FREQUENCY SHIFTS OF HIGH FREQUENCY P-MODES

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ABSTRACT

Frequency shifts (the difference between the frequencies from one time to another during the solar cycle) of high frequency ($\nu > 4$ mHz) p-modes are calculated for a non-magnetic polytrope convection zone model over which lies an isothermal chromospheric atmosphere threaded by a uniform horizontal magnetic field. The relevant observations of such frequency changes are discussed. Simultaneous changes in field strength and chromospheric temperature result in frequency shifts (theoretically calculated) that are similar to those observed.

Keywords: Magnetic fields, p-modes, solar cycle

1. INTRODUCTION

It is well known that p-mode frequencies ($\nu < 5$ mHz) are solar cycle dependent. The changes are reported for both low degree (Elsworth et al. 1990; Anguera Gubau et al. 1992) and high degree modes (Libbrecht and Woodard 1990; Woodard and Libbrecht 1991; Woodard et al. 1991). Recently, Elsworth et al. (1994) pointed out that the variations are similar for low and high degree which might suggest that the sources of such changes could lie in the surface layers rather than at depth.

The observations of Libbrecht and Woodard (1990) showed that if the change in frequency, $\Delta \nu$, of the solar p-mode, from years 1986 to 1988 and 1989, is plotted as a function of 1986 frequency, $\nu$, then $\Delta \nu$ first increases with frequency up to about 4 mHz but then turns over and falls sharply towards zero shift. These observations were averaged over degrees between 4 and 140. Jain and Roberts (1993) showed that simultaneous increases in chromospheric field strength and temperature can result in frequency shifts of the observed form, though less steep (thus suggesting the need for further refinements in the theoretical model).

A recent observation has entered the record of p-mode frequency shifts - the shifts in the frequencies are also found in high frequency modes (Ronan et al. 1994a, b). Earlier observations were confined to modes of frequency $\nu < 5$ mHz, where as the observations by Ronan et al. are in the range $4 < \nu < 6.5$ mHz. Ronan et al. compared the p-mode frequency data for 1991.4 with the data for 1987.9 averaged over degrees 30 to 150. They show that frequency shifts at high frequency ($> 5$ mHz) are opposite in sign to those recorded at low $\nu (< 5$ mHz) by Libbrecht and Woodard (1990), the shift is of order 10 $\mu$Hz near the photospheric cut-off frequency ($\approx 5.4$ mHz). Considering these two different data sets, it appears that the frequency shifts of the p-mode as a function of frequency exhibit a wave-like pattern with a crest occurring at about 4 mHz and a trough at about 5.5 mHz. This wave-like behaviour in the frequency shift is interesting. Moreover, the reported negative frequency shifts, of order $-10 \mu$Hz, are much stronger than those found at low $\nu$. These observations raise two basic questions: why are the p-mode ridges extended to such high frequencies, and what causes a solar cycle dependent frequency shift?

So far two explanations have been put forward to answer the first question. According to the first explanation, by Balmforth and Gough (1990), the chromosphere - corona transition region acts as an outer reflecting boundary to trap the high frequency acoustic waves. Alternatively, the acoustic waves with frequency greater than the acoustic cut-off frequency $\nu_{ac}$, i.e. $\nu > \nu_{ac}$ propagate away from temperature minimum and then get reflected at transition region.

The second explanation was proposed by Kumar and Lu (1991). According to these authors, the discrete spectra could be due to constructive interference in the photosphere, between acoustic waves directed from the source and ones that are bounced back from the lower turning point in the convection zone. They rule out the possibility of a strong acoustic reflection at the transition region (as suggested by Balmforth and Gough) on two accounts. Firstly, they argue that there is no observational evidence for frequency spacings between adjacent eigenmodes at a fixed $l$ to be irregular, which is theoretically expected if there is a significant reflection in the chromosphere or corona (Kumar 1993). Secondly, there is strong evidence of propagating acoustic waves with $\nu \geq 5$ mHz in the photosphere and lower chromosphere. The time-distance diagrams presented by Duvall et al. (1993) suggests that high frequency $p$-modes are not reflected off the corona. They interpret their observational results in favour of the scenario proposed by Kumar and Lu.

The analysis of Lou (1993), on the original data of Duvall et al. (1991), show changes in the frequency interval, $\Delta \nu_p$, in the range 4-17 $\mu$Hz for $50 < l < 150$. Lou (1993) also shows, with the aid of a very simple theoretical argument, that acoustic waves can be strongly reflected at the transition region, as suggested by Balmforth and Gough. Thus, together with the observational evidence
for \( \Delta \nu_p \). Lou (1993) favours the suggestion by Balmforth and Gough. He also suggests the possibility of acoustic waves with large \( l \) values being evanescent around the temperature minimum. However, the lack of observational evidence for \( \Delta \nu_p \) to change across \( \nu_{ac} \) for \( l < 50 \) and the rather simple physical considerations to demonstrate the reflections of high frequency \( p \)-modes around transition region, still leaves the puzzle of high frequency \( p \)-modes unsolved.

In an isothermal atmosphere with no magnetic fields, the acoustic cutoff frequency \( \nu_{ac} \) allows modes with \( \nu > \nu_{ac} \) to propagate within the atmosphere, whereas as the modes with \( \nu < \nu_{ac} \) are evanescent. At the temperature minimum of the Sun, \( \nu_{ac} \approx 5.3 \) mHz. However, magnetism changes this simple view of wave propagation and trapping (see for example, Jain et al. 1995).

In this paper, I examine frequency shifts for a model consisting of a field-free, polytropic convection zone above which lies an isothermal atmosphere threaded by a uniform horizontal magnetic field. In the presence of such fields, the Alfvén speed increases with height, causing all waves to be reflected whatever their frequency. Thus, in a uniform magnetic field, trapped \( p \)-modes exist for frequency above the acoustic cut-off frequency of 5.3 mHz. It should be noted that the real Sun does not have a uniform fields and the Alfvén speed in its atmosphere is bounded; but the Alfvén speed nonetheless increases rapidly with height to reach a value of order \( 10^3 \) km s\(^{-1}\) in the corona, very much larger than either the Alfvén speed or the sound speed (about \( 7 \) km s\(^{-1}\)) in the low chromosphere. Accordingly, we must expect the magnetism of the solar atmosphere to have an appreciable effect on waves that penetrate into it.

2. THE MODEL

The model shown in Figure 1 is studied in an attempt to find out how the \( p \)-mode frequencies are influenced by a horizontal magnetic field. The model introduces two distinct layers, an upper layer representing the magnetic chromosphere and a lower one representing the convection zone. The reference level \( z = 0 \) corresponds to the base of the chromosphere (i.e. the temperature minimum). The chromosphere and the convection zone are represented by the regions \( z < 0 \) and \( z > 0 \), respectively, with \( z \) increasing downwards. The medium is stratified by gravity, \( g \). The chromosphere is taken to be isothermal, with temperature \( T_c \), within which is embedded a uniform horizontal magnetic field of strength \( B_x \). This region models the chromospheric canopy. The region below the magnetic chromosphere, modelling the convection zone, is assumed to be field-free and to have a temperature \( T_p (1 + z/z_o) \) varying linearly with depth \( z \). Here \( z_o \) is the temperature scale-height and \( T_p \) is the temperature at the top \( z = 0 \) of this region.

For an isothermal chromosphere \( (z < 0) \), with a uniform magnetic field \( B \) in the \( x \)-direction, vertical component of velocity \( v_z (z) \exp i (\omega t - kx) \), with angular frequency \( \omega \) and horizontal wavenumber \( k \), is governed by the equation (see Jain 1992):

\[
\{ \mathcal{A}_1 + \mathcal{A}_2 \exp (-\mathcal{A}_3 z) \} \frac{d^2 v_z}{dz^2} + \mathcal{A}_4 \frac{dv_z}{dz} + \mathcal{A}_5 \left( \frac{\omega^2 - k^2 c_{ip}^2}{c_{ip}^2} \right) v_z = 0,
\]

where

\[
\begin{align*}
\mathcal{A}_1 &= \omega^2 c_{ip}^2, \\
\mathcal{A}_2 &= v_z^2 c_{ac}^2 (\omega^2 - k^2 c_{ac}^2), \\
\mathcal{A}_3 &= \frac{1}{B_x}, \\
\mathcal{A}_4 &= (\gamma - 1) g^2 k^2 + \omega^2 (\omega^2 - k^2 c_{ac}^2). 
\end{align*}
\]

Here \( B_x \) is the scale height within the isothermal atmosphere, \( c_{ac} \) is the sound speed, \( v_z \) is the Alfvén speed at the base of this region, \( \gamma \) is the adiabatic index, and \( g \) is the local gravitational constant.

Figure 1: The equilibrium structure of the model, consisting of a field-free thermally stratified 'convection' zone in \( z > 0 \) and an isothermal magnetic atmosphere in \( z < 0 \).

In the non-magnetic lower region \( (z > 0) \), representing the convection zone, the governing equation (for \( \Delta \equiv \text{div} \mathbf{v} \)) is

\[
\frac{d^2 \Delta}{dz^2} + \left( \frac{c_{ip}^2 + \gamma g}{c_{ip}^2} \right) \frac{d \Delta}{dz} + \left( \frac{\omega^2 - k^2 c_{ip}^2}{c_{ip}^2} \right) \Delta = 0.
\]
a prime denoting the derivative with respect to depth \( z \); 
\( c_{sp}'(z) \) is the square of the sound speed in the field-free region. For a linear temperature profile,

\[
c^2_{sp}(z) = c^2_{sp} \left( 1 + \frac{z}{z_0} \right), \quad z > 0,
\]

the solution of Equation (3) was first given by Lamb (1932). In terms of \( v_s \), we have

\[
(\omega^4 - g^2 k^2) v_s e^{k(z+z_0)} = C_1 \left\{ \left[ k c^2_{sp}(\omega^2 + g k) - \gamma g \omega^2 \right] U(-a, m + 2, 2kz + 2kz_0) \right. \\
-2a\omega^2 c^2_{sp} k U(-a + 1, m + 3, 2kz + 2kz_0) \right\}. \tag{5}
\]

Here \( C_1 \) is an arbitrary constant, \( U \) is a confluent hyper-geometric function, and \( m \) is the polytropic index. The parameter \( 2a \) is given by

\[
2a = \frac{\omega^2 m + 1}{g k} + \left( m - \frac{m + 1}{\gamma} \right) \frac{g k}{\omega^2} - (m + 2). \tag{6}
\]

The solution (5) is obtained by demanding a finite kinetic energy density at large depths, i.e. by requiring that \( \frac{1}{2} \rho(z)v_s^2 \to 0 \) as \( z \to -\infty \), for equilibrium density \( \rho(z) \).

3. NUMERICAL SOLUTIONS

Here, Equation (1) is solved numerically, for frequencies beyond 5 mHz. For physically real solutions, we require that \( v_s \) and \( dv_s/dz \) tend to zero as \( z \to -\infty \). The other conditions are that \( v_s \) and the total pressure perturbation are both continuous across the interface \( z = 0 \).

The cyclic frequency \( \nu \equiv \omega/2\pi \) for each mode \((n,l)\) is calculated for a given chromospheric temperature \( T_c \) and magnetic field strength \( B_c \), producing a base frequency \( \nu(B_c, T_c) \). The cyclic frequency \( \nu \) is repeated for a higher chromospheric temperature \( T_c' \) \((> T_c)\) and magnetic field strength \( B_c' \) \((> B_c)\), for the same pair \((n,l)\), thus producing cyclic frequencies \( \nu(B_c', T_c') \). The calculated frequency change \( \Delta \nu \) is the difference between these two sets: \( \Delta \nu \equiv \nu(B_c', T_c') - \nu(B_c, T_c) \). The parameters \( c_{sp}, m, \gamma \) and \( z_0 \), describing the convection zone \((z > 0)\), are fixed (see Jain and Roberts, 1993), and the degree \( l \) of the mode is determined by \( k^2 = \ell(\ell + 1)/R^2 \), where \( R \) is the solar radius.

4. RESULTS AND CONCLUSIONS

Figure 2 illustrates the calculated frequency shift \( \Delta \nu \) (in \( \mu \)Hz) as a function of base frequency \( \nu \) (in mHz) for modes of degree \( l = 100 \). The chromospheric magnetic field strength and temperature for the base frequency are taken as \( B_c = 20 \text{ G} \) and \( T_c = 4170 \text{ K} \). The calculations are repeated for a higher magnetic field strength \( B'_c = 30 \text{ G} \), and the chromospheric temperature \( T'_c \) is varied between 4170 K and 8000 K, as indicated in the figure caption.

![Figure 2: The frequency shift \( \Delta \nu \) as a function of frequency \( \nu \), for degree \( l = 100 \). The base frequencies \( \nu(B_c, T_c) \) are calculated for a chromospheric magnetic field \( B_c = 20 \text{ G} \) and a chromospheric temperature \( T_c = 4170 \text{ K} \). The mode frequencies \( \nu(B'_c, T'_c) \) are calculated for a magnetic field strength \( B'_c = 30 \text{ G} \) and the various curves shown correspond to chromospheric temperatures of (a) \( T'_c = 5000 \text{ K} \), (b) \( 6000 \text{ K} \), (c) \( 7000 \text{ K} \), and (d) \( 8000 \text{ K} \). It can be seen from Figure 2 that frequency shifts at a given frequency decrease with increasing temperature \( T'_c \). This is clear for frequencies greater than 4 mHz but can also be seen for lower frequencies if we magnify the low frequency regime of the curves (see Jain and Roberts, 1993). The downturn occurs at around 4 mHz and the magnitude is of few mHz where as the upturn occurs at a higher frequency and the magnitude is of few MHz. Thus frequency shifts produce an oscillatory behaviour (with downturn and upturn) as a function of frequency. The observations of Ronan et al. (1994) show a decrease in the frequency shifts up to about 5.5 mHz, with a magnitude of about 10 \( \mu \)Hz near 5.5 mHz, and an increase after that to reach zero around 6.5 mHz. The observed frequency shifts also increase with degree \( l \) although the exact relationship is not so clear. The present theoretical model is too simple to account for all the quantitative details of the observations. For example, the broad line widths in the observed p-mode spectra for the high frequency p-modes suggest that the damping effects not considered here may be important. 

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In conclusion, magnetism is important in the discussion of solar $p$-modes. As shown in Jain et al. (1993), it modifies the acoustic cut-off frequency and possibly, offers an explanation for solar cycle changes in $p$-mode frequencies.

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REFERENCES

7. Jain, R., Hindman, B. W. and Zweibel, E. G. 1993, see the other poster.