JILA REPORT # 38

RELATIVISTIC STABILITY OF A MAGNETOTURBULENT MASSIVE STAR

by

A. G. Facholczyk

IMPORTANT NOTICE

This report from the JILA contains unpublished material of a preliminary nature, or of transitory interest, and may not have been subjected to critical technical review. Such reports are issued sporadically as preprints, contract reports or unpublished communications. As such their reproduction, quotation, or citation in the open literature is not permitted without express permission of the author or the Chairman of the JILA. If this material is identified as accepted for publication in the open literature all citations should be made to that journal. If this constitutes a technical report to a sponsoring agency that agency may reproduce this material for further distribution within the government and to agency contractors.

University of Colorado
Boulder, Colorado
June 22, 1965
RELATIVISTIC STABILITY OF A MAGNETOTURBULENT MASSIVE STAR

A. G. Pacholczyk
Joint Institute for Laboratory Astrophysics
Boulder, Colorado

It has been well established by Chandrasekhar (1964) and by Fowler (1964) that a massive spherically symmetric star contracting through equilibrium configurations will become relativistically unstable long before it reaches the gravitational radius $R_g$. For a polytropic star with index $n = 3$ and mass of $10^8$ solar masses the instability radius is equal to about $10^4 R_g$. Such a massive object cannot reach the temperature of $6 \times 10^7 \,^oK$ needed for nuclear reactions to provide the energy which is radiated away before it becomes unstable. Moreover, because of a very short time scale of subsequent gravitational collapse no significant fraction of nuclear energy can be radiated away during the collapse phase.

Recently Layzer (1965) has shown that a sufficiently large rotating mass can generate a magnetoturbulent field whose energy is comparable to its own rest energy.

It is the purpose of the present note to point out that a Hoyle and Fowler's (1963) type massive object, which has developed a magnetoturbulent field through the mechanism proposed by Layzer (1965), can achieve the hydrogen burning stage long before it reaches the relativistic instability point. It is easy to show, following the approximate analysis by Fowler (1964), that the total energy of such a magnetoturbulent massive star in the post-Newtonian
The approximation is
\[
E = -\left(\frac{3}{8} \beta + \frac{3k}{4k + 1}\right) \frac{R}{R} + 1.3 \left(\frac{R}{R}\right)^2
\]
where \( \beta \) is the ratio of the gas pressure to the total pressure in the star and \( k \) is the ratio of the kinetic to the magnetic energy in the magnetoturbulent field. The equation (1) is written for the stage in which the rotational energy of the star is practically converted into the energy of the magnetoturbulent field. At this stage the magnetic energy is increased to the value of \( k/(2k + 1) \) times the gravitational energy (Layzer 1965). Being interested only in the order of magnitude estimation we are neglecting here the influence of the magnetoturbulent layer on the structure of the star, assuming it to be a wholly convective polytrope of index \( n = 3 \) (Hoyle and Fowler 1963). A quantitative theory of magnetoturbulent massive stars is needed to proceed in a more precise way. The relativistic term in (1) will lead to instability as soon as \( \partial E/\partial R < 0 \), i.e., when the radius \( R \) of the star decreases below its critical value
\[
R_{\text{crit}} \lesssim \frac{3.5}{k/(2k + 1) + \beta/2} R_g
\]
For the case with \( k \lesssim 1 \) and \( \beta \lesssim 10^{-3} \) we have \( R_{\text{crit}} \lesssim 10 R_g \). The radius of a massive object in a hydrogen burning phase is given from the structure of the star (Hoyle and Fowler 1963) as
\[
R_H \lesssim 5 \times 10^{5} R_g \left(\frac{M}{M_\odot}\right)^{-1/2}
\]
The ratio \( R_H/R_{\text{crit}} \) is larger than unity for stars with masses \( M \) smaller than \( 2.5 \times 10^9 \) solar masses \( M_\odot \). Such stars will not collapse until they
pass through the hydrogen burning phase releasing up to $3 \times 10^{61}$ ergs of energy from nuclear sources in a period of $10^6$ years. The quoted numbers correspond well to those required to explain the observed radiation of extragalactic radio sources. It should be noted, however, that the lack of an adequate mechanism of loss of the angular momentum by the system (the angular momentum cannot be absorbed by the magnetoturbulent field, Layzer 1965) may lead to the amount of the energy transferred into the magnetoturbulent field smaller than the amount given by Layzer (1965). In this case the resulting upper limit on the mass of a stable configuration will be lower than the quoted one.

A conversation with Drs. John Cox and Ian Roxburgh is gratefully acknowledged.

*The decrease of the instability radius as compared to the case of a massive star without magnetoturbulent field is due to the kinetic energy associated with the magnetoturbulent field. The presence of a magnetic field in general increases the radius of instability (Pacholczyk 1965). The stabilizing role of kinetic energy in the problem of relativistic instability of rotating large masses was considered by Roxburgh (1965).


References


