Remarks on the H⁻ Equilibrium in Stellar Atmospheres

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The importance of the charge-exchange process \( \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n_s, p, d) \) is compared to that of the associative detachment process. It is shown that (1) the first process is dominant in main sequence A stars, and in lower effective temperature stars with gravity smaller than the main sequence stars; (2) in the sun, the charge-exchange process is of minor importance at every depth.

INTRODUCTION

In their study of the dissociation equilibrium of \( \text{H}^- \) in stellar atmospheres, Lambert and Pagel (1968) mention that the charge neutralization (or charge-exchange) reaction,

\[ \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n_s, p, d), \]

the rate constant of which they adopt from calculations by Bates and Lewis (1955), may become important at F0 and in the deeper layers of a star like the sun. The cross section of this reaction has now been measured by Rundel et al. (1969), who found it to be between two and three times larger than the computed one in the relevant range of energies.

The corresponding rate constant at low energies given by these authors is \( D_{\text{H}^-, \text{H}^+} = 5 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1} \), as compared with the associative detachment rate constant \( D_{\text{H}^-, \text{H}^+} = 1.3 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1} \) (Schmeltekopf et al. 1967); both rates are practically independent of temperature.

Competition between the two reactions, i.e., charge exchange and associative detachment, is expected to occur in regions of those atmospheres, or parts of atmospheres, where the temperature is high enough so that protons have a significant density, although \( \text{H}^- \) is still present. One then wants to: (1) look at the variation of the rates for the two reactions at all depths along the spectral sequence; (2) compare for a given atmosphere, e.g., the solar one, the departure coefficients \( b_{\text{H}^-} \) and the net rates for collisional processes at different levels in the low chromosphere, both when the charge-exchange reaction is taken into account and when it is not; and (3) later on estimate how much the temperature distribution \( T_e(r) \) is affected, under radiative equilibrium, by the Cayrel effect (Cayrel 1963, Gebbie and Thomas 1970), when the \( \text{H}^- \) equilibrium is dominated by the charge-exchange reaction, and when it is dominated by associative detachment.

VARIATION OF THE RATES OF CHARGE EXCHANGE VERSUS ASSOCIATIVE DETACHMENT IN THE SPECTRAL SEQUENCE

At photospheric levels, in a series of LTE model stellar atmospheres (Carbon and Gingerich 1969), one computes the ratio \( n_{\text{H}_1} D_{\text{H}^-, \text{H}^+}/n_{\text{H}_1} D_{\text{H}^-, \text{H}^+} \), where \( n_{\text{H}_1} \) is the density of neutral hydrogen, and \( n_{\text{H}^-} \) the density of protons. The results for \( \log g = 4 \), \( g \) being the gravity of the atmosphere, are given in Figure I for effective temperatures \( T_{\text{eff}} \) ranging from 6000 to 10,000 K. The LTE ratio of \( \text{H}^- \) bound-free absorption to Paschen bound-free absorption is also plotted. The same quantities are given for a solar photospheric model (Gingerich et al. 1970), referred to as HSRA.

The charge-exchange reaction is seen to dominate over the associative detachment at all photospheric levels in LTE atmospheres with \( T_{\text{eff}} \) greater than 8000 K, \( \log g = 4 \), and for models having lower \( T_{\text{eff}} \) in the deep photosphere.

In the continuum in non-LTE models, the control of the temperature distribution by the radiative or the collisional processes is determined by the \( \text{H}^- \) equilibrium, as soon as the \( \text{H}^- \) opacity is larger than that due to neutral hydrogen; this condition, as well as dominance of photoionizations over collisional ionizations, delineates regions in optical depth for each model, where a non-LTE
rise in temperature will occur through the Cayrel effect. Figure 1 shows, for example, that for all \( \tau_{5000} < 0.5 \) in the \( T_{\text{eff}} = 8000 \) K model, one expects such a rise when radiative ionizations begin to exceed collisional ones; it happens that also at the same depths, the charge-exchange reaction still exceeds the associative detachment. For cooler models, on the other hand, such as \( T_{\text{eff}} = 7000 \) K, \( \tau_{5000} > 0.3 \) delimits the region where charge exchange is important, but at those depths it is likely that the collisional ionization of \( \text{H}^+ \) will dominate over radiative ionization, and then the Cayrel mechanism will no longer be operating. In the sun’s photosphere, the charge-exchange reaction is never important, whatever the depth.

Therefore, the influence of the charge-exchange reaction at photospheric levels will be greatest in A stars (\( T_{\text{eff}} = 8000–10,000 \) K), and especially in late A type stars (see Figure 1). Its effect being to increase the collisions, the Cayrel radiative rise of temperature will occur more superficially than if only associative detachment is considered.

Moreover, at a given effective temperature, the ratio \( n_\text{H}_2D_{\text{H}_2} - n_\text{H}D_{\text{H}_2} \) is very sensitive to gravity, being exactly like the ratio \( n_\text{H}_2/n_\text{H} \).

The charge-exchange reaction is then dominant in superficial layers even for stars of \( T_{\text{eff}} \) lower than 7000 K, when \( T_{\text{eff}} \) is decreased by 10 or 100 relative to the main sequence characteristic gravity.

Because the relevant rate constants are independent of \( T_{\text{eff}} \), the relative importance of charge exchange versus associative detachment is not likely to be changed substantially in a non-LTE atmosphere, where at a given depth \( T_e \) has risen relative to the LTE value but the ionization degree has not, at least as long as photoionizations dominate.

THE CHARGE-EXCHANGE REACTION IN THE SOLAR CHROMOSPHERE

When one goes toward smaller optical depths in the solar chromosphere, the \( \text{H}^+ \) opacity exceeds all other sources of opacity for \( \tau_{5000} \) greater than \( 5 \times 10^{-6} \) or \( 10^{-5} \), depending on the adopted model. There, as electron scattering becomes the main part of the absorption coefficient, \( \text{H}^+ \) is still as important an absorber as \( \text{H}_2 \) until \( \tau_{5000} \simeq 10^{-5} \).

In order to compare the variation with height of the two most important processes ruling the equilibrium of \( \text{H}^+ \), two models of the solar chromosphere are considered: (1) the Thomas and Athay model (1961), designated TA; (2) the Noyes and Kalkofen model (1970), designated NK. Between 400 and 1800 km (in the Noyes and Kalkofen scale where \( h = 0 \) for \( \tau_{5000} = 1 \)) these two models differ: at a given height, the first model is hotter by several hundreds of degrees. and
its electronic density is higher by a factor which reaches almost 10 around \( h = 1000 \text{ km} \). Below 500 km, in its height scale, the TA model is known to provide an upper limit for the temperature.

The equations of statistical equilibrium for \( \text{H}^- \) and \( \text{H}_2 \) have been solved under the following assumptions: the radiation field is described by \( J_{\nu} = W B_\nu(T_e) \), with a dilution factor \( W = \frac{1}{2} \) and with \( T_e = 5400 \text{ K} \) for wavelengths in the Balmer continuum, \( T_e = 5800 \text{ K} \) for wavelengths in the Paschen continuum (Noyes and Kalkofen 1970); there are no radiative processes governing the equilibrium of \( \text{H}_2 \); the free-free opacity of \( \text{H}^- \) is neglected; bound-free opacity of \( \text{H}^- \) is taken from Doughty et al. (1966). Induced emission has been taken into account.

The following processes are considered:

(A) \( \text{H}^- + h\nu \rightarrow \text{H} + e^- \) photodetachment  
(B) \( \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n\ell) \) associative detachment  
(C) \( \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n\ell) \) charge exchange  
(D) \( 3\text{H} \rightarrow \text{H}_2 + \text{H} \) 3-body collisions

The collisional detachment reaction \( \text{H}^- + e^- \rightarrow \text{H} + 2e^- \) is negligible, according to the rate constant derived from Bely and Schwartz (1969).

The equations of statistical equilibrium, respectively for \( \text{H}^- \) and \( \text{H}_2 \), are:

\[
\begin{align*}
n_{\text{H}^-} n_{\text{H}_2} D_{\text{H}^-} & = -n_{\text{H}^-} n_{\text{H}_2} D_{\text{H}^-} \left( 1 - \frac{b_{\text{H}^-}}{b_{\text{H}}^-} \right) \\
 & = 4\pi \left[ \frac{n_{\text{H}^-}}{b_{\text{H}}} \int_{v_0}^\infty \frac{\sigma_{\text{H}^-}(\nu)(1 - e^{-h\nu/kT_e}) B_{\nu}(T_e) d\nu}{h\nu} \\
 - W n_{\text{H}_2} \left\{ \int_{v_0}^\infty \frac{\sigma_{\text{H}^-}(\nu) B_{\nu}(T_e) d\nu}{h\nu} \\
 - \frac{1}{b_{\text{H}}} \int_{v_0}^\infty \frac{\sigma_{\text{H}^-}(\nu) B_{\nu}(T_e) e^{-h\nu/kT_e} d\nu}{h\nu} \right\} \right] \tag{1}
\end{align*}
\]

\[
\begin{align*}
n_{\text{H}^-} n_{\text{H}_2} (1 - b_{\text{H}^-} b_{\text{H}}^-) + n_{\text{H}}^2 D_{\text{H}^+} (1 - b_{\text{H}}^-) & = 0 \tag{2}
\end{align*}
\]

where \( n_{\text{H}^-}, n_{\text{H}_2}, D_{\text{H}^-}, \) and \( D_{\text{H}^+} \) are as defined above, \( n_{\text{H}} \) is the density of \( \text{H}^+ \), \( T_e \) is the electronic temperature, \( D_{\text{H}^+} \) is the rate of the triple collision process (D), and \( v_0 \) is the threshold frequency for the reaction (A).

The cross section of the reaction \( \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n\ell=2) \) is much smaller than that of \( \text{H}^- + \text{H}^+ \rightarrow \text{H}(1s) + \text{H}(n\ell=3) \), in the range of impact energies to be considered (Bates and Lewis 1955). Hence we write the net rate of the charge-exchange reaction by taking this reaction as leading to the formation of \( \text{H}(1s) + \text{H}(n\ell=3) \) only.

Tables 1 and 2 give, for the TA and the NK models, respectively, the net rates of associative detachment and of charge exchange, the departure coefficients \( b_{\text{H}^-} \) and \( b_{\text{H}} \), the non-LTE opacity at 5000 A due to Paschen and \( \text{H}^- \) bound-free absorption, i.e., \( \kappa_{\text{H}^-} = n_3 \sigma_3 \left[ 1 - (1/b_{\text{H}^-}) e^{-h\nu/kT_e} \right] \) and \( \kappa_{\text{H}^+} = n_3 \sigma_3 \left[ 1 - (1/b_{\text{H}}) e^{-h\nu/kT_e} \right] \), where \( n_3 \) is the population of the third level of hydrogen, \( \sigma_3 \) and \( \sigma_3 \) are the be photoionization cross sections at 5000 A for the \( n = 3 \) level in \( \text{H} \) and for \( \text{H}^- \).

From the extension of the NK model by the reference photosphere of Gingerich et al. (1970), the correspondence between the geometric scales in the two chromospheric models is \( z_{\text{ref}} = 10^{-3} \) at \( h = 420 \text{ km} \) in the NK model and at \( h \geq 0 \text{ km} \) in the TA model.

In the TA model, the \( b_{\text{H}^-} \) have been computed from the solution for the H\(_z\) source function, knowing \( b_z \); Dr. W. Kalkofen has kindly provided the \( b_{\text{H}^-} \) for the NK chromospheric model. In both cases, the hydrogen atom is represented by three discrete levels plus a continuum, and the \( b_{\text{H}^-} \) are affected by the neglect of the P\(_z\) line in the statistical equilibrium equations for \( \text{H} \).

Consideration of Tables 1 and 2 leads to the general conclusion that, although the charge-exchange process is more important than the associative detachment at heights over 500 km in the TA model, over 949 km in the NK model (which represents, in fact, almost the same physical altitude), the net rate for the charge-exchange reaction is more than 10 times smaller than the photoionization or recombination rates (see Figure 2). Hence, even when the charge exchange dominates the associative detachment in the sun, the situation is the same as already pointed out by Cayrel (1963) and by Gebbie and Thomas (1970): the \( \text{H}^- \) equilibrium depends almost exclusively on the radiative processes at low chromospheric levels. Taking into account the charge-exchange reaction will not influence the energy balance in \( \text{H}^- \) nor the temperature structure in the low chromosphere, whatever the height. It does change the \( b \) factors only by some per cent in the sense that any increase of the collisional processes drives \( b_{\text{H}^-} \) and \( b_{\text{H}} \).
TABLE 1
Thomas and Athay (1961) model

<table>
<thead>
<tr>
<th>$h$ (km)</th>
<th>$T_e$ (K)</th>
<th>$n_e$ (cm$^{-3}$)</th>
<th>$\tau_{5000}$</th>
<th>$b_1$</th>
<th>Net rate of associative detachment $n_{D_1-} + n_{e} (1-b_{23} b_{24})$</th>
<th>Departure coefficients $b_{31}$ no charge exchange</th>
<th>Departure coefficients $b_{31}$ no charge exchange</th>
<th>$\lambda = 5000$ A Bound free opacity $\kappa_3$ (cm$^{-1}$)</th>
<th>$\kappa_{H}$ (cm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5050</td>
<td>6.36 x 10$^{11}$</td>
<td>1.3 x 10$^{-3}$</td>
<td>0.90</td>
<td>1.951 x 10$^{10}$</td>
<td>2.081 x 10$^{7}$</td>
<td>1.747 x 10$^{4}$</td>
<td>1.04</td>
<td>1.05</td>
</tr>
<tr>
<td>100</td>
<td>5250</td>
<td>5.04 x 10$^{11}$</td>
<td>5.5 x 10$^{-4}$</td>
<td>1.04</td>
<td>2.561 x 10$^{10}$</td>
<td>2.645 x 10$^{6}$</td>
<td>1.591 x 10$^{4}$</td>
<td>1.15</td>
<td>1.16</td>
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<tr>
<td>200</td>
<td>5460</td>
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<td>2.7 x 10$^{-4}$</td>
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<td>2.129 x 10$^{10}$</td>
<td>2.189 x 10$^{6}$</td>
<td>2.062 x 10$^{4}$</td>
<td>1.36</td>
<td>1.37</td>
</tr>
<tr>
<td>300</td>
<td>5680</td>
<td>4.58 x 10$^{11}$</td>
<td>1.4 x 10$^{-4}$</td>
<td>1.40</td>
<td>1.286 x 10$^{10}$</td>
<td>1.325 x 10$^{6}$</td>
<td>3.094 x 10$^{4}$</td>
<td>1.64</td>
<td>1.67</td>
</tr>
<tr>
<td>400</td>
<td>5910</td>
<td>4.63 x 10$^{11}$</td>
<td>8.1 x 10$^{-5}$</td>
<td>1.60</td>
<td>2.655 x 10$^{10}$</td>
<td>6.443 x 10$^{6}$</td>
<td>4.091 x 10$^{4}$</td>
<td>1.96</td>
<td>2.02</td>
</tr>
<tr>
<td>500</td>
<td>6150</td>
<td>4.72 x 10$^{11}$</td>
<td>4.6 x 10$^{-5}$</td>
<td>1.81</td>
<td>2.599 x 10$^{10}$</td>
<td>2.672 x 10$^{6}$</td>
<td>3.842 x 10$^{4}$</td>
<td>2.29</td>
<td>2.38</td>
</tr>
<tr>
<td>700</td>
<td>6550</td>
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<td>2.2 x 10$^{-6}$</td>
<td>2.55</td>
<td>2.090 x 10$^{10}$</td>
<td>2.117 x 10$^{6}$</td>
<td>3.416 x 10$^{4}$</td>
<td>3.07</td>
<td>3.15</td>
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<tr>
<td>850</td>
<td>6840</td>
<td>3.69 x 10$^{11}$</td>
<td>1.7 x 10$^{-6}$</td>
<td>2.68</td>
<td>7.343 x 10$^{7}$</td>
<td>7.433 x 10$^{5}$</td>
<td>3.719 x 10$^{4}$</td>
<td>3.36</td>
<td>3.46</td>
</tr>
</tbody>
</table>

In columns 6, 8 and 14, $b_{31}$ is taken from column 9.
† At this height, the charge-exchange process starts to dominate.

TABLE 2
Noyes and Kalkofen (1970) model

<table>
<thead>
<tr>
<th>$h$ (km)</th>
<th>$T_e$ (K)</th>
<th>$n_e$ (cm$^{-3}$)</th>
<th>$\tau_{5000}$</th>
<th>$b_1$</th>
<th>Net rate of associative detachment $n_{D_1-} + n_{e} (1-b_{23} b_{24})$</th>
<th>Departure coefficients $b_{31}$ no charge exchange</th>
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<th>$\lambda = 5000$ A Bound free opacity $\kappa_3$ (cm$^{-1}$)</th>
<th>$\kappa_{H}$ (cm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>552</td>
<td>4200</td>
<td>1.59 x 10$^{11}$</td>
<td>1.27 x 10$^{-4}$</td>
<td>0.99</td>
<td>3.696 x 10$^{10}$</td>
<td>3.696 x 10$^{6}$</td>
<td>5.443 x 10$^{7}$</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>676</td>
<td>4700</td>
<td>6.03 x 10$^{10}$</td>
<td>2.11 x 10$^{-5}$</td>
<td>0.31</td>
<td>3.257 x 10$^{10}$</td>
<td>3.058 x 10$^{6}$</td>
<td>6.301 x 10$^{7}$</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
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<td>5100</td>
<td>5.89 x 10$^{10}$</td>
<td>9.86 x 10$^{-5}$</td>
<td>0.48</td>
<td>2.665 x 10$^{10}$</td>
<td>2.796 x 10$^{6}$</td>
<td>1.388 x 10$^{7}$</td>
<td>1.21</td>
<td>1.22</td>
</tr>
<tr>
<td>949</td>
<td>5510</td>
<td>6.76 x 10$^{10}$</td>
<td>5.97 x 10$^{-6}$</td>
<td>0.73</td>
<td>1.382 x 10$^{10}$</td>
<td>1.413 x 10$^{6}$</td>
<td>1.791 x 10$^{7}$</td>
<td>1.61</td>
<td>1.63</td>
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<tr>
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<td>4.39 x 10$^{-6}$</td>
<td>0.94</td>
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<td>6180</td>
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<td>3.19 x 10$^{-6}$</td>
<td>1.15</td>
<td>7.358 x 10$^{10}$</td>
<td>2.429 x 10$^{6}$</td>
<td>1.735 x 10$^{7}$</td>
<td>2.40</td>
<td>2.44</td>
</tr>
<tr>
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<td>6540</td>
<td>6.76 x 10$^{10}$</td>
<td>2.31 x 10$^{-6}$</td>
<td>1.27</td>
<td>1.380 x 10$^{10}$</td>
<td>1.391 x 10$^{6}$</td>
<td>1.926 x 10$^{7}$</td>
<td>2.88</td>
<td>2.92</td>
</tr>
<tr>
<td>1476</td>
<td>6970</td>
<td>6.76 x 10$^{10}$</td>
<td>1.84 x 10$^{-6}$</td>
<td>1.34</td>
<td>4.482 x 10$^{10}$</td>
<td>4.515 x 10$^{6}$</td>
<td>2.063 x 10$^{7}$</td>
<td>3.24</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Same notes as in Table 1.
closer to their LTE value. Obviously, in the photosphere, the associative detachment remains the overwhelming collisional process governing H\(^-\) (see again Figure 1).

One may notice that the net rate
\[ n_1 D_{\text{H}^- \text{H}} (1 - b_{\text{H}}/b_{\text{H}^-}) \]
is negative in the NK model for \( T_e = 4200 \) and 4700 K, which means that the photoionization rate is greater than the radiative recombination rate in the corresponding layers.

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**REFERENCES**


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Note added in proof

The cross section for the charge exchange reaction has been measured by Moseley, Aher et al. and Peterson (*Phys. Rev. Letters*, 1970, 24, 435) for energies as low as 0.15 eV, and remeasured by Gally and Harrison (J. Phys. B, 1970, 3, L25) in the keV range. The rate constant \( D_{\text{H}^- \text{H}^+} \) is \( 1.7 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1} \) at 5000 K, \( 1.4 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1} \) at 10,000 K. The results of the first part of this paper are then a little weakened. I am grateful to Dr. K. T. Dolder for drawing my attention to the quoted papers.