Eötvös Experiment with a Fluid Fiber

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Recent work aimed at an improved Eötvös experiment is described. Copper and tungsten test masses are contained in a cylindrical float supported by the buoyancy of water which is held at its maximum density point. Electrostatic forces are used to keep the float centered and to provide the torsion constant. Sources of noise associated with convection currents, magnetic fields, seismic noise, changing gradients in the gravitational field, long term drifts in the position detector, and Brownian motion are described, and recent experimental results are presented.

A necessary condition for general relativity and other metric theories of gravity is that the ratio of gravitational to inertial mass must be independent of the composition of the material. This condition was experimentally tested by Eötvös, Pekar and Petočé† who showed that the fractional difference in ratio of gravitational to inertial mass for a wide variety of substances was less than $3 \times 10^{-9}$. This experiment was repeated in 1964 by Roll, Krotkov, and Dicks‡ who showed that the fractional difference in the ratio of gravitational to inertial mass for gold and aluminum was less than $3 \times 10^{-11}$, and in 1972 by Panov and Braginsky§ who reported a fractional difference of less than $0.9 \times 10^{-12}$.

All of these experiments were performed using a traditional torsion balance; but, despite a significant amount of effort, none of these experiments reached the sensitivity that would be expected if they were limited by the Brownian motion of the torsion balance. Other sources of noise have dominated the Brownian motion. The torsion fiber itself is a source of noise, and the torsion balance is extremely sensitive to seismic noise. Also, the sensitivity of the torsion balance is limited because the torsion fiber not only supports the test masses but also provides the torsion constant. An attempt to increase the sensitivity of the torsion balance by increasing the size of the test masses would also require an increase in the radius of the torsion fiber. Increasing the radius of the torsion fiber, however, places even more severe requirements on the position detector.

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In an effort to circumvent the limitations of the traditional torsion balance we have separated the two functions of the fiber: we support the test masses by floating them on a fluid and use electrostatic forces to provide the torsion constant. A diagram of the apparatus is shown in Figure 1. The copper and tungsten test masses are enclosed in a hollow, cylindrical, aluminum container which is 10 in. (25.4 cm) in diameter and 9.5 in. (24.1 cm) high. Six cylindrical copper test masses (each of which has a mass of 509.8 g) are arranged in a 10 cm radius semicircle on one side of the float. Six tungsten test masses (each with a mass of 509.8 g) are arranged in a 10 cm radius semicircle on the other half of the float. If there is a difference in the ratio of gravitational to inertial mass for copper and tungsten, then there will be a slight difference in the acceleration of the copper and tungsten test masses toward the sun, and this will evidence itself as a 24 hour variation in the angular position of the float. The phase of this signal will depend on the orientation of the test masses.

Above the cylindrical float are six mirrors arranged in a hexagonal pattern so that we may monitor the position of the float in any one of six different orientations. Above these mirrors is an aluminum plate with seven 1 in. diameter aluminum balls that serve as electrodes to keep the floats centered and to provide the torsion constant. Only three of the electrodes are shown in this diagram. There is a matching set of seven aluminum balls that is attached to, but electrically insulated from, the top of the container. A 77 Hz ac voltage, which is typically 1 kV rms, is applied to the center electrode to keep the float centered. The 77 Hz voltage on the six outer electrodes, which are arranged in a hexagonal pattern, may be varied to change the torsion constant. With a 0.125 in. gap between the electrodes the float is critically damped (torsion constant 1 dyne-cm) at a torque electrode voltage of approximately 40 V rms.

One of the difficult problems associated with using a fluid to support the test masses is that convection currents within the fluid tend to rotate the float and introduce noise into the system. For this reason we have chosen water as the

![Figure 1](image-url)
fluid to support our test masses because it has the unusual and here very useful property that it has a maximum in its density at 3.98°C. If the temperature of the entire apparatus is stabilized at this point, then small variations in the temperature of the water do not change the density of the fluid. Thus there are no buoyant forces to drive convection currents when operating at 3.98°C.

The initial tests with water at its maximum density point were encouraging. As the temperature approached 4°C, there was a significant decrease in the rotational noise of the float. In recent experiments, the temperature of the entire apparatus changes by less than ten millidegrees centigrade over periods of 24 hours, and the temperature is kept within 0.1°C of the maximum density point. The apparatus is cooled by circulating water around an outer container. The temperature of this outer container is measured with a thermistor and controlled by sending a small current through a Litz wire heater. Temperature variations with periods of less than one day are further reduced by the insulation between this outer container and the inner container that holds the float and the water.

To maintain the free operation of the float we have had to add a wetting agent (Photo-Flo) to the water. Measurements of the effect of a one part in two hundred concentration of this wetting agent on the density of water show that its addition changes the maximum density point by less than 0.1°C.

Variations in the magnetic field strength at the float are a potential source of noise. One of the early (test) versions of this float contained a large piece of brass which had magnetic impurities such that a change of 0.04 G in the external dc magnetic field produced a torque of $10^{-4}$ dyne-cm - $10^{-4}$ times the magnitude of the Eötvös torque that we are looking for. In subsequent versions of the float, the only materials that were used were OFHC copper, tungsten, aluminum, RTV, epoxy, titanium, glass and Buna-N O-rings. In addition, all parts were machined with cemented carbide tools in an attempt to insure that no magnetic chips were introduced into the material. With these precautions, a change in the magnetic field of 1 G produces a torque of less than $10^{-7}$ dyne-cm on the float.

Eddy currents produced in the float by the 10 mG rotating magnetic fields that exist in the laboratory at 60 Hz give rise to a magnetic dipole moment that leads the rotating magnetic field by 90°. The interaction of this magnetic dipole moment with the rotating magnetic field produces a constant torque of approximately $5 \times 10^{-5}$ dyne-cm on the float in a direction opposite to that of the rotating vector for the magnetic field. The magnitudes of these rotating magnetic fields were measured with two sets of pick-up coils outside the apparatus and their effects reduced by magnetically shielding the entire apparatus with a 0.030 in. thick piece of Neenetic magnetic shield. In this way, we were able to reduce the magnitude of the rotating magnetic fields until they made no significant contribution to the torque on the float.

Another possible source of noise associated with magnetic fields results from the different susceptibility of the copper and tungsten test masses. If there is a magnetic field as well as a gradient in the magnetic field, then the force on the copper test masses will have a different magnitude and be in the opposite direction to the force on the tungsten test masses. For a magnetic field of 1 G, a field gradient of $10^{-6}$ G/cm will produce a torque of $10^{-8}$ dyne-cm. Although field gradients much larger than this may be produced and this effect may easily be observed, with sufficient care magnetic materials may be kept far enough away that this source of noise is not significant.

Changing gradients in the gravitational field are also a potential source of noise. Considerable attention was devoted to constructing the float so that it was cylindrically symmetric, but no matter how carefully this is carried out there will inevitably be small asymmetries in the mass distribution. To reduce
the spherical components of the mass quadrupole moment tensor (hereafter referred to as quadrupole moments) in the float, we created "large" gravitational field gradients using 200-pound stacks of lead bricks, measured the torque on the float, then disassembled the float and added mass to reduce the mass quadrupole moments. This procedure, described in detail in the appendix, though time consuming, provides a systematic method for measuring and reducing the mass quadrupole moments. Going once through this procedure reduced the largest mass quadrupole moment by 75%, with the result that that each of the four mass quadrupole moments was less than 100 g-cm² (compared to a moment of inertia of 10⁶ g-cm²). Further iterations using this procedure could be used to further reduce the residual quadrupole moment.

One serious limitation of the torsion balance which has evidenced itself in other measurements is its sensitivity to seismic noise. By comparison, tests with our surrogate (fluid) fiber show that it is remarkably insensitive to seismic noise. For a period of several weeks there was no observable correlation between the rotational noise of the float and the vertical seismic noise that was measured in the same room. The only evidence of any sensitivity to seismic noise is a possible response to an impulsive torque, which caused a deflection of 2 arc sec with a torsion constant of 1 dyne-cm, at the same time as an earthquake that registered a 6 on the Richter scale at a distance of 1000 miles. However, this particular event could have been coincidental rather than real: other large earthquakes have not produced similar identifiable responses. Tests to determine whether the day-to-day tilt of the floor (1 arc sec) affected the position of the float have also showed no significant effect. This insensitivity to seismic noise may be understood by two features of this apparatus that are very different from the torsion balance: the other oscillation modes of the float (the bobbing mode and the rocking mode) are highly damped, and the position of the float with respect to its container is independent of the acceleration of the container.

The requirements on the sensitivity and long term stability of the position detector depend on the torsion constant. Usually the torque electrode voltages were adjusted to provide a torsion constant of 1 dyne-cm. Under these conditions the apparent motion of the float will be less than the one-day Brownian motion if the drift in the position detector is less than 10⁻⁸ rad. The position detector is an autocollimator which is mounted on a stand outside the apparatus at room temperature. This stand is rigidly attached to the table that supports the apparatus. The light source for the autocollimator is a Texas Instruments T1E516C light-emitting diode which is usually operated (at 1/10 of maximum power) so that it emits a total of 30 mW of light at 0.94 μm. This LED is mounted on a large aluminum block at the focus of an 11-in. focal length achromatic lens. The output light from this lens, after reflecting from one of the fixed mirrors on the float, returns back through this lens and is focused down onto a split photocell (two silicon photodiodes which have a common cathode and are mounted on the same substrate). The current from these two photodiodes is amplified and then subtracted. The difference signal is a measure of the rotation of the mirror on top of the float. To avoid problems associated with temperature coefficients in the operational amplifiers and other sources of long term drift, the current to the light emitting diode was modulated at 50 Hz and the difference signal was measured with a phase-sensitive detector. The signal from the autocollimator is linear to within several percent over a range of 100 arc sec.

The stability of the autocollimator, the stand it rests on, and the mechanical stability of the torque electrodes can be tested by increasing the torque electrode voltage, or by operating the float in a feedback mode where the torque

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electrode voltage is electronically adjusted to keep the float in the same position. Tests to date indicate that the stability of the autocollimator is better than 10^{-7} arc sec. There is, however, some evidence that a problem may exist with the mechanical stability of the torque electrodes; this "geometry" question is still under investigation.

The fundamental limit to the torque that can be measured is due to the Brownian motion of the float. The mean square torque in a bandwidth Δf having a given phase is

\[ \langle \tau^2 \rangle = 2k_B T b \Delta f \]

where \( k_B \) is Boltzmann's constant, \( T \) is the absolute temperature, and \( b \) is the damping coefficient in the equation of motion

\[ I \frac{d^2 \theta}{dt^2} + b \frac{d \theta}{dt} + k \theta = \tau \]

where \( I \) is the moment of inertia (1.14 × 10^6 g·cm²), \( k \) is the torsion constant, and \( \tau \) is some external torque acting on the system. The damping is primarily due to the water. The measured value of the damping coefficient is 2×10^3 dyne·cm·sec which agrees well with the value calculated by assuming that the damping is all due to the water.

The float is usually operated so that the line dividing the copper test masses from the tungsten test masses lies in an east-west direction. Under these conditions the torque on the float at a 24 hour period is

\[ \tau = M a R \eta \cos \delta \]

where \( M \) is the size of one of the test masses (3.06 kg), \( a \) is the acceleration toward the sun (0.62 cm/second²), \( R \) is the effective moment arm of the test masses (6.0 cm), \( \delta \) is the declination of the sun and \( \eta \) is a measure of the fractional difference in the ratio of gravitational to inertial mass for copper and tungsten:

\[ \eta = \frac{\frac{m_C}{m_I}}{\frac{1}{2} \left( \frac{m_C}{m_I} + \frac{m_C}{m_I} \right)} \]

Then, setting the signal torque equal to three times the noise torque, the minimum value of \( \eta \) that can be measured with a 3σ error is

\[ \eta_{\text{min}} = \frac{3/2k_B T b \Delta f}{M a R} \]

Using the values for our present apparatus, we have

\[ \eta_{\text{min}} = 1.1 \times 10^{-11}/\sqrt{\text{Day}} \]

For a given size apparatus this limitation is insurmountable. However, as the linear size of the apparatus, \( D \), increases the minimum measurable value of \( \eta \) decreases as \( D^{-5/2} \).

Before beginning a data run it is only necessary to calibrate the autocollimator and determine the torsion constant of the float. The autocollimator is calibrated by measuring the voltage output from the phase sensitive detector as the autocollimator is rotated and measuring the angle of rotation with a commercial autocollimator. The torsion constant is determined by measuring the oscillation
period and the damping time of the float at several different torque electrode settings. If the moment of inertia of the float is known, then the torsion constant may be determined as a function of the torque electrode voltage. The moment of inertia was measured by supporting the entire float in an air bearing and measuring the oscillation period with a deliberate mass unbalance. There is also a 13% correction to this moment of inertia because of the water that rotates with the float. This correction was determined by using a digital computer to solve the Navier-Stokes equation for our apparatus. Measurements of the torsion constant show that it is proportional to the square of the applied voltage as expected.

A typical data run consists of monitoring the float position, electrode voltages, temperatures, and other parameters that might affect the position for several weeks. A plot of the float position vs time for a two week stretch is shown in Figure 2. The amplitude of the 24-hour signal, which has a maximum and minimum at sunrise and sunset, is $\theta = 5.6 \times 10^{-8}$ rad, and the amplitude of the torque with this period and phase is $6.8 \times 10^{-8}$ dyne-cm. However, the method of assigning an uncertainty to this number is not clear cut. One approach is to take the rms amplitude of the position at five frequencies from 0.67 cycles/day to 1.33 cycles/day, with the exception of the 1.0 cycle/day amplitude, as a measure of the uncertainty. With this approach, the fractional difference in the ratio of the gravitational to inertial mass is

$$\eta = (0.6\pm2) \times 10^{-11}.$$

On the other hand, the amplitude of the 24-hour component of the position which is 90 degrees out of phase with the expected signal is larger than the noise at nearby frequencies. Since the temperature, the line voltage, gravitational gradients, and many other possible sources of noise have a 24-hour component, this

![Figure 2](image-url)
result is not surprising. Since, however, one cannot rule out a comparable amplitude in phase with the signal, it is equally reasonable to take this amplitude as a measure of the uncertainty. This gives

$$\eta = (0.6\pm 4) \times 10^{-11}$$

No attempt has been made to estimate a 3σ error.

This result is still an order of magnitude away from the limit that would be expected if the motion of the float was predominately determined by Brownian motion at periods of 24 hours. Down to frequencies of 10 cycles/day a power spectrum analysis shows the motion is limited by the Brownian motion of the float. There are, however, below this frequency apparently some non-Brownian-motion sources of noise that are not well understood. Our plans are to continue operating the present apparatus in an effort to understand these torques. In addition, we are constructing a float that is five times larger, which should give a 3σ, one day, Brownian motion limit in the parameter $\eta$ of $2 \times 10^{-13}$.

APPENDIX

Procedure for Tuning Out the Mass Multipole Moments

The gravitational interaction energy between some mass distribution on the float $\rho(r')$ and some external mass distribution $\rho(r)$ is

$$W = -G \int dV' \int dV \frac{\rho(r') \rho(r)}{r - r'}$$

Here, $r$ is a vector from the center of mass of the float to an external volume element, $dV$, and $r'$ is a vector from the center of mass of the float to a volume element, $dV'$, in the float. This interaction energy may be expanded in terms of Associated Legendre Polynomials as

$$W = -G \int dV' \int dV \rho(r') \rho(r) \sum_{\ell,m} (-1)^m (\ell + 1) \frac{\ell + 2}{\ell} \frac{1}{\ell + 2} p_{\ell m}(\cos \theta) e^{-i m \phi}$$

The torque on the float may be found by taking the derivative of this interaction energy with respect to a rotation of the float about the vertical axis

$$\tau = \frac{dW}{d\phi} = \sum_{\ell,m} (-1)^m \ell (\ell + 1) \frac{dV'}{dV} \rho(r') (r') \frac{p_{\ell m}(\cos \theta)}{\ell + 1} \frac{1}{\ell + 2} e^{-i m \phi}$$

With some algebraic manipulation this expression may be rewritten as

$$\tau = \sum_{\ell=2}^\infty \sum_{m=1}^{2m(\ell-m)!} \frac{2m(\ell-m)!}{(\ell+m)!} [c_{\ell m} S_{\ell m} - c_{\ell m} S_{\ell m}]$$

where $s_{\ell m}$ and $c_{\ell m}$ are associated with the mass distribution of the float

$$s_{\ell m} = \int dV' \rho(r') (r') \frac{p_{\ell m}(\cos \theta)}{\ell + 1} \sin m \phi$$

$$c_{\ell m} = \int dV' \rho(r') (r') \frac{p_{\ell m}(\cos \theta)}{\ell + 1} \cos m \phi$$

and $S_{\ell m}$ and $C_{\ell m}$ are associated with mass distributions external to the float

$$S_{\ell m} = G \int dV \rho(r) \frac{1}{r^2} \frac{p_{\ell m}(\cos \theta)}{\ell + 1} \sin m \phi$$

$$C_{\ell m} = G \int dV \rho(r) \frac{1}{r^2} \frac{p_{\ell m}(\cos \theta)}{\ell + 1} \cos m \phi$$
\[ C_{\ell m} = G \int d\Omega \rho(r) \frac{1}{r^{\ell+1}} P_{\ell}^{m}(\cos \theta) \cos m\phi \]

Note that the index \( m \) in the expression for the torque now runs from 1 to \( \ell \), and the index \( \ell \) runs from 2 to infinity. The terms associated with \( m = 0 \) produce no torque, and the Associated Legendre Polynomials with negative values of \( m \) are multiples of the corresponding Associated Legendre Polynomials with positive values of \( m \). The term with \( \ell = 0 \) has been dropped because the only possible value of \( m \) is zero, and this term produces no torque. The terms with \( \ell = 1 \) have been dropped because if the axis of rotation passes through the center of mass, these terms are zero.

The procedure used to tune out the mass quadrupole moments in the float consisted of finding a position for the lead bricks which made a significant contribution to only one of the terms \( S_{\ell m} \) or \( C_{\ell m} \). While measuring the mass quadrupole moments, care had to be taken to ensure that the higher order moments did not make a significant contribution to the torque. The torque produced by placing lead bricks in this position is then measured, and the corresponding multipole moment, \( S_{\ell m} \) or \( C_{\ell m} \), may be calculated. Once the value of the mass quadrupole moments had been measured, we disassembled the float and added mass to reduce the moments.

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