Saturation in Fraunhofer Lines

GENTLEMEN,—

Gussmann's reply\(^1\) to our comments\(^2\) on the applicability of his proposed interpretation\(^3\) of saturation in Fraunhofer lines leaves something to be desired both algebraically and physically.

Gussmann suggests that the term \(dS_i/d\tau_c\) in our equations (4)–(6) is a misprint for \(dS_i/dt_c\) because his derivation of an expression for the non-LTE weighting function includes the latter rather than the former. Had he checked the algebra and the logic of our representation, he would have found this to be not a misprint but a careful choice of meaningful quantities. After integrating the first term by parts, we can rewrite our equation (1) in the form

\[
G[\tau_c,\tau_i] = \int_{\tau_c}^{\infty} \left[ dS_i/dt_c \right] e^{-\tau_c} dt_c + [S_c - S_i] e^{-\tau_c}
\]  

(1)
giving the line-profile,

\[ I_e - I_i = \int_{0}^{\infty} G[\tau_e, \tau_i] e^{-\eta} \, d\tau_i. \]  

(2)

(As before, for simplicity, we consider only the centre of the disk.) One can vary the form of this expression by integrating (2) by parts, including one or both terms of G in the integration. If one includes only the first term in G, one obtains equation (6) of our letter:

\[ I_e - I_i = \int_{0}^{\infty} \left\{ [dS_j/d\tau_e] \left[ i - e^{\eta} \right] e^{-\eta} + [S_e - S_i] \left[ dt_i/d\tau_e \right] e^{-(\eta + \theta)} \right\} \, d\tau_e. \]  

(3)

If one includes both terms of G, one obtains Gussmann’s equation (3) of his letter:

\[ I_e - I_i = \int_{0}^{\infty} \left[ dS_j/d\tau_e + (S_e - S_i) \right] \left[ i - e^{\eta} \right] e^{-\eta} \, d\tau_e. \]  

(4)

Which expression one uses depends upon what one wants to do with it.

Historically, the weighting function was introduced because it could be tabulated once and for all for a given atmospheric model, under the then conventional LTE approach. Everything pertaining to the line was contained in the \( \exp[-\tau_j] \, d\tau_j \) terms. When one admits non-LTE effects, and saturated lines are the examples \textit{par excellence} where such non-LTE effects must enter, the non-LTE weighting function divides into two parts, one depending only on the atmospheric model \( \text{[if } S_j = B_s(T_j) \text{]} \), the other depending in part but very heavily upon the line. Consequently, any reformulation that scrambles these two parts of the weighting function simply scrambles the physical picture.

We tried to “unscramble” the situation further by suggesting an even stronger separation of \( G(\tau_e, \tau_i) \) into two parts, one depending only on \( \tau_e \), the other only on \( \tau_i \), with the exception of the optical diminution factor \( \exp[-\tau_j] \) in our equations (1a) (7), and (2a). There, the weighting function simply represents the difference between specific intensity in the continuum and the line source function at each atmospheric point. It has the same interpretation in an LTE or non-LTE situation, and the difference in its value reflects directly the difference between \( S_j \) in the LTE and non-LTE situations, the specific intensity of the continuum not changing between the two situations for a given atmospheric model. We suggested that \( G(\tau_e, \tau_i) e^{\eta} \) might be regarded as an “equivalent” source function because it behaves, algebraically and physically, exactly in this way.

By contrast, Gussmann’s algebraic juggling to reduce the non-LTE weighting function to a formal identification with absorption-tube profiles masks any semblance of a physical picture. As a glance at either of the alternatives (3) or (4) will show, the components of the weight-term are thoroughly scrambled. A prime example is \( dS_j/d\tau_e \), a hybrid quantity since the essence of the non-LTE effect is the strong dependence of \( S_j \) upon line-scattering, which is in itself a strong non-LTE quantity, rather than a dependence of \( S_i \) upon \( \tau_e \), which would be the LTE situation where \( S_i \) depends only upon \( T_e(\tau_e) \). Indeed, \( dS_j/d\tau_e \) has largely formal meaning: formal because \( dS_j/d\tau_e \) is formally integrated over \( \tau_e \) in Gussmann’s
expression (4), whereas physically the expression \( [dS_i/d\tau_c] d\tau_c \) should be thought of as \( [dS_i/d\tau_c] [d\tau_c/d\tau_l] d\tau_l \), or \([dS_i/d\tau_l]d\tau_l \) integrated over \( d\tau_c \). We see the lengths to which such formalism can be carried by noting his expression\(^1\) for the intensity in the continuum:

\[
I_c[\omega] = S_i[\omega] + \int^\infty_0 [dS_i/d\tau_c + (S_c - S_i)] e^{-\omega \tau_c} d\tau_c. \tag{5}
\]

It is physically meaningless to express such a dependence of \( I_c[\omega] \) upon a line source function; algebraically, the only reason expression (5) can be so written is because of the identity:

\[
o = S_i[\omega] + \int^\infty_0 [dS_i/d\tau_c - S_i] e^{\omega \tau_c} d\tau_c, \tag{6}
\]

which introduces into (5) the pseudo-dependence of \( I_c \) upon \( S_c \). It is interesting that Gussmann uses equation (5) to construct the non-LTE extension of his absorption-tube interpretation of saturation.

We have already commented on the balance of Gussmann's remarks, and see no reason to alter those comments. As a conclusion to this stimulating exchange of letters, and with our thanks to Dr. Gussmann, we would like to emphasize that the purpose of the saturation-weighting-function theory, as suggested in its LTE formulation by Pecker in 1951, is now quite obsolete, owing to the generalized use of high-speed computers, which eliminates both the need for interweaving the two essential integrations (over wavelength and over depth) and the advantage of pre-calcualting a few more or less universal functions. We think, therefore, that in carrying out these calculations, one should forget about this type of formulation, however rough or fine it may be.

We are, Gentlemen,

Yours faithfully,

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References
