Cepheid Pulsations

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CEPHEID VARIABLE STARS have been of interest to astronomers ever since the light changes of Delta Cephei were discovered in 1784 by the English amateur John Goodricke. Today, several thousand Cepheids are known in the Milky Way galaxy, in the Large and Small Magellanic Clouds, and in other nearby galaxies. Of high intrinsic luminosity, Cepheids are very important in setting up the extragalactic distance scale.

About half a century ago, astronomers established that the cause of a Cepheid’s light variations is expansion and contraction of the star, which causes concurrent changes in surface temperature and color. Since the pioneering studies of stellar pulsation by A. S. Eddington, much work has been done, and recently the mechanism of Cepheid variation has become understood in considerable detail.

Earlier it had been believed that Cepheids were eclipsing binary stars. Spectrographic studies showed that for each variable the radial-velocity curve was almost a mirror image of the light curve. But in an eclipsing binary, minimum light should occur when the star of greater surface brightness is undergoing eclipse and has no orbital motion in the line of sight. Instead, Cepheid light and velocity curves indicate that minimum light occurs about the time the recessional velocity is largest, a phenomenon not easily explained by any double star hypothesis.

Later work confirmed the pulsation theory, as shown by the following example for Delta Cephei. At two times during its cycle, the star’s color is the same, and presumably the temperature of the emitting surface is also the same. Thus, if Delta Cephei is pulsating, any difference in luminosity would be due to a difference in the star’s surface area (and radius) at the two times. The radius variations of the star derived by interpreting the photometric data in this way agree reasonably well with the radius range of 1,300,000 kilometers deduced from the spectroscopic radial-velocity curve. The same test for pulsation has been applied successfully to other Cepheids.

The light curve and radius changes of Eta Aquilae, a 4th-magnitude Cepheid, are shown here, based upon observations and analyses by J. B. Oke and others. During each cycle of 7.18 days, the pulsation causes the intrinsic luminosity to vary by over 0.5 magnitude, as the surface of the star moves alternately outward and inward by almost six percent of the mean radius.

Maximum luminosity occurs a tenth of the period later than minimum radius does, rather than simultaneously as expected from simple theory. Such skew light and radius curves are typical for Cepheids with periods from one to 50 days, although only those between seven and 10 days have pronounced bumps on the descending portions of their light curves.

Theory predicts and observation confirms a close relationship between period and density for Cepheids. Period in days, multiplied by the square root of mean density in solar units, equals a number close to 0.04. Roughly, the pulsation period of a star is the time required for a sound wave to travel through it. Hence, large tenuous stars have longer periods than do small dense ones. The known periods of Cepheid variables indicate low mean densities—of the order of 10$^4$ gram per cubic centimeter, just about what the modern theory of stellar evolution predicts for giants like Delta Cephei and Eta Aquilae.

When plotted in a Hertzsprung-Russell diagram (see also page 332, SKY AND TELESCOPE, June, 1966), the Cepheids all lie in a region above and to the right of the main sequence $A$ and $F$ stars. That is, the Cepheids are more luminous and some-
what cooler than these stars. The related RR Lyrae and W Virginis variables pulsate too, but they have smaller masses than the classical Cepheids and belong to Population II rather than Population I.

In the March, 1967, issue (page 153), Margherita Hack shows the evolutionary tracks of giant stars across the H-R diagram. Those for stars of five to nine times the sun’s mass cross the Cepheid instability region at least once, and sometimes there are several crossings at different luminosities during the same star’s lifetime.

Evolution and pulsation theories have clarified the famous period-luminosity law, whereby the intrinsic luminosity of any Cepheid is given by its observed period. Discovered originally for variables in the Small Magellanic Cloud, this historic relation has been used to obtain distances of other galaxies containing Cepheids of known apparent magnitudes and periods.

But as the period-luminosity chart suggests, the rule is none too accurate, because two Cepheids with identical periods can differ in luminosity by a magnitude or more. Also, in the course of its post-main-sequence evolution, extending over a few million years, a Cepheid very gradually expands or contracts at almost constant mean luminosity, slowly changing its period by sometimes many days. In spite of these difficulties, Cepheids remain important yardsticks for the distances of galaxies.

**How Cepheids Pulsate**

In a pulsating star such as a Cepheid, the radial motions are significant only in the outer half of the star, which includes less than 1/1,000 of the stellar mass (because the central concentration is so high). The task of the theoretical astrophysicist is to obtain from pulsation theory the properties of this time-dependent outer envelope, which is then to be combined with a model stellar interior.

Hydrodynamic studies of stellar pulsations begin with the basic ideas of the conservation of mass, momentum, and energy. Imagine the star divided into a series of concentric layers, each containing a fixed fraction of the mass. The overall motion of each shell of gas is determined by the amount of imbalance of the outward-pushing pressure gradient and the inward force of gravitational attraction. In a nonpulsating star, these forces are equal at all times and the layer as a whole is stationary. Here we will treat only radial pulsations, in which all parts of each spherical shell participate equally. During the motion, the conservation-of-energy equation determines the temperature changes at all levels in the star.

In the calculations, energy is allowed to flow from one mass shell to the next by radiation, conduction, and convection. Compared to the steady outward flow of energy in the star, the variable energy flow is always small, except in the outer 10 or 15 percent of the radius. In these outer regions, appreciable energy is alternately dammed up and released, and it is the phasing of this effect that determines whether or not the star can continue to pulsate for an appreciable length of time.

If extra energy is pent up in the outer layers when the star is smallest, it will somewhat increase the pressure in these layers, giving an extra boost to the expansion phase of the cycle. Moreover, release of this energy when the star is largest lowers the pressure in the outer layers, removing some of the support against gravity, so the ensuing contraction also receives added impetus. Hence, the pulsations are amplified slightly at each swing, and their amplitude tends to grow slowly. In this case, the outer layers of the star are acting as a thermal dynamic heat engine, transforming some of the radiant energy flowing outward through the stellar layers into mechanical energy of pulsation; the pulsations are “driven.”

On the other hand, if energy is dammed up when the star is largest and released when it is smallest, the pressure behavior during the cycle will be just the reverse of what we have described, and the pulsations will tend to be damped. This process is referred to as radiative damping. In Cepheids, the phasing of the damping up and release of energy is such that the outer regions of the star produce damping, while the interior produces damping. The driving predominates over the damping, thus maintaining the pulsations.

Although the nuclear reactions occurring in the central regions of the star are the basic source of the energy it radiates, they play no direct role in exciting the pulsations. According to many detailed calculations, the pulsations are vanishingly small in the central regions. As we have noted, these massive interior portions remain essentially stationary, while the outer parts of the star expand and contract.

**The Modern Approach**

In recent studies of stellar pulsation, astrophysicists have used elaborate tables of the physical conditions inside a star: pressure, energy, radiative opacity, and conductivity. Since the existence of radial oscillation depends sensitively on the relative amounts of driving and damping, the material properties must be precisely known for even rough comparison of a calculated model with an actual star.

Early theoretical work was centered around adiabatic pulsations, in which each mass shell neither loses nor gains heat during the course of the cycle. As we have seen, this happens to be a good approximation for most of the mass inside a Cepheid.

A further restrictive approximation in this early work was that all variations of position, temperature, density, pressure, and luminosity in the mass shells be very small compared with the average values. This so-called linear approximation, used in combination with the adiabatic assumption, allowed the period of
The solid curves show a model by the authors with gamma effect included; dashed curves are without gamma effect. Minimum radius occurs at the time of half period, which is about that of highest temperature. Zone 17’s equilibrium value is nearly 70,000 Kelvin, while zone 30, in the He+ ionization region, is near 40,000 Kelvin. The heat of contraction goes mostly into removing the second electron from once-ionized helium nuclei (He+), and little is left over for increasing the gas temperature.

Thus, a mass shell containing partial ionization is cooler than its surroundings during compression, and it absorbs heat. The added heat increases the pressure during the subsequent expansion giving an extra push that tends to build up the pulsation amplitude. This regulation of the local temperature by the second ionization of helium is the gamma effect.

In the chart at left, note that the oscillations in zone 30 are smaller than they would be if there were no gamma effect, whereas in zone 17 they are larger. The dashed curves also show decay from cycle to cycle, since there is no gamma effect to drive the oscillations.

**Kappa Effect.** The opacity of stellar material normally decreases as the temperature increases, and during compression the radiation flows more easily. This tends to damp the star’s oscillations. But in the He+ ionization region, the opacity is largest at maximum compression (because the number of absorbing ions has been increased), thus damping up energy and causing increased pressure during expansion.

**Radius Effect.** Finally, there is a tendency to lock up energy during maximum compression merely because of the convergence of the mass shells toward the center when the radius is at a minimum. This effect is very small, especially when the pulsation amplitudes are quite low.

The linear adiabatic theory shows that when the He+ ionization region is at a suitable mass level in the star, these three pulsation-driving effects can, in fact, overcome the dissipation in the deeper interior, causing the pulsations to grow. In certain cases, the ionization of hydrogen nearest the surface can also permit the star to oscillate, as Eddington pointed out a quarter-century ago.

This theory also permits us to investigate the pulsations of a star in modes other than the fundamental. When oscillations in a higher mode are taking place, there are one or more levels inside the star where there is no motion, while above and below them material is rising and falling and changing in physical condition. However, the periods of these harmonics can be computed from the linear adiabatic theory as well.

**The New Calculations**

Using fast electronic computing machines, the authors and others at Los Alamos Scientific Laboratory, and R. F. Christy at California Institute of Tech-
The authors' shallow-envelope models calculated on nonlinear, nonadiabatic theory, shown at several stages in their history. The horizontal time coordinate is marked in units of the pulsation period, and the numbers on the curves denote the successive cycles since the start of the calculations. At the left, the oscillations build up, at the right they die out.

Unfortunately, the current nonlinear, nonadiabatic integrations are still lacking in certain details. The observed light is influenced by the thin atmosphere of the star, where the formulas for radiation diffusion are not very satisfactory. Furthermore, the layer at whose base hydrogen is fully ionized and at whose top it is not ionized is actually very thin compared with the mass shells used in the integrations.

Therefore, the exact shape of the light curve, and the lag of its maximum relative to the time of maximum compression, may not be well determined. Nevertheless, the light curves computed with a few optically thin mass shells for the outer layers qualitatively resemble observed light curves. Finally, the effects of convection on the time variations of the energy flux have been calculated from a simple theory, which may not give correct results for the cooler stars, where the energy flow by convection is important.

We show above some results for two shallow-envelope models. The star is assumed to have a solid boundary only 15 percent of the radius inward from its outside, and at this level a surface of constant luminosity is introduced. The zoning is so coarse that hydrogen ionization cannot contribute to the instability. It is evident that the first model is unstable, small perturbations growing with time, at a rate that increases the radial-velocity amplitude by a factor of 2.7 every 100 cycles. After almost 1,000 cycles the pulsation amplitude levels off.

The second case is for a decaying oscillation, with other conditions like those in the first. Here, however, the He II ionization region is too near the surface for its driving effect to occur in enough stellar
material to overcome the interior radiative damping. For both models, the "noise" decays as the oscillation proceeds. The behavior of a shallow-envelope model after the oscillations have become of constant amplitude is detailed in three charts. They show for five different levels in a Cepheid the periodic changes in radius, velocity, and luminosity (expressed as bolometric magnitude). Although we can see various phase relationships in the star's interior, the surface fluctuations, especially in luminosity, are not reliable for this model, which has a relatively massive, opaque zone on top.

Christy has computed models of RR Lyrae variables with optically thin mass zones on top. He obtains the correct phase relationships between light and radial velocity for these Population II Cepheids that have periods shorter than one day. In our calculations here at Los Alamos, we have succeeded in obtaining the observed phase relations for classical Cepheids also.

Turning back to the H-R diagram, we can plot there the results of nonlinear, nonadiabatic integrations of shallow-envelope models by David S. King, to see how closely they match the instability strip in which the observed pulsations of Cepheids take place (dashed outlines).

Left: Changes in absolute bolometric magnitude (scale at upper left) for the 50-layer model, with the curves displaced to avoid overlapping. Note the largest variation is for an inner layer, while the outermost (50) has a "stillstand" on the descending branch of the light curve, just as observed for many Cepheids in the sky.

Right: Both stable (circles) and unstable Cepheid shallow-envelope models by D. S. King indicate an instability strip nearly coinciding with the observed strip (dashed lines). The masses are solar units.

All illustrations with this article, except as otherwise noted, are from the authors. He chooses three mass values, and in each of these shows solid dots if pulsations are maintained (unstable stars), open circles if they die out (stable stars). He has assumed a mixture of elements proposed by L. H. Aller, valid for stars of solar composition, but the lack of closer coincidence with the observed instability strip may be due to his shallow envelopes, neglect of hydrogen ionization as a driving source, and the neglect of energy flow by convection.

The physical causes of stellar pulsation in the Cepheid, RR Lyrae, and possibly also the W Virginis variables, now seem to be understood in some detail. Perhaps the application of these numerical integration techniques, combined with a time-dependent theory of convection, will allow insight into the redder pulsating variables, such as the Mira-type stars, which are still only rather poorly understood.