ENERGY EXCHANGES BETWEEN TWO ESCAPING ELECTRONS

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Abstract
The final-state Coulomb interaction between three or more outgoing charged particles from a reaction can cause exchanges of energy and angular momentum between them. Such exchanges may be present for example when the reaction products consist of two electrons receding from a positive ion. The experimental evidence for the existence of such effects in threshold ionization and threshold autoionization are reviewed, and their possible theoretical descriptions are discussed.

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1. INTRODUCTION

This talk will be concerned with the post-collisional exchanges of energy that may occur between the outgoing particles of a reaction, when these particles consist of two electrons receding from a positive atomic ion. These outgoing particles may be produced by electron impact ionization of neutral or negatively ionized atoms, or by photo-double ionization or photo-double detachment of positive, neutral or negative atoms.

The monopole Coulomb interaction between the outgoing charged particles of a reaction has a special role to play in the time development of those final states which consist of three or more outgoing charged particles. Even after these particles are outside the reaction zone in which their identities and internal energies are established the long-range Coulomb interaction continues to act in changing the partitioning of the available energy and angular momentum of the system, and the final kinetic energies and angular momenta of the individual particles are not fully determined until they are completely free from each other. For example, in the case of two electrons receding from a heavy (and stationary) singly charged positive ion the Hamiltonian, once the electrons are further than a few atomic units from the ion, is

\[
H = H^{(\text{internal})} - \frac{1}{2} \nabla_1^2 - \frac{1}{2} \nabla_2^2 - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{1}
\]

where \(\mathbf{r}_1\) and \(\mathbf{r}_2\) are the position vectors of the two electrons. Although strictly it is not possible to partition the initial potential energy terms between the two electrons, and therefore a definite total energy cannot be ascribed to each, there is nevertheless in some sense a continual exchange of energy between the two electrons through the last term in (1), and the final total energies of each are not known until this term becomes zero. The last term also causes the forces acting on the electrons to be nonradial, leading to exchanges of angular momenta. The final state Coulomb interaction therefore causes in effect a post-collisional exchange of energy and angular momentum in such reactions.

The situation is quite different when there are only two outgoing charged particles, since the total energy of each of them is fully determined as soon as they are outside the reaction zone. Also in this case the monopole Coulomb interaction is unable to change the relative angular momentum of the two particles. Even in the case of three or more outgoing charged particles the exchanges of energy would not usually be
noticeable in most reactions, because the Coulomb energy between the particles, once they are otherwise free of each other, is usually so much smaller than their total energies that the effects of the continuing Coulomb interaction are not significant.

There are two cases, however, in which the final state Coulomb interaction may become more important. The first of these occurs near electron impact ionization thresholds or photon impact double-detachment thresholds, for which the total energy of the outgoing particles (apart from the internal energy of the residual ion) is approximately zero. In these reactions the slowly receding electrons may spend a considerable time interacting with each other and with the Coulomb field of the ion, and the resulting changes in the distribution of energy and angular momentum between them may become significant. The second example occurs in the near-threshold electron impact excitation of short-lived autoionizing states of atoms. In these reactions the inelastically scattered electron recedes slowly from the excited neutral atom, but at the time of autoionization of the atom into an electron and an ion, the Coulomb interaction energy between the scattered electron and these other two charged particles may still be comparable with its kinetic energy, again leading to observable changes in the final energies of both the scattered and ejected electrons. It is these two examples that will be discussed in this talk. Other examples, such as the near-threshold inner-shell ionization of atoms, followed by multiple Auger electron emission, may also exist.

Although for the present purposes we shall often be referring to the time evolution of the final state, and shall also sometimes be alluding to classical descriptions, it should be remembered that a more conventional approach, at least from the theoretical point of view, would be to consider the solutions of the time-independent Schrödinger equation which represents the reaction. It seems however that this latter approach, although undoubtedly the correct way to solve these scattering problems, is at the present time somewhat limited in its ability to take account of long-range final state interactions between more than two particles. It is this very point that provides the interest and motivation for the present studies.

2. THRESHOLD ELECTRON-IMPACT IONIZATION

We shall primarily be considering the reaction

\[ e + A^{m+} \rightarrow A^{(m+1)+} + 2e \]  

(2)
near to its threshold, where \( m \geq 0 \). The photo-double detachment reaction

\[
\nu + A^{(m-1)+} \rightarrow A^{(m+1)+} + 2e
\]

where \( m \geq 0 \), also provides the same products, but high resolution studies of such reactions have not yet been made.

These near-threshold ionization processes have been treated theoretically by several authors, using a variety of quantum mechanical, semi-classical and purely classical methods, the main intent of most of this work being to establish the exponent \( n \) of the power law dependence of the total ionization cross section \( \sigma \) on the excess energy \( E \) above threshold,

\[
\sigma \sim E^n
\]

The value of \( n \) for single ionization of neutral atoms has variously been found to range from 1.0 to 1.5, sometimes being a function of \( E \) itself.

The earliest treatment, that of Wannier, in which classical methods were used to obtain the value 1.127 for \( n \), occupies a special place in this field. The long-range final state interactions are correctly included in this treatment, as has been emphasized by Rau and Fano and Lin, but although it seems to be rigorous, apart from the assumption of quasi-ergodicity in the initial conditions, it has not received general approval. In the earliest quantum mechanical treatment the asymptotic wavefunction was taken to be the product of two outgoing Coulomb waves, which leads to the value 1.0 for the exponent \( n \). The effective ionic charges \( z_1 \) and \( z_2 \) for the Coulomb waves are defined by

\[
-\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{|r_1 - r_2^*|} = -\frac{z_1}{r_1} - \frac{z_2}{r_2}
\]

and although \( z_1 \) and \( z_2 \) are dependent on the initial directions and momenta of the electrons they do not vary with time, which implies that final state interactions of arbitrarily long range are not included. This therefore constitutes an important difference between the two approaches, and we will follow Rau and Fano in assuming that it is this difference that is essentially responsible for the different values of \( n \).

The Wannier exponent 1.127 has also been obtained more recently by semi-classical (Peterkop) and quantum mechanical (Rau) methods. The quantum mechanical treatments of Temkin and co-workers and Kang and Kerch give values of \( n \)
somewhat greater than 1.0, while the classical trajectory
integrations$^{14-17}$ give approximately 1.127, as indeed they
must. Experimentally, it is difficult to differentiate be-
tween the exponents 1.0 and 1.127, but recent studies$^{20-24}$
favor the latter value. In the experiments of Marchand
et al.$^{24}$ for example, n has been found to be $1.16 \pm 0.03$ for
the near-threshold electron impact ionization of helium.

To understand why the final state Coulomb interactions
increase the value of n it is helpful to consider what Rau$^6$
has described as "dynamic screening," or what in other words
amounts to the changes with time of the effective charges $z_1$
and $z_2$ of equation (5). If two electrons start in "symmetric"
orbits (by which is meant $r_1 = r_2$, $\vec{r}_1 = \vec{r}_2$ and $\vec{\theta}_1 = -\vec{\theta}_2$) they
will retain this symmetry for all future times and will both
eventually escape with the same kinetic energy $\frac{1}{2}E$. If on
the other hand the starting condition is nearly symmetric,
but with electron 2 being slightly faster and further from the
ion than electron 1, then it will be better screened from the
ion than is electron 1 (i.e. $z_2$ will be smaller than $z_1$) and
will therefore experience a lesser attraction, and its ini-
tial advantage in speed and distance will increase with time.
Put in other words, the potential energy part of the
Hamiltonian [Eq. (1)] is always a maximum at $r_1 = r_2$ (for
all values of $\theta_{12}$), and therefore deviations from the symme-
tric orbit will increase with time.

This radial correlation instability is most effective
when the two electrons have similar initial kinetic energies,
and it tends to reduce the probability of the two electrons
having the same or similar final kinetic energies. The insta-
bulity leads in effect to an exchange of energy between the
electrons, and in extreme cases electron 1 may finish with a
total energy which is negative, remaining bound to the positive
ion and thus removing flux from the ionization channel. The
instability becomes more effective as the available excess
energy E becomes smaller, since although the electrons always
start with high kinetic energies when near the ion they must
spend longer times and travel further distances from the ion
before the Coulomb energy becomes sufficiently smaller than E
for them to be free. When $E = 0.1$ eV for example, this stage
is not reached until the electrons are further than about 200 Å
from the residual ion, which takes a time greater than about
$6 \times 10^{-13}$ s. The net effect of the dynamic screening is
therefore to reduce the probability of ionization at small
values of E, whilst having a smaller effect at higher ener-
gies. In other words the exponent n is made slightly
greater than unity, actually giving a zero slope to the ion-
ization cross section at the threshold itself. More complete
and quantitative accounts of these effects can be found in the
works of Wannier,$^{12}$ Rau,$^6$ Fano and Lin$^{18}$ and Fano.$^{19}$
As well as the instability in the radial correlation there is an accompanying stability in the angle $\theta_{12}$ between the final directions of the two escaping electrons, since the electrostatic forces always tend to push this angle towards $\pi$, at the same time tending to impart equal and opposite angular momenta to the outgoing electrons. In terms of the excess energy $E$ the spread $\Delta \theta_{12}$ of angles about $\pi$ is of the order of $E^{1/4}$ in the Wannier model, which implies a maximum angular momentum of each electron of the order of $E^{-1/4}$ (where $E$ is measured in atomic units).

Although these various ideas and interpretations are not yet proven there is some support for them in the experiments of Cvejanović and Read, who attempted to measure the correlations in energy and angle of the two escaping electrons. This was achieved by a time-of-flight coincidence technique, thereby exploiting the slow velocities of the electrons and avoiding the usual difficulties of the more conventional techniques. For small excess energies $E$ (≤1 eV) in electron-helium ionization they were able to measure the probability distribution $P(E_1, E_2)$ for the partitioning of the energy $E$ between the two electrons, which is predicted to be uniform (i.e. independent of $E_1$ or $E_2$) by most theories of threshold ionization. In fact the experimental results are consistent with its being uniform over the whole range of values of $E_1$ and $E_2$, although for technical reasons this could not be definitely established for values of $E_1$ or $E_2$ less than about 0.05 eV. Also the angular correlation function $P(\theta_{12})$ was found to have a width $\Delta \theta_{12}$ which decreases as $E$ decreases, as predicted by the Wannier-Peterkop-Rau theory.

These authors also obtained information of a quite different type through the development of a new threshold detection technique which enabled electrons of nearly zero energy to be detected with high energy resolution $\Delta E$ (≤50 meV) and high efficiency ($\approx 50\%$). If the only electrons to be detected are those having energies $E_{1,2} \leq \Delta E$, and if the probability distribution $P(E_1, E_2)$ is independent of $E_1$ and $E_2$, then the yield of detected electrons is proportional to $\sigma \times (\Delta E/E)$, which has the power law dependence $E^{n-1}$. Figure 1 shows a spectrum obtained by this technique. It can be seen that the value of the exponent $(n-1)$ above the ionization threshold is indeed consistent with that expected from the Wannier theory. An analysis of this and other spectra has shown that the exponent for the yield above the ionization energy is $0.131 \pm 0.019$. This result does not however constitute a definite proof of the Wannier law since the uniformity of $P(E_1, E_2)$ for values of $E_1$ and $E_2$ less than $\Delta E$ has not been definitely established, as mentioned above.
Figure 1. A spectrum showing the measured yield of very low energy (≤50 meV) electrons resulting from electron impact of helium. The curve drawn through the points above the ionization threshold is proportional to $E^{0.127}$, where $E$ is the energy excess above the ionization energy. Reproduced from Cvejanović and Read, J. Phys. B 7, 1841 (1974), with permission.

An equally important part of this spectrum, as pointed out by Fano, is the region below the ionization energy, where peaks corresponding to the threshold excitation of Rydberg states of helium can be seen. These states have large radii [$\sim n^2$ or $1/(-E)$] and can only be produced when it is possible for one electron to reach these large distances while the other recedes to infinity, and therefore there is time for the final state interaction to cause appreciable exchanges of energy and angular momentum. The net effect is that the probability of exciting these states near their thresholds is reduced, and that those which are excited tend to have a large range of orbital angular momenta. In fact it can be seen from the spectrum that as the ionization energy is approached these peaks merge together and that a mean yield drawn through the separate and merged peaks has a shape which approximately mirrors that of the yield above the ionization energy.
In summary one may say that the decrease, or cusp, in the yield of very low energy electrons immediately above and below the ionization energy exists because the post-collisional exchange of energy between the outgoing electrons decreases the probability of their both having a small final energy. There should also be an associated increase in the range of angular momenta of the two electrons in this region, as discussed above, but there is no experimental evidence for this yet.

More recently, the cusp at the ionization threshold has also been seen by Spence, who used a modified version of the trapped well technique to detect inelastic and ionization electrons over a range of final energies. These experiments, as well as those of Cvejanović and Read, are consistent with the Wannier law holding up to about 2 or 3 eV above the ionization energy, in the case of electron-helium scattering.

3. THRESHOLD EXCITATION OF AUTOIONIZING STATES

In the previous example we were concerned with the final state interaction between two electrons that leave the reaction zone (that is, the region of short-range interactions in which the identities and internal states of the outgoing particles are determined) at essentially the same time. In this second example we shall be concerned with two electrons of greatly different energies that leave the reaction zone at substantially different times, the slower electron leaving first and the faster one being delayed by its temporary capture as part of an autoionizing state of the target atom. As before, we shall adopt a simple description by emphasizing the time evolution of the final state and attributing the observed effects to final state interactions, since although this model may have weaknesses a more exact description does not yet exist (see Taylor and Yaris and the discussion in section 4 below).

Consider the electron impact excitation of a double excited autoionizing (or pre-ionizing) state of helium

\[ \text{e} + \text{He} \rightarrow \text{He}^{**} + \text{e}_1 \]

\[ \rightarrow \text{He}^+ + \text{e}_2 \]

and suppose that the energy \( E_1 \) of electron 1 is less than a few eV. The lower lying states \( \text{He}^{**} \) have lifetimes of \( 5 \times 10^{-5} \) s or longer and the ejected electrons \( \text{e}_2 \) have energies \( E_2 \) of 33.2 eV or more. In the case of the shorter lived states it is possible for \( \text{e}_1 \) to have traveled less than a few tens of Angstroms before the time of break-up of the autoionizing atom, and then the Coulomb energy between \( \text{e}_1 \) and \( \text{e}_2 \) at this
instant may be significant compared with \( E_1 \). This may then lead to observable energy exchanges between the electrons, causing their final energies to be different from the nominal values \( E_1 \) and \( E_2 \).

There are two simple but equivalent ways of describing the origin of the energy exchanges, if \( e_2 \) can be regarded as being very fast compared with \( e_1 \) (that is, if the "sudden approximation" is assumed). Before the autoionization event \( e_1 \) is receding from the neutral atom as an expanding charge cloud. The ejected electron \( e_2 \) has an additional initial potential energy \( \Delta E \) caused by its being immersed in the Coulomb field of this cloud and it retains this part of its total energy in traveling through the cloud, therefore having a final kinetic energy equal to \( E_2 + \Delta E \). The detailed probability distribution of the values of \( \Delta E \) depends on the lifetime \( \tau \) of the autoionizing state, and on the nominal energy \( E_1 \) of the scattered electron, but clearly the magnitude of \( \Delta E \) increases as \( \tau \) and \( E_1 \) are decreased. Alternatively, from the point of view of the scattered electron, the neutral atom from which it is receding suddenly changes, at the moment of autoionization, into a positive ion, causing it to be retarded and to have the final energy \( E_1 - \Delta E \). The total energy of the system must of course remain unchanged.

The distribution of energy exchanges \( \Delta E \) can be found experimentally either by measuring the loss of energy of \( e_1 \) or the gain of energy of \( e_2 \). The latter method was in fact the means by which the effect was first noticed (Hicks et al.\(^{29} \)) in electron-atom scattering. Some of their results are shown in Fig. 2. It can be seen that the energies of three of the four ejected electron peaks increase as the energy \( E_1 \) is decreased, the lifetime of the fourth state (\( 2s2p^3P \)) being too long to show the effect. More recently (Spence\(^{30} \)) the energy exchange \( \Delta E \) has also been observed as a loss of energy of electron \( e_1 \).

An analogous effect in the excitation of autoionizing states of He by impact with He\(^+\) ions of energy 1 to 4 keV had been noticed much earlier by Barker and Berry\(^{31} \); in these experiments the scattered ion, which is the analogue of the scattered electron \( e_1 \) and has a velocity of the same order, interacts with the ejected electron \( e_2 \) and causes it to have a decreased energy (the difference in the sign of \( \Delta E \) being simply caused by the difference in sign of the charge of the scattered particle). Using a simple classical interpretation of a post-collision Coulomb interaction between the scattered particle and ejected electron they found that the probability distribution for the energy loss \( \Delta E \) is given by
Figure 2. Spectra of ejected electrons from the four lowest autoionizing levels of helium (2s$^2$ 1S, 2s2p $^3$P, 2p$^2$ 1D and 2s2p $^1$P), at an ejection angle of 70°. The figures above the spectra show by how much the incident energy exceeds the energy of the autoionizing states. The displacement of the three broader peaks to higher ejected electron energies can be clearly seen as the incident energy is decreased. Reproduced from Hicks, Čvejanović, Comer, Read and Sharp, Vacuum 24, 573 (1974), with permission.

\[ P(\Delta E)\, d(\Delta E) = \frac{b}{(\Delta E)^2} \exp\left(-\frac{b}{\Delta E}\right) d(\Delta E) \]  

(7)

where
\[ b = \frac{1}{\nu \tau} \]  

(8)

and where $\nu$ is the velocity of the scattered particle. The resulting line shape of the ejected electron energy peaks is
asymmetric, with a long tail towards large values of $\Delta E$. The most probable value of $\Delta E$ is

$$\Delta E_{\text{peak}} = \frac{1}{2} b$$

(9)

and the full width at half maximum of the distribution is 1.07$b$. Barker and Berry studied the $(2s2p)^1P$ state of He, and found that the dependence of $\Delta E_{\text{peak}}$ on the ion velocity $v$ is as given by Eqs. (8) and (9), but with a value of $\tau$ somewhat smaller than the known lifetime of the state. These classical expressions have also been used\textsuperscript{29,32-34} to interpret the energy shifts $\Delta E_{\text{peak}}$ seen in electron-helium scattering,\textsuperscript{29} but again the fits to the experimental data have required values of $\tau$ which are too small.

In quantum mechanical terms the classical model of Barker and Berry becomes the "shake-down" model.\textsuperscript{35,36} This particular nomenclature has been chosen in analogy with the well-known "shake-off" and "shake-up" events which occur in the field of Auger electron spectroscopy when the outer electrons of an atom have to adjust to the sudden removal of an inner electron (see for example Carlson\textsuperscript{37,38}). Before the autoionization event the scattered electron has the wave function $\psi_{k\ell}$, where $k$ is its momentum ($=\sqrt{2E_1}$, in atomic units) and $\ell$ its angular momentum. This may be approximated by its asymptotic form $Y_\ell^m(\vec{r}) j_\ell(kr)$ and it must be modified by the appropriate factor\textsuperscript{39} [$=\exp(-r/2\tau) = \exp(-r/2k\tau)$] to allow for the loss of amplitude as the autoionizing state decays. After the autoionization event the final wavefunction for $E_1$ is the analogous outgoing Coulomb wavefunction $\phi_{k'\ell}$, where $k'$ is the final momentum. The change in charge at the time of autoionization causes a "shake-down" of the free scattered electron from $\psi_{k\ell}$ to a continuum state $\phi_{k'\ell}$ of lower energy, and in the sudden approximation (i.e. $E_1 \ll E_2$) the probability that it will find itself with the final momentum $k'$ is proportional to

$$P_{k'\ell}(k) = |q_{k'\ell}(k)|^2$$

(10)

where the overlap integral is

$$q_{k'\ell}(k) = \int \phi_{k'\ell}^* \psi_{k\ell} \, d\vec{r}$$

(11)

There is no change in the angular momentum of the electron since it experiences what is essentially a monopole perturbation, if the sudden approximation is valid. This shake-down model is approximate in many respects,\textsuperscript{35,36} but it is nevertheless able to explain some features of the experimental
observations (see also below). Recently for example the model has been used\textsuperscript{40} to obtain the probability distribution $P(\Delta E)$, and it has been found that this is well approximated by the classical expression (7), still being very asymmetric with a long high energy tail, as can be noticed for example in some of the peaks shown in Fig. 2.

A new development occurred in this field when it was realized\textsuperscript{32,33,41} that if the energy exchanges are large enough, the scattered electron $e_1$ may finish with a negative total energy, becoming bound to the residual ion to form an excited state of the neutral atom. This process can be illustrated schematically as

\[
\text{e} + \text{A} \rightarrow \text{A}^{**} + \text{e} \rightarrow \text{A}^+ + \text{e}
\]  

(12)

and it constitutes, in effect, an extra exchange mechanism for forming excited states of the atom A when the incident energy is near the energies of autoionizing states of the atom. Smith et al.\textsuperscript{33} tested this idea by studying the excitation cross sections of various Rydberg states of He at incident energies in the region of the first four autoionizing states, and indeed found structures having a magnitude of a few percent of the normal excitation cross section. These structures have positions that vary with the energy of the excited state being observed, and shapes that are unlike the Fano-Beutler shapes of resonant structures. Heideman et al.\textsuperscript{41,42} have also conjectured that this same process may be responsible for certain structures seen in the optical excitation functions of helium at energies near 60 eV. More recently, a detailed study has been made\textsuperscript{36} of the structures in excitation cross sections of various states of He, Ne and Ar: Fig. 3 shows some of the results for He.

The shake-down model can readily be tested against these new structures. The scattered electron is shaken down from its initial wavefunction $\Psi_{k\ell}$ to a bound state wave function $\phi_{n\ell}$ of the neutral atom. Since the excited states that are populated in this way tend to be Rydberg states with high values of $n$, King et al.\textsuperscript{36} were able to approximate the $\phi_{n\ell}$ by the appropriate hydrogenic wave functions, and they found that the calculated values of the overlap integrals $q_{n\ell}(k)$ have shapes that are similar to those of their experimentally observed structures. This is illustrated in Fig. 4, which shows the functions $q_{n0}(k)$, after convolution with a Gaussian function of width (FWHM) 100 meV to simulate the finite energy resolution of the experimental measurements. It can be seen
Figure 3. Excitation functions at 0° for the states of helium having the principal quantum number \( n \) from 3 to 8 (and including all possible values of the quantum numbers \( S, L \) and \( J \) for each value of \( n \), except for the lowest spectrum, which is for \( 3^{1}P_1 \) only). The autoionizing state which is responsible for these structures is thought to be the \( 2p^2 \, ^1D \) state at 59.90 eV. Reproduced from King, Read and Bradford, J. Phys. B, in press, with permission.

that the theoretical curves of Fig. 4 contain a sharp feature (dip or peak) just above threshold, followed by a broader feature of the opposite polarity (peak or dip), followed by yet broader features at higher energies, and that the experimentally observed structures of Fig. 3 are similar, but with the later and broader features being masked (if they exist) by the energy dependent transmission function of the apparatus. The fact that the observed structures are similar to the overlap integral \( q_{\ell}(k) \) rather than its square \( P_{\ell}(k) \) is presumably caused by the new mechanism interfering
Figure 4. Calculated values of the overlap integral $q_{n0}(k)$ as a function of $E_1(=\hbar^2k^2/2m)$, for $n = 4$ to $8$, and after convolution with an apparatus function having a Gaussian shape and a width of 100 meV (FWHM). The lifetime $\tau$ has been taken to be $9.1 \times 10^{-15}$ s, (that of the $2p^2\,^1D$ state of helium), and the phase shift $\delta$ of the scattered electron has been taken to be $0^\circ$. The magnitude and sign of the normalization of these curves are arbitrary. Reproduced from King, Read and Bradford, J. Phys. B, in press, with permission.

coherently with a large direct contribution to the excitation cross section. A more detailed comparison shows that the shake-down model also agrees reasonably well with the observed structures in other atoms, although many details (such as the choice of phase shift of the scattered electron, and the energy dependence of the excitation cross section of the autoionizing state) have yet to be properly considered.

To summarize, the post-collisional interaction between the scattered and ejected electrons causes an energy exchange between them which has been observed in studying (i) the gain in energy of the ejected electron $e_2$, (ii) the loss in energy of the scattered electron $e_1$ when this electron has a final total energy which is still positive, and (iii) the formation of excited states when $e_1$ has a final energy which
is negative. The shake-down model seems to give the magni-
tudes and distributions of these energy exchanges reasonably
well, but with some details remaining to be explained. One
further type of investigation, namely that in which the elec-
tron $e_1$ is detected when its final energy is nearly zero,
has yielded a threshold spectrum that may contain additional
interference and resonance effects and therefore cannot yet be
used to obtain further information about the shake-down pro-
cess. More detailed information on these various effects and
their interpretation can be found in a recent review of the
shake-down effect.

4. SUMMARY

Two types of reaction have been discussed, and in both
there is some evidence for exchanges of energy between the
outgoing particles of the reaction after they have left the
inner zone in which their identities and internal states are
determined. In the case of threshold ionization there are
also post-collisional exchanges of angular momentum. Special
emphasis has been placed on these exchanges, at the expense
of other aspects of the reactions, but before concluding it
is necessary to regain a more balanced view.

In a sense even the concept of a post-collision interac-
tion does not exist, since a reaction or collision is not
complete until all the interactions have vanished. A complete
theory would include all the final state interactions, of
whatever range, and there would then be no need to invoke any
further time-dependent processes. Current theories indeed
attempt to do just this. However, the phenomena that we have
been discussing involve such long ranges (hundreds of
Angstroms in the case of threshold ionization and tens of
Angstroms in the case of threshold autoionization) that it
must be difficult to do this in practice.

For example, Taylor and Yaris have proposed that the
energy displacements seen in threshold autoionization should
be explicable in terms of resonances. They suggest that the
effects seen near the energy of the $(2s)^2 \, ^1S$ state of He could
be due to a series of resonances such as $(2s^2np)^2P$, the higher
members of which could have energies above that of the
$(2s)^2 \, ^1S$ state. These resonances may have an appreciable
overlap with the decay channel $1smpk$, if $n \approx m$, and could
then lead to structures in the excitation functions of the
$1smp$ states. The overlapping of many such resonances could
give shapes that are different from the usual Fano-Beutler
shapes of isolated resonances. In this model the autoionizing
state $(2s)^2 \, ^1S$ is never formed as an intermediate step, al-
though it is present as the core of the resonant state. The
model might also explain the energy shifts and asymmetric profiles seen in ejected electron spectra. Although this model may ultimately be the correct one to use, it seems probable that it will give convergent results only if many levels of several resonance series are included in the calculation. It also seems possible that a superposition of the assumed resonant states, which could well have widths greater than their spacings, might in effect look very similar to a single electron receding from a decaying (2s)$^2$ 1S core, particularly since this represents the dominant decay channel. The difference in the models would then become largely semantic.

In general it seems that long-range final state interactions between more than two particles cannot easily be included in current theoretical techniques, and that new methods, such as a flip-over to classical descriptions at large distances, or a wave-packet approach with Fourier analysis to regain specific energies, need to be developed to do this.

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