JPL Technical Memo

Proceedings of Conference on Experimental Tests of Gravitational Theories

1971

123

The Lunar Laser Ranging Experiment

P. L. Bender, Joint Institute for Laboratory Astrophysics; R. H. Dicke and D. T. Wilkinson, Princeton University; C. O. Alley and D. G. Currie, University of Maryland; J. E. Faller, Wesleyan University; J. D. Mulholland, Jet Propulsion Laboratory, California Institute of Technology; E. C. Silverberg, McDonald Observatory; H. E. Plotkin, Goddard Space Flight Center; W. M. Kaula, University of California at Los Angeles; and G. J. F. MacDonald, University of California at Santa Barbara

I. Introduction

The emplacement of the Apollo 11 retro-reflector package on the lunar surface has made possible very accurate measurements of the lunar distance (Refs. 1-5). A continuing program of range measurements to the package at nearly all phases of the moon is being carried out by the McDonald Observatory under NASA support. Returned signals have also been obtained by the Pic du Midi Observatory in France and by the Air Force Cambridge Research Laboratories (AFCRL) Lunar Laser Observatory near Tucson, Arizona. It is hoped that several other lunar ranging stations will be in operation within the next year or two, including ones in Japan, Hawaii, Russia, and the southern hemisphere.

The present observing program at the McDonald Observatory consists of three observing periods on most nights when the weather permits, except for a period of 5 days around the new moon. One observing period is near the time of meridian transit for the moon, and the others are 3 or 4 h earlier and later. Several runs of about 50 shots each are normally fired during each observing period.

The ruby laser system being used at present gives 3-J pulses with a repetition rate of one every 3 s. The total pulse length between the 10% intensity points is 4 ns. The rms variation in the observed transit time due to the laser pulse length and the jitter in the photomultiplier receiving the returned signal is 2 ns. The present overall accuracy of the measured transit time is 2 ns, and improvement to less than 1 ns requires only refinement in the calibration procedures.

The uncertainty in the range correction for the effect of the atmosphere was shown some time ago (Ref. 6) to be less than 6 cm out to zenith angles of 70 deg. This result was based on using the surface value of the atmospheric refractive index as a predictor for the correction, as is often done in radio work. Recently, H. S. Hopfield (Ref. 7) pointed out that very much better corrections for the optical case can be obtained by using the surface pressure as the predictor. It now seems clear that the total error in the range correction for zenith angles of up to 70 deg will be less than 1 cm except under rare atmospheric conditions.

With data from two or more well located observing stations, the lunar range can be corrected accurately for the effects of polar motion and fluctuations in the earth's rotation rate. Very accurate corrections can be made for the earth tides at each station. It appears that the use of lasers giving roughly 0.1-ns pulse lengths is highly desirable. With them, single-shot ranging accuracies of about 3 cm are expected.

II. Model for Discussing Accuracy of Lunar Results

The actual lunar range results presumably will be analyzed by fitting a numerical integration for the lunar motion such as that of Garthwaite, Holdridge, and Mulholland (Ref. 8) to the data. All the planetary effects are then included, and the lunar libration parameters and the location of the retro-reflector package with respect to the lunar center of mass can also be determined. However, in order to analyze the accuracy to be expected from the lunar ranging program, it is desirable to use a somewhat simplified model. For this model, we neglect the planetary perturbations and use the Hill-Brown solution (Ref. 9) to the main problem for the motion of the moon. The planetary masses and orbits appear to be well enough known so that uncertainties in them will not affect the results. Terms higher than second degree in the lunar gravitational field are neglected, and the theory of Eckhardt (Ref. 10) for the lunar librations is used. Effects of the earth's bulge, including precession and nutation, are neglected. Finally, we assume that the polar motion and angular position of the earth are well enough known so that changes in the errors in these quantities over a few days are small.

The parameters in the model are as follows:

a, e, e', i, a_1 -- parameters in Brown's theory corresponding to the mean lunar distance, lunar eccentricity, earth's eccentricity, inclination of the lunar orbit, and ratio of the lunar to solar distance. The a_1 also includes a factor $(E - M)/(E + M)$, where E and M are the masses of the earth and moon, and this factor will give the main uncertainty in a_1 when a has been determined fairly well.

ℓ, D, F, ℓ'; L -- Brown's four-angle variable and the mean longitude L of the moon; ℓ, D, and F are, respectively, L minus the mean longitude of perigee, of the sun, and of the node, and ℓ' is the mean longitude of the sun minus that of its perigee.

x_1, x_2, x_3 -- rectangular coordinates of the retro-reflector package with respect to the lunar center of mass measured along the moon's principal axes (x_1 is in the mean direction of the earth, and x_3 is toward the north lunar pole).

β, γ -- lunar libration parameters; $\beta \equiv (C - A)/B$, $\gamma \equiv (B - A)/C$, where A, B, and C are the lunar moments of inertia about the three principal axes.

σ, λ, z -- cylindrical coordinates of the observing station; ρ is the distance from the axis of rotation, λ the longitude, and z the distance above the equatorial plane.

ϵ -- inclination of the equatorial plane to the ecliptic.

As is well known from spacecraft tracking results, the differences between measurements before, during, and after meridian passage determine $\sigma \cdot \cos \delta$ and λ, where δ is the declination of the moon. The period of the terms from which $\sigma \cdot \cos \delta$ and λ are determined is 25 h, and it turns out that the shortest period terms which have to be determined accurately in the lunar motion and librations have periods of about 14 days. The accuracy of determining these two quantities thus will not be appreciably degraded by uncertainty in the lunar part of the problem once the lunar parameters have been moderately well determined. If data from only one station are available, some loss in accuracy will result from having to determine $\sigma \cdot \cos \delta$ and λ, but with several well located stations, little accuracy will be lost. There are thus in effect only 17 parameters instead of 19 which have to be determined from the lunar range terms with periods of longer than 1 day. Three of these, e', ℓ', and a_1 (actually a_1/a) are probably better determined from planetary radar or spacecraft tracking.

We take a right-handed earth-centered coordinate system for the observing station location, with the x-axis pointed from the center of mass of the earth to the center of mass of the moon. The y-axis is in the ecliptic plane, and the z-axis in the northern hemisphere. For the retro-reflector location we take a similar coordinate system, except that it is centered on the moon and rotated 180 deg about the z-axis. The topocentric distance to the reflector is then given by

$$\rho^2 = (r - x_E - x_M)^2 + (y_E + y_M)^2 + (z_E - z_M)^2$$

where r is the center-to-center distance, x_E, y_E, z_E are the earth station coordinates in the new system, and x_M, y_M, z_M are the reflector coordinates. Expanding this gives

$$\rho = \left[r - x_E - x_M\right] + \frac{1}{2}\left[\frac{\left(r_E^2 - x_E^2 + r_M^2 - x_M^2\right)}{(r - x_E - x_M)}\right] + \left[\frac{(y_E y_M - z_E z_M)}{(r - x_E - x_M)}\right] + \theta\left[\frac{\left(y_E^4 + \cdots\right)}{(r - x_E - x_M)^3}\right]$$

where r_E and r_M are the radii of the earth and moon at the observing station and reflector locations. If we write D as

$$\rho = \left[r - x_E - x_M\right] + \left[\frac{(y_E y_M - z_E z_M)}{a}\right] + C$$

we can show that the uncertainty in C affects only the constant term in ρ. The second bracketed term is always less than 3×10^4 m in amplitude, so that only the largest terms in it need to be considered. Thus, with some relatively small corrections, the conclusions concerning the accuracy achievable from the lunar range measurements are almost the same as if only the first bracketed term in ρ were present.

Using any of the above forms for ρ, we can calculate the partial derivatives of ρ with respect to each of the parameters. Each partial derivative consists of a sequence of terms of different frequencies. How well the corrections needed to the different parameters can be separated depends on whether the partial derivatives involve the different frequencies in substantially different ways. A slightly conservative estimate of the accuracy of the results can be obtained by taking 17 of the frequency terms in the partial derivatives which appear to affect the results most and considering that only the coefficients of these terms in the range residuals are determined by the observations. The problem can then be inverted to find the accuracy of each parameter, given the accuracy with which the residuals at the different frequencies are determined.

In the above model, it may seem that the mean motions for the five angle variables in the problem should be included as parameters. However, if the corresponding phase angles are determined separately from the earlier and the later parts of the data, then the changes in the phase angles give a measure of the errors in the mean motions. Similarly, differences in phase angles from three sections of the data can be used to determine the secular acceleration. While additional parameters for the corrections to the mean motions and the secular accelerations would be included in a final analysis, they do not seem necessary at present.

III. Checks on Gravitational Theory

For several centuries the motion of the moon provided one of the most stringent tests of gravitational theory. With the very accurate range measurements now being made, it appears that this will again be the case. The most general test, of course, is to see which if any of the existing theories can be made to fit the observations by adjusting the known parameters in the problem. Whether or not a given deviation from an existing theory can be detected depends on whether the residuals it causes have the same frequencies as do the residuals caused by errors in some of the parameters to be determined.

One general relativistic correction term which enters in the motion of the moon and should be measurable has been pointed out by Baierlein (Refs. 11 and 12). It has an expected amplitude of about 1 m and a time dependence of the form cos 2D, where D is the difference in the mean longitudes of the sun and the moon. The only non-Newtonian part of the metric which plays a role in determining the coefficient of this term depends on Baierlein's parameter η'. The cos 2D term would have an amplitude of 1.5 m instead of about 1 m if η' were zero instead of one. The Einstein theory and the Brans-Dicke theory with $\omega = 5$ give a difference in amplitude for this term of about a centimeter.

Isolating the roughly 1-m term with cos 2D time dependence requires some care, since the main term of this form due to the strong perturbation of the moon by the sun has an amplitude of about 3000 km. The values determined for a, e, and x_1 are all strongly affected by the observed amplitude of the cos 2D term. In effect, the constant term in the range depends mainly on $a - x_1$, and the cos ℓ term depends mainly on $e \cdot a$. The cos 2D term depends mainly on a and is the most efficient term to use in decoupling the three parameters. However, with about a factor 5 loss in accuracy, one can regard the cos 2ℓ term as being used with the constant and cos ℓ terms to determine the three parameters. The resulting expected value of the cos 2D term can then be compared with the observed value to look for the relativistic term.

A second effect to look for has been suggested by Nordtvedt (Refs. 13-16). This is a possible difference between the gravitational mass and the inertial mass of a large body. Under Einstein's theory, the ratio is exactly unity, but in other theories it can have a slightly different value. The question raised by Nordtvedt is whether the gravitational self energy of the body enters in the same way for both types of mass. If it does not, then he predicts that the anomalous acceleration of the earth toward the sun will lead to an extra term in the earth-moon distance, with time dependence cos D. For the maximum plausible effect, where the gravitational self energy does not contribute at all to the gravitational mass, Nordtvedt gives a magnitude of roughly 24 m for this term. The scalar-tensor theory predicts an effect which is smaller than this by a factor $(\omega + 2)$. With $\omega = 5$, this would yield a term of roughly 3-m amplitude.

The experimental separation of Nordtvedt's term from other terms of the same frequency seems quite feasible. The largest such term present in the lunar motion has an amplitude of 110 km and is proportional to $a \cdot a_1$. With a small increase in the accuracy of a, the main limitation will come from uncertainty in the moon-earth mass ratio, which enters in the definition of a_1. For an uncertainty of 3×10^{-5} in this ratio, as obtained from the tracking of space probes, the corresponding cos D term can be calculated to 7 cm. The effect of uncertainties in the other parameters in the problem is even smaller. Thus, the equality of gravitational and inertial mass for the earth apparently can be checked to high accuracy by the lunar ranging experiment. The cos D term is fortunately well separated in frequency from the other terms in the problem which have to be determined.

A third way in which a departure from Einstein's gravitational theory would affect the motion of the moon has been discussed by Dicke (Ref. 17). The scalar-tensor theory with $\omega = 5$ predicts a decrease of roughly 3×10^{-13} to 3×10^{-11} per year in the gravitational constant, depending on the present mass density of the universe. This would lead to a secular deceleration of the lunar motion and an increase in the lunar distance. The total secular deceleration of the moon could be determined very accurately from the lunar range measurement in a period of 5 to 10 years (Ref. 18), but there now appears to be doubt about whether the tidal part of the deceleration can be found well enough to permit separation of the scalar-tensor theory effect. Van Flandern (Ref. 19) has recently obtained an apparent secular acceleration of the moon which is quite different from the previously accepted value derived from the work of Spencer Jones. In discussing possible sources of systematic errors in the earlier work, Van Flandern states: "In particular, corrections applied to the observations to smooth out the effects of changes in observing procedure are sufficient by themselves to alter the derived acceleration by 100%." Whether future re-analysis of the material used by Spencer Jones could give the tidal deceleration with the necessary accuracy is thus doubtful.

References

1. Alley, C. O., et al., J. Geophys. Res., 70, 2267-2269, 1965.
2. Faller, J., et al., Science, 166, 99-102, 1969.
3. Alley, C. O., et al., Science, 167, 368-370, 1970.
4. Alley, C. O., et al., Science, 167, 458-460, 1970.
5. Faller, J. E., and Wampler, E. J., Scientific American, 38-49, Mar. 1970.
6. Alley, C. O., and Bender, P. L., in Continental Drift, Secular Motion of the Pole, and Rotation of the Earth, Symposium No. 32 of the International Astronomical Union, 86-90, Edited by W. Markowitz and B. Guinot, Reidel, Dordrecht, Holland, 1968.
7. Hopfield, H. S., Radio Science, 6, 357-367, 1971.
8. Garthwaite, K., Holdridge, D. B., and Mulholland, J. D., Astron. J., 75, 1133-1139, 1970.
9. Brown, E. W., Mem. Roy. Astron. Soc., 57, 1-145, 1908.
10. Eckhardt, D. H., The Moon, 1, 264-275, 1970.
11. Baierlein, R., Phys. Rev., 162, 1275-1288, 1967.
12. Krogh, C., and Baierlein, R., Phys. Rev., 175, 1576-1579, 1968.
13. Nordtvedt, Jr., K., Phys. Rev., 169, 1014-1016, 1968.
14. Nordtvedt, Jr., K., Phys. Rev., 169, 1017-1025, 1968.
15. Nordtvedt, Jr., K., Phys. Rev., 170, 1186-1187, 1968.
16. Nordtvedt, Jr., K., Phys. Rev., 180, 1293-1298, 1969.
17. Dicke, R. H., in The Earth-Moon System, 98-164, Edited by B. G. Marsden and A. G. W. Cameron, Plenum Press, New York, 1966.
18. Alley, C. O., et al., The Application of Modern Physics to the Earth and Planetary Interiors, Proceedings of the NATO Advanced Study Institute, 523-530, Edited by S. K. Runcorn, John Wiley and Sons, Inc., London, 1969.
19. Van Flandern, T. C., Astron. J., 75, 657-658, 1970.