Damping of Oscillations in Accretion Disks Due to Viscosity

J. P. Cox and B. L. Everson
Joint Institute for Laboratory Astrophysics
University of Colorado and National Bureau of Standards
Boulder, CO 80309

ABSTRACT

A general method for calculating the effects of nonadiabatic, viscous perturbations is outlined. This is then applied to a "classical" steady-state accretion disk, with vertical oscillations. The oscillation periods obtained are on the order of the quasi-periodic oscillations observed in dwarf novae. The observed decay timescales of these oscillations may then be compared with the derived damping constant to estimate the viscosity present in the accretion disk. The derived viscosities needed to give the observed damping are of the order of eddy viscosities; in other words, the alpha parameter of Shakura and Sunyaev is of order unity.

I. INTRODUCTION

The large-scale properties of steady-state accretion disks about compact objects have now been well-described for a decade (Pringle and Rees 1972, Shakura and Sunyaev 1973, Lynden-Bell and Pringle 1974, Pringle 1981). However, deriving the parameters involved from first principles is exceedingly complicated (see, e.g., Stewart 1975, 1976). The discovery of quasi-periodic oscillations in cataclysmic variables by Robinson and Nather (1979) is quite significant because these are clearly outside the scope of a steady-state disk model.

Typical periods for quasi-periodic oscillations are between 30 and 150 sec and their decay times are 3-5 periods. Utilizing an adiabatic nonviscous perturbation method, Cox (1981) showed that the periods of such oscillations should be on the order of those of Keplerian orbits in the region where they arise, regardless of the detailed structure of the disk, provided that self-gravity of the disk is not important. The fact that this region spans a range of radii would presumably account for quasi-periodicity.

Here we consider nonadiabatic, viscous perturbations with a separation between the basic oscillation frequency and its deviations due to nonadiabaticity and viscosity. Most importantly, the fact that these deviations are complex allows a determination of the decay timescale in terms of arbitrary viscosity parameters. These may then be compared to observations to gain an insight into the values of such parameters in the region of the accretion disk where quasi-periodic oscillations are presumed to occur.
II. BASIC EQUATIONS AND METHOD

We take the pressure tensor to be

$$P_m = [P + (\frac{2}{3} \eta - \zeta)(\nabla \cdot \nabla)]_m - 2\eta S_m$$  \hspace{1cm} (1)

where $P$ is the isotropic pressure, $\eta$ is the dynamic viscosity, $\zeta$ is the bulk viscosity (both viscosity coefficients are of unspecified physical nature), $I_m$ is the identity tensor, $\nabla$ is the velocity, and

$$S_m = \frac{1}{2} [\nabla \nabla + (\nabla \nabla)^T]$$  \hspace{1cm} (2)

is the velocity strain tensor. (The tilde denotes the transpose.)

The equation of motion

$$\rho \frac{dv}{dt} = -\nabla \cdot P + \rho \tilde{f}$$  \hspace{1cm} (3)

where $\tilde{f}$ is the sum of all "body" forces per unit mass, may then be written as

$$\frac{d^2 \tilde{f}}{dt^2} = -\frac{1}{\rho} \nabla \nabla P - \nabla \psi + \tilde{F}_c.s. + \tilde{f}_{vis}$$  \hspace{1cm} (4)

where $\psi$ denotes the gravitational potential due to disk material, $\tilde{F}_c.s.$ the force per unit mass due to the central star, and $\tilde{f}_{vis}.$ the force per unit mass arising from viscous effects.

By taking the Lagrangian variation and the time derivative of eq. (4) and assuming that the variations are proportional to $e^{i\omega t},$ we eventually obtain the cubic equation

$$-\omega^3 d + \omega^2 e + \omega f - g = -i \int \tilde{f}_m^* \left[ \frac{d \tilde{f}_{vis.}}{dt} \right] \rho \ dt$$

$$+ i \int \tilde{f}_m^* \left[ \frac{d}{dt} \left( \frac{1}{\rho} \nabla (\delta P_{n.a.}) \right) \right] \rho \ dt$$  \hspace{1cm} (5)

where $d, e, f,$ and $g$ are volume integrals over the "equilibrium" configuration, involving the perturbation $\tilde{f}_m(\varepsilon \delta \tau)$ and its derivatives, and $\rho$ denotes the density in the "unperturbed" system. Here the viscous and nonadiabatic pressure ($\delta P_{n.a.}$) contributions appear explicitly on the right-hand side.

If the right-hand side of eq. (5) vanishes, we obtain an oscillation angular frequency $\Sigma$ close to that found by Cox (1981).

By letting $\omega = \Sigma + \omega'$, we obtain the deviation due to nonadiabaticity and viscosity.
III. APPLICATIONS

First, to test the method, we have applied it to purely spherically symmetric radial motion of a star, as in Cox and Giuli (1968, §27.6 d), and have obtained exact agreement with the results reported there.

Second, we have applied the method to damped, vertical oscillations of an accretion disk. As a nonpulsating steady state model, we have taken a standard model that is isothermal in the vertical direction (Pringle and Rees 1972, Shakura and Sunyaev 1973, Pringle 1981).

The approach being used is a generalization of that in Aizenman and Cox (1975). Consequently, the results are expressed as rather complicated integrals involving the unknown eigenfunction $\xi$. Because $\xi$ is not known, we have tried the simplest forms possible: a constant and a linear function of distance from the orbital plane. This procedure should be adequate for the present order-of-magnitude considerations. As one would expect, a constant perturbation perpendicular to the orbital plane will produce very little damping due to nonadiabaticity or viscous coupling. However, for a linear perturbation, we do encounter appreciable damping, but with no significant change in the basic oscillation period. It is interesting to note that, with this functional form of $\xi$, the “dynamical” terms (those that arise directly from the viscous forces) turn out to be very small; most of the damping is due to the "energy" terms (those that arise from the viscous heating).

For the purposes of numerical applications, we have chosen the gammas to be equal to $5/3$ and constant; a typical Population I chemical composition; and an "unperturbed" $z$ (vertical)-distribution in hydrostatic equilibrium, with gravity arising only from the central star, and isothermal in $z$. The only free parameters are: (1) The angular mode (the $m$-value) of the perturbation; (2) the mass accretion rate; and (3) the radius (or radii) at which the oscillations occur. For (1), we have taken small $m$-values, between $-2$ and $+2$; for (2), we have assumed an accretion rate of $10^{-9}$ solar masses per year (the results are very insensitive to the accretion rate); for (3), we have used the radii suggested by Robinson and Nather (1979): $1.5 \times 10^3$ cm and $4.2 \times 10^9$ cm. We have used eq. (3.23) of Pringle (1981) to calculate the disk temperatures (needed for the disk thicknesses, cf. eq. [3.16] of Pringle 1981). For the oscillation periods we have used the results of Cox (1981).

As our calculated values are only of order-of-magnitude significance and involve a number of assumptions, we shall not present detailed numerical results here. Our main conclusion is that, to give the observed damping times of $\sim 3$-5 periods of the quasi-periodic oscillations, it is necessary that the viscosities presumably responsible for the damping have values appropriate to eddy viscosities, viz., $v \sim 10^{12}$-$10^{14}$ cm$^2$ sec$^{-1}$ for the kinematic viscosity. These values correspond to $\alpha \sim 0.2$-$3$ ($\alpha$ is the "alpha parameter" introduced by Shakura and Sunyaev 1973).
IV. DISCUSSION

The values of $v$ and $\alpha$ derived here are consistent with expectations obtained from other considerations. These values imply that, if quasi-periodic oscillations are disk oscillations, and if these oscillations are damped primarily by viscosity, the viscosity needed must be an eddy viscosity with about the value usually assumed on the basis of other considerations for accretion disks. In other words, $\alpha \sim 1$ for the viscosity which is assumed to cause the damping. It is not likely that these order-of-magnitude results would be changed significantly by more realistic eigenfunctions.

We thus conclude that such a procedure is a viable method for estimating values of $v$ and $\alpha$ in an oscillatory region of an accretion disk.

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References