 6600$, and a resonant transmission of ~2%. The interferometer is suspended by two stainless steel ribbons (1x0.01 cm$^2$), one at each end, inside a thick-walled vacuum envelope. A low-drift servo loop with ac thermistor sensing stabilizes this aluminum wall temperature to better than 5 $\mu$K over a 1 day period. The experiment is located inside the "Quiet House" (20) which provides a >20 dB thermal and acoustic barrier. The inside air temperature is stabilized by Peltier-cooled panels which pump the laser discharge heat out via a slow water flow. This yields a stability better than 1 mK over times of 1 hour and better than 10 mK over a 1 day period, in the face of room temperature variations of ±1 degree.
The "hard-seal" He-Ne laser provides about 1mW of red light at 6328 Å. The beam passes through an isolation stage formed by a pair of Faraday isolators and one acousto-optic modulator (AOM). An ADP phase modulator crystal (EOM) is driven at ~1 kV pk-pk at a modulation frequency of 25 kHz via a resonant step-up transformer. Additional isolation (via frequency shift) of the laser beam reflected by the Fabry-Perot interferometer is provided by a second AOM. The transmitted light is frequency shifted by a third AOM to avoid fringes of the photo-detector's scattered light and the output mirror of the cavity. The ac output of the photodetector, after lockin detection, provides the error signal for the servo-system. Based on the 10 µW of fringe signal and an unity gain frequency of 5 kHz, the shot noise limit of the cavity-locked laser is expected to be ~1 miliHz, while the observed (19) frequency noise at short times (~1 s) is less than 50 mHz using first harmonic detection.

Figure 2 shows a two day segment of the recorded heterodyne beat frequency. The uniform Zerodur creep, which is about 185 kHz, has been subtracted. We suppose residual temperature changes working via the I_2 pressure shift or the cavity expansion may cause part of the observed (small) variations.

From a three week record of the beat frequency data, we made predictions/correlations based on measured Quiet-House residual temperature variations. The model fitted (for the I_2) a small prompt term (~1.1 kHz/K) and (for the cavity) a "thermal integrator" of 24 hr time-constant (~320 Hz/K). The correspondence is good, but not perfect. It is clear that thermal control is still a problem. See Fig. 2.
FIG. 2. Heterodyne beat frequency residuals. Uniform cavity drift of 1.08 Hz/s removed. Fast noise is from the I2 reference. Slow drifts may be partly thermal.

Our first KT data begin on July 27, 1986, and end September 22, 1986. This includes ~15 days of uninterrupted data. Another block of data begins October 31, 1988, ends April 31, 1989, and includes more than 90 days of uninterrupted data. Analysis proceeds as follows: We first average the 4320 beat-frequency samples (=2 days) in blocks of 9 which reduces the sampling rate to 10/hr. We next remove from the 2 day data set the uninteresting linear trend from cavity creep. We note this procedure may remove some power (<10%) from a hypothetical sinusoidal signal at 1 cycle per day. We then decimate the data set (like Fig. 2) to a final sample rate of 1/hr. This leaves us with the remaining frequency residuals due to uncontrolled environmental perturbations, as well as to a possible "aether effect."

We have examined the frequency residuals for a sidereal signal by several methods, with similar results. One method is back-folding the data to form a 1 year record of more or less continuous data. Instead, here we Fourier analyze the full nearly-3-year-long record by putting zero values into the data gaps. (The amplitude scale has been corrected x 11.3 for the attenuation due to the zero-fill procedure.) The amplitude spectrum for Fourier frequencies up to 3 cycles per day is interesting. For frequencies <1/2 cycle per day the observed amplitudes steeply decline due to the removal of the linear trend from the data. For Fourier frequencies above a few cycles per day, the spectrum falls off rapidly as expected for a model which has thermal disturbances acting on a reference cavity with a time constant of ~1 day.

Examination of Fig. 3A indicates an enhanced noise level near 1.0 cycles per day. It is easy to imagine a strong driving term at the solar frequency, perhaps phase-shifted and broadened by variables such as cloud cover, weekend work schedules, etc. With breaks in the data, the usual windowing procedure is ineffective, so even a bright line input will corrupt adjacent frequency bins. To find the transfer function, we tried adding a strong solar or sidereal signal in the time domain. The half-amplitude FT width was ±3 bins. Removing the best solar sinewave drops the sidereal amplitude from 31.7 to 14.9 Hz. See Fig. 3B.

To test if the increased noise level was due to leakage from a "real" signal at the sidereal frequency, we removed this best-fitting sinewave in the time-domain as before. The spectrum was essentially unchanged from Fig. 3A. We conclude that the broad noise buildup around 1 cycle/d is due to solar -- not sidereal -- input.
For this report we work conservatively with the unmodified data of Fig. 3A. Further, we ignore the phase information implicit in Eqs. (2) & (3). Working near (±4 bins) the sidereal bin, the quadrature amplitudes are found to be normally distributed, with standard deviation $\sigma_n = \sqrt{\langle A_n^2 \rangle} = \sqrt{\langle A_0^2 \rangle} = 29 \text{ Hz}$. The sidereal amplitude, 31.7 Hz, gives a normalized measured value $x_m = 31.7/29 = 1.09$.

We now wish to analyze our results with the hypothesis that we have a sinusoidal signal of amplitude $P$ in the presence of random noise. From tabulated values of the integral of the error function, we find there is 10% probability for finding a realized value beyond $x_{10} = 1.28$. Thus from a simple analysis we would conclude that there is less than 10% probability that there was a real signal as large as $X = x_{10} + x_m = 1.28 + 1.09 = 2.37$ units of $\sigma_n$, i.e. $P < 68.7 \text{ Hz}$.

However, inspection of Fig. 3A shows that one local peak ($A=65 \text{ Hz}$) falls in bin 1094, just conjugate to the sidereal frequency of interest, bin 1100, i.e. symmetric around the solar bin (1097). We offer the following, less optimistic scenario: Suppose that the modulation processes (weather, cloud-cover, etc.) that spread out the solar forcing-function produce symmetrical "sidebands" around the solar bin. Then we can view the upper "weather" sideband, also of $A = 65 \text{ Hz}$, as partially cancelling a putative sidereal amplitude to give the realized value of 31.7 Hz. A pessimistic estimate would assume ideal out-of-phase cancellation, producing the value $65 + 31.7 = 96.7 \text{ Hz}$ for this sidereal amplitude. We regard this scenario as less than 10% probable, and can therefore believe that our experiment shows, with a probability > 90%, that there is no sidereal signal as large as 96.7 Hz (rounded to 100 Hz).

The present result, setting an upper limit to a possible "aether effect" amplitude of $P < 100 \text{ Hz}$, corresponds to a fractional frequency amplitude $\Delta \nu / \nu < 2 \times 10^{-13}$. From Eqs. (3),(4) our experimental result can be expressed in the form $2(\beta - \alpha - 1) u < 50 \text{ m/s}$. Using the value $u = 377 \text{ km/s}$ one obtains $(\beta - \alpha - 1) < 6.6 \times 10^{-5}$. This limit enables us to deduce separately $\beta = (1/2) \pm 2 \times 10^{-5}$ and $\delta = 0 \pm 2 \times 10^{-5}$. Taken with the already known value (12-14) $\alpha = -(1/2) \pm 1 \times 10^{-7}$, we see that the three parameters defining the transformation equations are now known to agree with Einstein's SR values to better than 70 ppm. This value represents a 280-fold improvement over the original
KT experiment. Further analysis, taking phase into account, may sharpen our limit.

The present experiment is not easily improved by a large margin, at least not for earth-based experiments. Some cavity problems would be minimized with a space-based experiment, e.g. cavity distortion due to body forces would be essentially eliminated. For an orbital period of ~90 min the effective thermal isolation of the reference cavity would be ~20 times better. With thermal control to 1/10 K, a factor of 100 improvement seems possible, if similar improvements of the I₂ stabilized laser are possible. We note that the flicker noise due to the I₂ laser is ~100 Hz (2×10⁻¹³), similar to the obtained value for Δν_beat. Since the pressure shift in the I₂ cell is ~15 kHz/K, a reservoir temperature stability of 10 μK will be needed. Intensity shifts and gas lens effects are also serious.

This report summarizes an improved Kennedy-Thorndike type experiment based on modern laser metrology. The heterodyne signal between a cavity-locked He-Ne laser and one stabilized on I₂ shows a fractional frequency amplitude at the sidereal frequency of Δν_beat/νc < 2×10⁻¹³ (90% confidence interval). This null result is more accurate by a factor of ~300 than the previous best measurement, by R. J. Kennedy and E. M. Thorndike in 1932! Following the reasoning of Robertson (6), the Lorentz transform of SR can now be based on experimental facts at the 70 ppm level.

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References

17. We note misprints in ref. 8: d² should read d⁻² in Eq. (6.15) and Eq. (6.17). Similarly 8 should read -δ in Eq. (6.18).