THEORETICAL DESIGN STUDY OF AN APPARATUS FOR THE
PRODUCTION OF A HIGH INTENSITY POLARIZED LITHIUM BEAM

by

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NOTICE
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Foreword

This report is a product of design studies for a new alkali atom beam apparatus, now being built for use in measuring differential electron-atom collision cross sections. Dr. Eminyan, a visitor to JILA on leave from the University of Paris, took responsibility for the design of the high intensity alkali beam source, based on the Laval nozzle, which will permit a much greater range of experimentation and, therefore, will greatly enhance the value of the completed apparatus. As will be evident to the reader, Dr. Eminyan made a quite comprehensive survey of the current literature and, for the benefit of the rest of the research group, summarized the physical considerations involved in the source design.

I have obtained Dr. Eminyan's permission to publish this part of the design study as a JILA Report, anticipating that the material contained in it will be generally useful, in view of the considerable interest in alkali beams by the research community.

Stephen J. Smith
Physicist, JILA
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1. **Introduction**

Atomic beams are generally formed by Knudsen effusion through a small oven aperture. The maximum attainable beam intensity is therefore limited by the effusion rate through the oven slit. These effusing atoms have a maxwellian distribution of velocity, no mass flow motion, and the highest beam intensity available by this process is of the order of $5 \times 10^{16}$ atoms/sec/steradian. In order to increase this intensity, it is possible to apply the proposal of Kantrowitz and Grey\(^{(1)}\), who replaced the oven slit by a miniature supersonic Laval nozzle.

The nozzle converts a part of the random internal energy of the oven vapor into directed mass motion. For Mach number $M$ sufficiently high, all the atoms leave the oven with a large flow velocity $U$ so that the random velocities are negligible compared to $U$ and the flow is precollimated in the direction of the mean motion. By this process, intensities as high as $1 \times 10^{19}$ atoms/sec/steradian with narrowed velocity distributions have been obtained recently\(^{(2)}\) for noncondensable gas. In 1965, Hundhausen and Pauly\(^{(3)}\) and more recently in 1967, Hundhausen and Harrison\(^{(4)}\) used a Laval nozzle to produce alkali metal beams. These last authors obtained a centerline intensity of $7 \times 10^{18}$ particles/sec/sr for a cesium beam using a chilled collimator to prevent the clogging of the aperture in a short time.

The purpose of this note is to make available the results of a design study of an apparatus for the formation of a high intensity polarized beam of lithium.

The elements of a basic continuum source are shown in Fig. 1.
The oven works at relatively high pressures from one to one hundred torr, so that a mass flow expands through the nozzle, to form a supersonic free jet. The skimmer or collimating slit collimates the beam before it enters a six pole magnet. The two components \( m_J = + \frac{1}{2} \) and \( m_J = - \frac{1}{2} \) of the lithium atom are separated by the inhomogeneous magnetic field which focuses the atoms in \( m_J = + \frac{1}{2} \) states toward the axis while atoms in \( m_J = - \frac{1}{2} \) states are forced away from the axis on hyperbolic trajectories.

2. Description

2.1 The oven

The oven (see Fig. 1) is composed of two separate chambers. It is designed to allow the temperature and pressure of the vapor to be varied somewhat independently, and to prevent the clogging of the nozzle. The small chamber A determines the temperature of the vapor which flows out through the nozzle while the pressure of the vapor in this chamber is determined principally by the temperature of the large chamber B in which the lithium is placed. The pressure versus the oven temperature is given for Li, K, Cs, by the equilibrium curves \(^5\) (Figs.2a, b, c). The capacity of the main chamber B is approximately 27 cm\(^3\) and can be filled with 14 g of lithium at 20 °C.

The furnace body and nozzle block are heated by separate variable transformers and tantalum wire wound on ceramic cylinders of characteristics:
Large cylinder    Small cylinder

number of turns       30          10

diameter of the turn  3.6 cm      1.65 cm

diameter of the wire  0.031 cm     0.020 cm

In order to calculate the mass consumption of lithium and the working time of the oven, the nozzle mass flow and rate can be expressed by

\[ G = \rho^* A^* a^* \quad \text{g/sec} \quad (2.1) \]

where the superscript * means the value at the throat of the nozzle, \( \rho \) the specific gravity of the vapor, \( A \) the throat area = \( \frac{\pi D^2}{4} \), and \( a \) the sound velocity. Considering a perfect gas in an isentropic flow, \( \rho^* \) and \( a^* \) in the sonic point (Mach number \( M = 1 \)) can be expressed versus the stagnation conditions \( p_o, T_o, \rho_o \)

\[ \rho^* = \rho_o \left( \frac{2}{\gamma+1} \right)^{1-\frac{1}{\gamma}} \quad (2.2) \]

\[ a^* = \frac{a_o}{\gamma+1} \quad (2.3) \]

where

\[ a_o = \sqrt{\frac{p_o}{\rho_o}} \quad (2.4) \]

For typical condition \( \gamma = \frac{5}{3} \), \( T_o = 1273 \text{ °K}, p_o = 35 \text{ torr}, D = 1.210^{-2} \text{ cm}, \)
the velocity of sound \( a^* = 1.5 \times 10^5 \text{ cm/s}, \) the mass flow
\[ G = \rho * A * a = 3.5 \times 10^{-5} \text{ g/s}, \text{ and with 14 g of lithium, the oven would work for about 100 hours.} \]

2.2 The skimmer or collimating slit

In order to prevent the clogging of the slit which is cooled by a liquid nitrogen trap, the collimating slit is composed of two rotating split wheels. The intersection of the two wheels gives an approximate square slit of 2 x 2 mm. The wheels rotate very slowly at a rate which must be fixed by experience, so that the shape of the skimmer does not change due to clogging.

2.3 The six pole magnet

The lithium atoms, with magnetic dipole moment of the order of one Bohr magneton are deflected and focused by an inhomogeneous magnetic field. The magnet used was described by Christensen and Hamilton\(^{(7)}\) and built by the Hewlett Packard Company. The magnitude of the magnetic field varies as

\[ H = H_0 \left( \frac{r}{r_o} \right)^2 \]

(2.5)

where \( H_0 = 9400 \) gauss, \( r_o = 1.5 \) mm, and the magnet length \( L_m = 76 \) mm.
3. Gas dynamics

3.1 The free jet

The flow from the high pressure oven chamber expands through the nozzle into a region of vacuum or of finite small pressure. If this pressure is small enough, there are two or more expansions, the first down to the Mach disk and the others downstream beyond the Mach disk. The overexpansion produced in front of the Mach disk permits much higher Mach numbers and better beam collimation than could otherwise be obtained for a given pumping capability.

Ashkenas and Sherman (8) summarize the free jet behavior. At large enough pressure ratios there is a flow in which the conversion from thermal to directed motion is nearly complete. When $p_0/p_1 \gg 1$, the skimmer is inside the Mach disk, the position of which ($x_M$) is given with good accuracy by:

$$\frac{x_M}{D} = 0.67 \left( \frac{p_0}{p_1} \right)^{1/2}$$

(3.1)

where $D$ is the throat nozzle diameter, $p_0$ is the stagnation pressure, and $p_1$ is the background pressure. For the hypersonic part of an ideal expansion, the Mach number in the skimmer plane, at a distance $x_S$ downstream of the nozzle can be expressed:

$$M_S = C \left( \frac{x_S-x_0}{D} \right)^{\gamma-1} + \frac{1}{2} \frac{(\gamma+1)}{\gamma-1} \left[ C \left( \frac{x_S-x_0}{D} \right)^{\gamma-1} \right]^{-1}$$

(3.2)

where $C = 3.26$ and $x_0/D = 0.075$ for a monoatomic gas. Furthermore,
for $\gamma = 5/3$, the speed ratio which determines the beam directivity, is given by

$$S_S = 3 \left( \frac{x}{S} \right)^{2/3},$$

(3.3)

where $S = \frac{U}{W}$, $U$ is the flow velocity and $W$ is the most probable random velocity:

$$W = \left( \frac{2kT}{M} \right)^{1/2}.$$  

(3.4)

The dependence of $M_S$ and $S_S$ on the axial distance $\frac{x}{S_D}$ is shown in Fig. 3. The impact pressure given by a Pitot probe in the flow axis is

$$p_1 = p_0 \left( \frac{\gamma+1}{\gamma-1} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{\gamma+1}{2\gamma} \right)^{\frac{1}{\gamma-1}} C \left( \frac{x-x'}{D} \right)^{-2}$$

(3.5)

where $p_0$ is the stagnation pressure, $x'_0/D = 0.06$ for $\gamma = \frac{5}{3}$, and the impact temperature is given by

$$T_1 = \frac{T_0}{1 + \left( \frac{\gamma-1}{2} \right) M^2_S}$$

(3.6)

where $T_0$ is the stagnation temperature. The dependences of $p_1$ and $T_1$ on the speed ratio $S$ are shown in Fig. 4 for a monoatomic gas.
In the ideal expansion, the flow velocity inside the Mach disk will be constant at nearly the terminal velocity, and the streamlines appear to radiate from a source at a distance $x_0$ downstream of the orifice. The density decreases along each streamline in proportion to the inverse square of distance \(8\),

\[
\frac{\rho}{\rho_0} = \frac{B}{r^2} \cos^2 \left( \frac{r \phi}{h} \right), \quad (3.7)
\]

where $\rho_0$ is the stagnation density, $\phi$ is the polar angle connecting the field point at distance $r$ (units of nozzle radius) from the apparent source at $x_0$, $B = 0.643$, and $h = 1.365$ for a monoatomic gas. The variation of density with polar angle $\phi$ at constant distance $r$ is approximately independent of $r$. On the axis of the flow field the density is given by

\[
\rho(x) = 0.161 \rho_0 \left( \frac{x}{D} \right)^{-2}, \quad (3.8)
\]

and is shown in Fig. 5. The isentropic particle flux density, introduced as a reference intensity by Hagen and Morton \(9\) becomes, in the hypersonic region, proportional to the particle flux $G$ through the nozzle

\[
I_{is} = 0.627 G \quad \text{particles/sec/sr} \quad (3.9)
\]
In the same typical conditions as used in section 2.1, the isentropic particle flux density would be $I_{is} \approx 2 \times 10^{18}$ particles/sec/sr.

3.2 **Intensity and speed distribution**

The previous formulas for the free jet are very useful in the beam generation problem because they provide the information necessary to characterize the final beam produced after the collimation.

An important fact must be noted. Using a lithium beam and a skimmer cooled by a liquid nitrogen baffle, there is no shock wave in front of the skimmer.

The sticking coefficient* variation with the wall temperature is not known for a Li beam. Nevertheless, energy considerations imply that this coefficient, in general, increases when the wall temperature decreases. Furthermore, a calculation based on the work of Mayer and Tracy* shows that the sublimation rates will never preclude cryogenic condensation. A very small number of atoms are reflected from the skimmer and the atom flow may be calculated in as straightforward a manner as that of a classical beam.

*The fraction of atoms which, upon collision with the surface, will condense.
3.2-1 The collisionless model

Kantrowitz and Grey\(^{(1)}\) calculated the atom flow through the collimator, provided that the skimmer action produced a collisionless beam without disturbance.

This model of calculation is based on the critical assumptions:

1. the free jet is ideal and the expansion is isentropic,
2. the flow at the skimmer is parallel, and
3. the flow downstream of the skimmer is collisionless.

For the region far away from the skimmer, \( x \gg x_s \), these yield the following basic performance equations. The total flow through the skimmer

\[
N_S = \frac{\rho_S a_0^\pi d_s^2}{4 \left[ 1 + \frac{(\gamma-1)}{2} M_s^2 \right]^{1/2}} \quad \text{particles/sec} \quad (3.10)
\]

where \( a_0 \) is the stagnation speed of sound, \( d_s \) is the skimmer diameter, \( M_s \) is the Mach number at the skimmer entry, and \( \rho_S \) is the density at the skimmer entry. The center line beam intensity\(^{(11)}\)

\[
I = \frac{1}{2\pi} \frac{\rho_S a_0^\pi d_s^2}{4} \frac{M_s (3 + \gamma M_s^2)}{\left[ 1 + \frac{1}{2} (\gamma-1) M_s^2 \right]^{3}} \quad \text{particles/sec/sr} \quad . \quad (3.11)
\]

In terms of speed ratio for a monatomic gas, eq. (3.11) is expressed by Hagena and Morton\(^{(9)}\) as

\[
I = I_{ls} \sin^2 \frac{a_S}{S} \left( S^2 \frac{S^2}{S^2} + \frac{3}{2} \right) \quad (3.12)
\]
where $I_{ls}$ is given by eq. (3.9), $S_S$ is the speed ratio at the skimmer orifice given by eq. (3.3), and $2\alpha_s$ is the angle at which the skimmer is seen from the nozzle. The same authors express the center line differential intensity as

$$\frac{dI}{dV_S} = \frac{I_{ls} \sin^2 \alpha_s}{\sqrt{\pi}} \frac{V_S^3}{S_S} \exp \left[ -\left( \frac{V_S - S_S}{S_S} \right)^2 \right] \quad (3.13)$$

where

$$V = \frac{v}{\sqrt{\frac{2kT}{m}}} \quad (3.14)$$

and $v$ is the particle velocity.

3.2-2 The flow divergence model

The collisionless model predicts an intensity proportional to the skimmer area. However, it is obvious that it predicts unreasonably high intensity ($I > I_{ls}$) if the skimmer is enlarged. This is due to the neglect of the divergent nature of the flow approaching the skimmer. In including this divergence, using the same notation, Hagen and Morton obtain the center line intensity,

$$I = I_{ls} \left[ 1 - \cos^2 \alpha_s \exp \left( -S_S^2 \sin^2 \alpha_s \right) \right] \quad (3.15)$$

As expected, eq. (3.15) no longer shows an unlimited increase of $I$ with either $S_S$ or $\sin \alpha_s$. For $S_S \sin \alpha_s > 2$ the intensity has practically reached its limiting value $I = I_{ls}$. For small skimmers, on the other hand, the flow divergence becomes negligible for $S_S \sin \alpha_s \lesssim \frac{1}{3}$. 
In that case eq. (3.12) is a sufficiently accurate approximation for eq. (3.15). This dependence of \( I/I_{is} \) on the axial distance \( x_s/D \) is shown in Fig. 6 for the two models.

The relationship between skimmer size, speed ratio and \( I/I_{is} \) can be used to define a characteristic skimmer radius \( r_s^* \), beyond which the intensity no longer increases:

\[
r_s^* = \frac{2x_s}{S_s}.
\] (3.16)

With eq. (3.3) we get the following dependence of \( r_s^* \) on skimmer distance and speed ratio:

\[
r_s^*/D = 0.67 \left( \frac{x_s}{D} \right)^{1/3}.
\] (3.17)

This formula indicates a continuous slow increase in the characteristic skimmer size with \( x_s \). In this special case where \( r_s = r_s^* \), Hagen and Morton\(^{(9)}\) obtain an expression for the centerline differential intensity

\[
\frac{dI}{dV_s} = I_{is} \sin^2 \alpha_s \left( S_s^2 + \frac{3}{2} \right) f(V_s) F(V_s)
\] (3.18)

with

\[
f(V_s) = \frac{V_s^3 \exp\left(-\left(V_s - S_s\right)^2\right)}{\sqrt{\pi} \ S_s \left( S_s^2 + \frac{3}{2} \right)}
\] (3.19)
and \( F(V_s) \), the velocity dependent divergence factor which approaches 1 as \( \alpha_s \) goes to zero, is given by
\[
F(V_s) = \frac{(2 S_s V_s - 1) - (2 S_s \cos \alpha_s - 1) \exp [2 S_s V_s (\cos \alpha_s - 1)]}{2 S_s^2 V_s^2 \sin^2 \alpha_s}. \quad (3.20)
\]

For the characteristic skimmer \((r_s = r_s^*)\), \( F(V_s) \) is reduced to
\[
F(V_s) = \frac{S_s}{4V_s}, \quad (3.21)
\]

and we may obtain
\[
\frac{dI}{dV_s} = \frac{I_s S_s^2 \sin^2 \alpha_s V_s^2}{4\sqrt{\pi}} \exp \left[-(V_s S_s)^2\right]. \quad (3.22)
\]

This shows that the divergence factor \( F(V_s) \) affects the differential intensity in such a way as to change the velocity dependent term from \( V^3 \) for the collisionless model to \( V^2 \) for the flow divergence model.

4. **The beam density**

The cross section profile of the atomic beam can be calculated from the trajectory equations inside the magnet \(^7\)
\[
\begin{align*}
  r_+ &= \theta_o \left[ L_1 \cos \left( \frac{\omega Z}{V} \right) + \frac{V}{\omega} \sin \left( \frac{\omega Z}{V} \right) \right] \quad (4.1) \\
  r_- &= \theta_o \left[ L_1 \text{ch} \left( \frac{\omega Z}{V} \right) + \frac{V}{\omega} \text{sh} \left( \frac{\omega Z}{V} \right) \right] \quad (4.2)
\end{align*}
\]
Here \( r^+ \) represents the distance of \( m_j = \pm \frac{1}{2} \) atoms from the central axis, \( z \) is the distance from magnet entrance, \( \theta_0 \) is the initial divergence angle, \( \nu \) is the particle velocity, \( L_1 \) is the distance between source and magnet entrance (see Fig. 1), and

\[
\omega = \left( \frac{2 \mu_0 H \nu}{m r_o} \right)^{1/2} \quad . \tag{4.3}
\]

In the expression for \( \omega \), \( m \) is the mass of the atom, \( \mu_0 \) is the Bohr magneton, \( H \) is the magnetic field in the magnet, and \( r_o \) is the magnet radius. The maximum acceptance angle for the \( m_j = +\frac{1}{2} \) state atoms is obtained by setting \( r_+ = r_o \), \( \frac{d r}{d z} = 0 \), and solving eq. (4.1) for \( \theta_0 \). This yields

\[
\theta_0^{(\text{max})} = \frac{r_o}{\left[ L_1^2 + \left( \frac{\nu}{\omega} \right)^2 \right]^{1/2}} \quad . \tag{4.4}
\]

The solid angle of acceptance for a source point for \( \theta_0 \max \ll 1 \) is

\[
\Omega_+ = \frac{\pi r_o^2}{L_1^2 + \left( \frac{\nu}{\omega} \right)^2} \quad . \tag{4.5}
\]

The beam width in the region of the focus can be calculated from the point source trajectory eq. (4.1), using \( \theta_0 = \theta_0 \max \), to determine the radial distance \( r_L \) and slope at the magnet exit. Assumption of a straight line trajectory outside the magnet allows prediction of the beam geometry, assuming a flow velocity for all the particles and typical conditions of Section (2.1):
\( \theta_0 \approx 0.015 \text{ rad.} \)

\( (r_+)_L = 0.11 \text{ cm.} \)

\( \frac{dr}{dx}_L \lesssim 10^{-2} \text{ rad.} \)

Since the beam is nearly parallel and the skimmer aperture about 0.2 cm in diameter, the width should be between 0.11 cm and \((0.2 + 0.11) = 0.31 \text{ cm,}\)
and the solid angle is

\[ \Omega_+ \approx 7 \times 10^{-4} \text{ steradian} \]

which is an approximate value for an extended source.

For the \( m_J = + \frac{1}{2} \) state atoms, the flow from the magnet is

\[ F = I\Omega_+ \text{ atoms/sec} \quad (4.6) \]

In order to illustrate these results, using the flow divergence model, we assume the following typical parameters: \( \frac{x_S}{D} = 80, \quad S_S = 60 \) (from Fig. 3), and \( \alpha_S = \frac{1}{10} \text{ rad.} \) From eq. 3.9 and 3.15 we obtain \( I \approx 2 \times 10^{18} \text{ atoms/sec/steradian,} \)
and from eq. (4.8) we then find \( \vec{r} \approx 1.4 \times 10^{15} \text{ atoms/sec.} \)
Therefore, the density should be between \( 2 \times 10^{11} \text{ and } 2 \times 10^{12} \text{ atoms cm}^{-3}. \)

5. General discussion

The collisionless model assumes an isentropic flow down to the skimmer. The high speed-ratio \( S \) and beam intensity \( (I > I_{is}) \) (§ 3.2-1) given by the free jet theory seem unreasonable.
Different authors\textsuperscript{(12,13)} have noted that the transition from isentropic to molecular flow may occur upstream of the skimmer provided the skimmer distance is not too small or the source density too high. The speed ratio $S$ and the Mach number $M$ approach a terminal value upon increasing $x_S$. This value corresponds to a "freezing" of the parallel temperature $T''$ along the beam axis. Anderson and Fenn\textsuperscript{(12)} have studied experimentally the kinetic freezing problem. By measuring the velocity distribution of the molecules, they have determined the functional dependence of the limiting Mach number on a characteristic Knudsen number $K_{n_o}$ for the expansion, based on the stagnation mean free path, $\lambda_o$, and the throat diameter, $D$:

$$K_{n_o} = \frac{\lambda_o}{D} \quad \text{(5.1)}$$

They found that for a free argon jet, the Mach number at which the expansion froze was approximately

$$M = 1.18 \left( K_{n_o} \right) - \frac{\gamma-1}{\gamma} \quad \text{(5.2)}$$

or

$$S = 1.07 \left( K_{n_o} \right) - \frac{\gamma-1}{\gamma} \quad \text{(5.3)}$$

with

$$\lambda_o = 7.310^{-20} \frac{T_o}{p_o^\sigma} \text{ cm} \quad \text{(5.4)}$$

where $T_o$ is the stagnation temperature in $^\circ K$, $p_o$ is the stagnation pressure in torr, and $\sigma$ the atomic collision cross section. From
eq. (5.3) we obtain

\[ S = 1.07 \left( \frac{10^{20}}{7.3} \frac{D p_o}{T_o} \right) \frac{\gamma - 1}{\gamma} \]  \hspace{1cm} (5.5)

The velocity ratio dependence with \( p_o/T_o \) for a lithium beam, is shown in Fig. 7 for \( D = 1.2 \times 10^{-2} \) cm and \( D = 2 \times 10^{-2} \) cm. For the typical conditions of Sec. (2.1), we have shown \( I/I_{1s} \) in Fig. 6, including the correction due to the freezing temperature. For \( x_S/D > 100 \) the Kantrowitz-Grey and Hagena-Morton models give approximately the same results. For a rather compact construction, \( x_S/D = 80 \) (\( D = 1.210^{-2} \) cm), the atom flux becomes \( F = \dot{I} \dot{n} f = 1.2 \times 10^{15} \) atoms/sec and the maximum density becomes \( 1.7 \times 10^{12} \) atoms cm\(^{-3} \) with \( f = 0.85 \) representing the freezing correction.

5.2 The nozzle geometry

Hundhausen and Harrison\(^{(4)}\) first used a Laval nozzle (convergent section \( 30^\circ \) half angle by 6 mm length, divergent section \( 30^\circ \times 0.6 \) mm) for obtaining their supersonic jet. However, these dimensions are not critical, and a convergent nozzle of other dimensions (for example, \( 30^\circ \) half angle x 1/4" length, hole diameter 0.004") would be easier to construct. An experimental study for \( CO_2 \) by Busol\(^{(14)}\) showed that there exists a certain relationship between the diameter, \( d_{ex} \) and the length, \( l \), of the supersonic part of the nozzle, which results in an increase of directivity of the beam and high density in the flow. This relationship is

\[ 0.5 < \frac{l}{d_{ex}} < 5 \]  \hspace{1cm} (5.6)
To verify this relation for the condensable beams it would be necessary to do an experimental study of the nozzle design, applying different convergent and Laval nozzle geometries, to obtain the optimum beam conditions.

5.3 The formation of dimers $\text{Li}_2$

The atom flux density which can be obtained by increasing the density in the oven is limited by the formation of dimers. This association of atoms is favored by the pressure. The number of dimers in thermodynamic equilibrium was calculated by Ditchburn\(^{(15)}\). He employed the method of partitions. Neglecting the vibrational states of the molecule and the state of the atoms other than the normal state he obtains the dissociation equation for the reaction.

$$A + A \leftrightarrow A_2 \quad (5.7)$$

$$\frac{C_M}{C_A^2} = \frac{4\pi^{1/2} h R^2}{(mkT)^{1/2}} e^{u_o/kT} \quad (5.8)$$

where $C_M$ = molecular concentration, $C_A$ = atomic concentration, $R$ = radius of gyration of the molecule, $h$ = Planck constant, and $u_o$ = energy of dissociation in ergs/mole. Since only a small fraction of the molecules is associated we may put:

$$C_A = L_o \frac{273}{T} \frac{P}{760} \quad (5.9)$$
where \( L_0 \) = Lochsmitt number and \( p \) = pressure (torr).

The associated fraction is

\[
\gamma = \frac{4\pi}{(mkT)^{1/2}} \frac{L_0}{T} \frac{273}{760} \frac{u_o}{kT} p e^{u_o/kT}
\]  

(5.10)

with the molecular constants \(^{16}\)

\[
R = 2.67 \times 10^{-8} \text{ cm}
\]

for Li\(_2\)

\[
u_o = 1.03 \text{ ev}
\]

\[
R = 1.95 \times 10^{-8} \text{ cm}
\]

for K\(_2\)

\[
u_o = 0.514 \text{ ev}
\]

If the mean free path in the beam is large at the oven aperture, the dimer concentration in the beam will be about the same as that in the oven chamber. The experimental results of Morgenstein \(^{17}\) in 1967 for potassium, confirm our hypothesis. Morgenstein produced a potassium beam using an oven with a Laval nozzle. He separated K from K\(_2\) with a Stern-Gerlach magnet. His results are summarized in Table 1, where we have also included the values calculated by the Ditchburn theory for the oven pressure and the oven and nozzle temperatures.
<table>
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<tr>
<th>OVEN Temperature (°K)</th>
<th>NOZZLE Temperature (°K)</th>
<th>OVEN Pressure (torr)</th>
<th>experiment γ(K₂) %</th>
<th>theory γ(K₂) % with T&lt;sub&gt;nozzle&lt;/sub&gt;</th>
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Table 1 shows good agreement between theory and experiment. The percentages of association calculated from the two temperatures provide limits within which all but six of the experimental points fall. The experimental errors for the measurement of the oven and nozzle temperatures, as those for the determination of the $K_2$ percentage, were not given by Morgenstein. Table I shows that the molecules are not completely equilibrated at the nozzle temperature since the percentage calculated at this temperature is less than observed. This can be explained by the fact that the nozzle chamber is not large enough for the molecules to have many collisions with the wall. Nevertheless, this theory gives a good estimation of the percentage of molecules obtained in the beam, and it should be applicable for lithium.

We have shown in Fig. 8 the theoretical curves for Li for saturated vapor conditions and for different pressures. In order to prevent the formation of dimers it seems necessary to work at a low pressure $p_o \leq 10$ torr, and to increase the temperature of the nozzle chamber. In these conditions the speed ratio $S$ is relatively small according to eq. (5.4), and the atom flux is reduced.

A compromise may be found by increasing the nozzle diameter, which increases $S$. Then, the intensity with a small $p_o/T_o$ ratio increases as is indicated in Fig. 7.
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FIG. 2a

VAPOR PRESSURE $P$ (Torr) vs. OVEN TEMPERATURE $T^\circ C$

- CS
- K
- Li
FREE JET

FIG. 4
\[ \frac{\rho \ T_0 \ (^\circ \text{K})}{p_0 \ \text{torr}} = 1.54 \times 10^{18} \left( \frac{x}{D} \right)^{-2} \]
$D = 0.12 \text{ mm}$
$D_s = 2 \text{ mm}$
$T_0 = 1273^\circ \text{K}$
$P_0 = 30 \text{ TORR}$

**Collisionless Model**

**Flow Divergence Model**

**Fig. 6**
REFERENCES


