Some implications for physics and geophysics of laser range measurements from Earth to a lunar retro-reflector

Introduction

Recent developments in modern physics and in modern engineering—quantum electronics as embodied in the atomic clock and the laser, and the ability to land scientific instruments on the Moon—make possible for the first time the precise measurement of distances from points on the Earth to points on the Moon. The general technique and implications for several areas of science were discussed briefly in an earlier publication. A more detailed discussion of the method and how it could improve our knowledge of geocentric longitude is to be published in the proceedings of the IAU/IUGG Symposium on Continental Drift, Secular Motion of the Pole, and Rotation of the Earth, held at Stresa, Italy, in March 1965.

Although the talk actually given at the NATO Advanced Study Institute in March 1967 was devoted mainly to the topics presented at Stresa (at the invitation of Professor Runcorn who was present at that earlier symposium), we would refer the interested reader to that publication and wish to amplify here some other geophysical and physical implications mentioned only briefly during the talk. These are (i) the possibility of increased understanding of the Chandler wobble through better measurement of the tipping of the Earth with respect to its axis of rotation, and (ii) a definitive test of the conjectured secular decrease of the gravitational constant.
**Technique**

*Retro-reflector*

The gain in the retro-direction of diffraction-limited corner reflectors is sufficiently large to permit a package (described below) of about 8 kg mass (including support and pointing mechanisms) to serve as an effective point benchmark on the Moon's surface. For a laser-illuminated spot of about 10 km diameter, the return from the corner reflector package will be much larger than that from the lunar surface and, in addition, will not suffer the time smearing of several microseconds produced by the curvature of the lunar surface.

The problems which arise in placing corners on the Moon stem from the fact that one must meet and simultaneously satisfy many different and sometimes conflicting requirements. In an ideal environment, the choice is relatively simple since, for a given geometry and allowed weight (payload), one maximizes the return signal by making a single-diffraction-limited retro-reflector as large as the weight restrictions and fabrication techniques will permit.

Two aspects of the practical problem which vitiate the above conclusion are (i) a sideways displacement of the returned laser beam because of the relative velocity between the Moon and the laser transmitter (velocity aberration), and (ii) the wide lunar temperature variation from full Moon to new Moon as well as the exposure of the reflector to an energy input from direct sunlight of 2 cal/min cm² during essentially half the time.

The velocity aberration (which displaces the centre of the returned diffraction pattern between 1.5 and 2 km) limits the diameter of a diffraction-limited retro-reflector to about 12 cm (4.5 in.) unless one employs two telescopes spatially separated from one another, one for transmitting and another for receiving. One of these must be movable along an arc of about 1.6 km to allow for the varying direction of the relative velocity vector, and both must have high precision of pointing and tracking. For a wide range of corner sizes, however, the loss in efficiency for a given total payload weight, which results from using a larger number of small diameter corners, is almost exactly compensated by the increased diffraction spreading which locates a combination transmitting-receiving instrument site further up on the side of the returned diffraction pattern. This results in essentially the same optical efficiency for a given total payload weight for corner sizes ranging from 1.5 in. to 4 or 5 in. Below the 1.5 in. size one experiences an overall loss in efficiency because the diffraction spreading is so large that the combined site is essentially at the centre of the returned diffraction pattern.

It is therefore possible to minimize the thermal gradients which would distort the diffraction pattern by choosing a size of 1.5 in., and using a sufficient number in an array to permit observation of a reflected signal. Actual tests in a simulated lunar environment on a single solid fused silica corner reflector have confirmed the conclusions of theoretical calculations which indicate that the proposed array consisting of some 90 corner cubes 1.5 in. in size will remain essentially diffraction-limited throughout the lunar day and night. Some sacrifice in returned signal is accepted by using total internal reflection, rather than aluminizing the back surfaces, in order to increase the lifetime and to prevent additional thermal distortion. An array of this
type is being constructed for earliest possible emplacement under the NASA Apollo lunar surface experiments programme.

Laser ranging system

The return to a laser radar transceiver from the corner reflector described above will be adequate for a round trip transit time measurement using range gating and single photoelectron detection techniques with currently available Q-switched ruby lasers. By processing the data from about 100 returns it will be possible to measure the time to about $10^{-8}$ sec which gives a relative time uncertainty of 15 cm/c. (The absolute uncertainty involves the value of the speed of light, c, which is known with a probable error of about three parts in $10^7$. Assuming the constancy of the velocity of light, the relative range is all that is needed.) Typical laser system parameters are

- pulse duration $10^{-8}$ sec
- energy $10 J$
- beam divergence $10^{-3}$ rad
  (angular radius from 2-cm diameter aperture)
- transceiver aperture 150 cm
- bandpass filter width 3 Å
  at 6943 Å
- detector quantum efficiency (multi-pass) $10\%$
- pulse repetition rate 1 pulse/3 sec

Continued progress in laser technology is expected to lead soon to field operational systems having near diffraction-limited performance and pulse lengths less than $10^{-9}$ sec. This would allow much smaller transmitting apertures and less elaborate signal processing.

Method of measurement and accuracy

The basic uncertainty in the range will be introduced by uncertainty in the total atmospheric delay. However, this can be predicted to about 6 cm for a zenith distance of 70° from a knowledge of local temperature and pressure. The technique is independent of angular errors introduced by fluctuations in the atmospheric index of refraction.

When accurate lunar range measurements have been made, the residuals from the best available ephemeris will be used in a least-squares analysis to determine improved values for the various lunar and geophysical parameters which affect the range. However, a simplified model can be used for estimating the expected accuracy with which the parameters can be determined. The model assumes that one range measurement $D_1$ from a given ground station is made at time $t_1$ about 4 hours before the time $t_m$ of minimum distance. A second measurement $D_m$ is made at about $t_m$, and a third $D_2$ at a time $t_2$ about 4 hours after $t_m$.

Uncertainties in the distance $R$ of the observing station from the Earth's axis of rotation and in the time $t_m$ at which the distance to the target is minimum will give mainly range residuals with a period of one day. Since the only important residuals due to errors in the lunar parameters will have periods of 14 days or longer, it is easy
to divide the problem. Measurements of the minimum distance $D_m$ made over an extended period can give improved values for a number of the lunar parameters and for the distance of the station from the equatorial plane of the Earth. The difference $D_1 - D_2$ plus the times $t_1$ and $t_2$ will give $t_m$, while the quantity $\frac{1}{2}(D_1 + D_2) - D_m$ can be used to find $R$.

The accuracy with which $t_m$ can be determined has been discussed elsewhere. It is easily shown that

$$\delta t_m = \frac{\delta (D_1 - D_2)}{2\omega R \sin \phi}$$

where $\omega$ is the Earth's rotation rate and $\phi = (\omega/2)(t_2 - t_1)$. For $\delta (D_1 - D_2) = 15$ cm, $R = 5 \times 10^8$ m, and $\phi = 60^\circ$, we find that $\delta t_m$ is about 0.25 msec. Similar measurements at two stations would give the difference in geocentric longitude to the equivalent of 15 cm in distance. Since atmospheric, timing, and Earth tide effects would be expected to give little systematic error when averaged over periods of a month or more, somewhat higher accuracy may be achievable for the average difference in geocentric longitudes. This is, of course, important in connection with the hypotheses of ocean floor spreading and continental drift. The accuracy with which the lunar motion can be determined from the measurements made near the time of minimum range is high enough so that the contribution to the uncertainty in the difference in geocentric longitude between two stations will be negligible.

**Motion of the Pole and the Chandler Wobble**

In this chapter we wish to concentrate mainly on the problem of the tipping of the Earth with respect to its axis of rotation (motion of the pole). For the past 60 years the International Latitude Service has coordinated and analysed observations of astronomical latitude carried out at approximately five stations located around the Earth at $59^\circ 8'N$ latitude. From these observations and from latitude and longitude determinations at other observatories, it is known that the position where the Earth's axis of rotation intersects the surface traces out a roughly elliptical path with a mean radius of about 5 m. However, the motion contains both an annual term and one with a period of about 14 months corresponding to a free nutation of the elastic Earth. As these terms get in and out of phase, the amplitude of the motion changes considerably. The annual term can be driven by various perturbations which have the proper frequency, but the means for exciting the 14-month 'Chandler wobble' term is not known. In addition there are both shorter period motions of the pole, including fortnightly, monthly, and semiannual tides, and slower motions consisting of an apparent secular motion of the pole and possibly a 24-year libration.

The analyses of the Chandler wobble which have been carried out would be consistent with a $Q$ for the Earth in the range of $10$ to $50$. However, the problem of noise in the observations is a severe one. It is normally assumed that the uncertainty in the monthly mean position of the pole is about a metre. The results can also be affected by systematic errors due to slow changes in the horizontal gradient of the atmospheric index of refraction or due to variations in the local vertical. Because of the noise it is not yet possible to say whether the Chandler wobble can be characterized by its power.
spectrum alone, as would be the case if it resulted from excitation of a damped resonance by many small disturbances with a broad range of frequencies. The other alternative would be occasional strong excitation followed by exponential decay. In addition, we do not yet know whether the damping takes place in (or due to interactions with) the atmosphere, the oceans, the mantle, or the core. The conclusion of Munk and MacDonald's discussion of damping still seems valid: 'The situation is appallingly uncertain.'

We shall assume that observations of the lunar range are made from three ground stations located at latitude \(B = 35^\circ\) N and at longitudes \(L_1, L_2,\) and \(L_3\) which differ by 120°. If the position of the pole moves a distance \(\mu\) towards longitude \(\lambda\) through an angle \(\mu\), then

\[
\Delta(L_2 - L_4) = \sqrt{3} \tan B \left( \mu \cos(\lambda - L_{12}) \right)
\]

\[
\Delta(2L_3 - L_1 - L_2) = 3 \tan B \left( \mu \sin(\lambda - L_{12}) \right)
\]

where \(L_{12} = (L_1 + L_2)/2\). Assuming that the uncertainties in determining the times \(t_m\) for the three stations are independent and each is equal to 0.25 msec, we find that the components of the polar motion can be found with an uncertainty of

\[
\delta(\mu \cos(\lambda - L_{12})) = \delta(\mu \sin(\lambda - L_{12})) = \frac{2 \cdot \omega}{\sqrt{3} \tan B} \delta t_m \sim 15 \text{ cm}
\]

This value is the proper one to use in discussing fortnightly or monthly tides. However, somewhat higher accuracy may be achievable for slower terms in the polar motion due to the averaging out of systematic atmospheric or other errors which have a monthly period. It thus appears that a regular, long-term programme of lunar range measurements may be capable of improving our knowledge of the Chandler wobble by a substantial amount.

In the above discussion it was assumed that the rotation rate of the Earth is known. Actually, observations of \(t_m\) at the same station on successive days will give the single-day rotation rate to 5 parts in \(10^9\). This will permit a check with improved accuracy on short-term variations in the rotation rate, but it seems unlikely that variations will be found with periods short enough to increase significantly the uncertainty in the polar motion. Although the rotation rate is measured with respect to the retro-reflector package on the Moon, the laser range measurements will give the lunar motion and librations with sufficient accuracy so that the rotation rate with respect to the Sun can also be obtained.

**Test for a Secular Decrease of the Gravitational Constant**

The existence of one or more corner reflectors on the Moon would permit a new check of general relativity. It has frequently been suggested that the gravitational 'constant' may not be constant, that gravitation may be steadily weakening in comparison with the strength of electromagnetism and other interactions. These theories are frequently misunderstood, it being thought that they are non-relativistic. Within the framework of standard relativistic field theory, all that is required for the validity of the Brans-Dicke cosmology is the existence of still another elementary particle, a
particle with neither spin, charge, nor mass. The massless and chargeless particles known or presumed to exist consist of the photon, electron, and muon neutrinos, and the graviton, particles with spin 1, 1/2, 1/2 and 2, respectively. The Brans–Dicke cosmology requires in addition the scalaron, a particle with zero spin. Actually, it is not the existence of the scalar particle which is critical, the quantum effects not yet being important, but rather the associated zero mass scalar field, treated classically.

It has been shown that within the framework of the formalism of Einstein’s general relativity the inclusion of the zero mass scalar field, plus assumptions of initial conditions and coupling, leads directly to the Brans–Dicke formalism. While this theory is general relativistic it is not general relativity, for part of the gravitational force is due to the scalar field and the whole of the gravitational effect is not geometrical as Einstein assumed.

A peculiar feature of the coupling of a particle (such as a proton or electron) to a scalar field is the resulting variability of the particle's mass, being a function of the scalar field variable. This results in the dimensionless gravitational coupling constant $Gm^2/he$ being a function of the scalar. When this result is expressed in ordinary centimetre, second, gram units of length, time, and mass, $G$ becomes a function of the scalar. Hence it changes with the change in the scalar that results from the expansion of the Universe.

It has been shown that the existence of a zero mass scalar field would not affect any non-gravitational physics. It has also been shown that the mass regulation effect mentioned above serves to adjust the strengths of the scalar and gravitational interactions automatically to make them the same order of magnitude.

An ideal way to test for the presence or absence of a secular decrease in $'G'$ would be to follow the Moon's motion with great precision for a period of years. The secular increase in the Moon's period to be expected within the framework of the Brans–Dicke cosmology is in the range of $(2 \text{ to } 6) \times 10^{-11}$ per year, if one takes the value of the parameter $\omega$ contained in the theory to have the value of about 6, as suggested by measurements of the solar oblateness and by other information. Based upon the telescope observations of the past 250 years, it is known that the tidal-induced increase in the Moon's period is $12.9 \times 10^{-11}$ per year, which is known to $0.6 \times 10^{-11}$ per year. This results from comparisons of the motion of the Moon with that of the Sun and of Mercury. The time scale is thus provided by planetary time. Inasmuch as it is believed that the tidal couple on the Moon is reasonably constant over a period of a few hundred years (this belief is strengthened by telescopic observations) the next decade should witness the same tidal-induced increase in the lunar period. However, a further increase of 2 to 6 parts in $10^{11}$ per year should occur not on a planetary but on an atomic time scale, if the secular decrease in the gravitational 'constant' should exist.

An accurate method for determining the rate of change of the lunar motion is to observe the phase of the cos $2D$ term in the lunar range. Here $D$ is the mean longitude of the Moon with respect to the Sun. The amplitude of this term is about $3 \times 10^6$ m. An error of $2.5 \times 10^{-8}$ rad in $D$ would give a residual in the observed range of the form sin $2D$ with amplitude $15$ cm. Such a term should be easily observable for the following reasons: (i) uncertainties in the other lunar parameters will not give this frequency, (ii) no systematic errors with this frequency are expected, and (iii) this term is well resolved in frequency from other terms expected in the residuals.
Laser range measurements from Earth to a lunar retro-reflector

If the difference between \( n \), the mean motion of the Moon, and \( n' \), the mean motion of the Sun, is given by \( n - n' = (n - n')_0 (1 - \beta t) \) with \( \beta = 4 \times 10^{-11} \) year\(^{-1} \), then

\[
D = D_0 + (n - n')_0 t - \frac{1}{2} \beta (n - n')_0 t^2
\]

From observations of the phase of the cos \( 2D \) term at times near \( t = 0, T, \) and \( 2T \), we can find \( \beta \). Its uncertainty is given by

\[
(n - n')_0 T^2 (\delta \beta) = 2 \delta \{ D(T) \} - \delta \{ D(2T) \} - \delta \{ D(0) \}
\]

Assuming that the uncertainties in \( D \) are independent and that each is \( 2.5 \times 10^{-8} \) rad, we find for \( T = 5 \) years that \( \delta \beta \approx 3 \times 10^{-11} \). It thus appears that the lunar deceleration predicted by the Brans–Dicke theory could be observed in about 10 years. The rotational rate of the Earth with respect to the Moon obtained by laser range measurements could also be used if the rotation rate of the Earth with respect to the fixed stars could be determined with sufficient accuracy\(^{10}\).

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References