72. Some Problems in Temperature Measurements from Line Spectra

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The problems dealt with in this article arise in the formulation of a general theory of spectral line formation and in its application. We shall not be concerned with details of procedures, such as the curve of growth, or model atmosphere computations. These are adequately treated in many places—see e.g., Aller,① Mustel,② Minaert,③ Unsöld.④ Our interests lie rather in the validity of the hypotheses underlying such treatments.

We may as well say at the outset that we give no simple set of rules whose application would result in a definitive temperature structure, even on the assumption that a temperature can be defined. The general philosophy of the approach is to attempt to sketch the development of a physically sound theory for use in computing the spectrum of a given model atmosphere, and to outline some current difficulties. By a study of the dependence of the shape and intensity of a spectral line on the atmospheric parameters, we may then be able to infer something about an atmosphere emitting a particular spectrum. A difficulty lies in our inability to test any theory by controlled experiments on the observed atmosphere. This results in a necessity to fall back on consistency arguments; in a stellar atmosphere, however, the parameters which may reasonably be introduced to describe the atmosphere are so numerous that consistency is apt to be a rather weak reed.

The source of energy which balances the radiative loss of a stellar atmosphere may be either radiative or mechanical. In either case, energy is supplied to the electrons and ions in some fashion and these, in turn, interact by elastic or inelastic collisions with one another and set up equilibrium distributions among the three energy modes (1) kinetic energy, (2) internal energy, and (3) radiation. In thermodynamic equilibrium the interaction among these modes at any point is complete and, as a consequence, the energy partition can be described in terms of a single parameter—the thermodynamic temperature. A condition that this particular partition applies is that the radiation be enclosed; this evidently cannot apply in a stellar atmosphere and, correspondingly, we must enquire how a temperature is to be defined under these conditions.

The equilibrium configuration of a gaseous atmosphere is specified by the equation of the rates into and out of each energy region associated with each energy mode. Such a specification is too general a problem to be feasible, at this stage, for a study of stellar atmospheres, and it has become customary to assume that the velocity distributions of the atmospheric constituents are Maxwellian. In a stellar atmosphere this seems reasonable because of the preponderance of redistributitional elastic collisions over inelastic ones—the latter, of course, tend to produce departures from the Maxwellian distribution since they occur at energies corresponding to the excitation potentials of the atomic energy levels. It is further usual to assume, following Bhatnager, Krook, Menzel, and Thomas,⑤ that the kinetic temperatures of the electrons and ions are equal. Nevertheless, the validity of these assumptions plays such a central role as to warrant deeper consideration in the future.

The distributional temperature is manifested in line profiles in two distinct ways. Firstly, by influencing the rates of electron collision excitations, it will play a part in determining the populations of the atomic levels, and so in influencing emission and absorption rates. Secondly, the thermal Dop-
pler effect produces broadening of the line absorption and emission coefficients. However, the influence is not at all straightforward; the atmospheric opacity, and, therefore, its density, and also macroscopic velocities, influences the line strengths and profiles. The problem of separating thermal and macroscopic velocities is a difficult one which has, to my knowledge, never been unambiguously solved in any single case.

The equilibrium radiation field is described by the equation of transfer, which for a plane parallel atmosphere can be written

\[ \mu \frac{dI_z}{dz} = -\langle \kappa_t + \kappa_e \rangle I_z + \langle \kappa_t + \kappa_e \rangle \]

In this equation, \( \mu = \cos \theta \), \( \theta \) being the angle made by the radiation with the outward normal; \( \kappa_t, \kappa_e, \xi_t \), and \( \xi_e \) are emission and absorption coefficients in the line and continuum; \( I_z \) is the specific intensity; and \( z \) the depth. By introducing the optical depth in the line namely,

\[ \tau_z = -\int \kappa_z(\nu) \, dz \]

Eq. (1) may be written,

\[ \frac{dI_z}{d\tau_z} = (1 + \tau_z) I_z - \langle \xi_t + \tau_z \rangle S_1 \]

where \( \tau_z = (\kappa_t/\kappa_z) \), and \( S_1, S_z \) are the source functions defined as ratios of the emission to absorption coefficients in the line and continuum, respectively.

Considering the physical approximations inherent in the theory, it is certainly justifiable to adopt the Eddington approximation to Eq. (3), namely,

\[ \frac{1}{x_c^2} \frac{dI_z}{d\tau_z} = I_z - \frac{S_1 + \tau_z S_z}{1 + \tau_z} \]

where \( x_c = (1 + \tau_c) \sqrt{3} \).

The solution of Eq. (3) is immediate for \( \tau \ll 1 \), when the emergent intensity is simply the integral of the emission (for an emission line). For an absorption line the situation is slightly more complicated; however, in both cases the line profiles will reflect directly those of the absorption or emission coefficient in the absence of macroscopic motions and if the atmosphere is uniform throughout. This fact has been widely used in attempts to determine thermal velocities from stellar spectra; in the solar case they have met with little success.

For a Maxwellian velocity distribution of the emitting atoms, the line shape should be Gaussian with a \( 1/e \) width given, for purely thermal broadening, by

\[ \Delta \lambda(1/e) = \frac{\lambda}{c} \sqrt{\frac{2kT}{m}} \]  

where \( T \) is the kinetic temperature of the atoms.

For weak solar emission lines, such as are observed in prominences, the profiles are indeed usually Gaussian; unfortunately, however, observations of lines of two or more different elements frequently give incompatible thermal velocities. In this case the discrepancy is commonly regarded as a manifestation of velocities of macroscopic elements. To account for the Gaussian line shape these are supposed to have a Gaussian distribution of velocities with mean velocity \( \xi \)—the so-called “micro-turbulent” velocity. In this case the \( 1/e \) width is

\[ \Delta \lambda(1/e) = \frac{\lambda}{c} \sqrt{\frac{(2kT)/m + \xi^2}} \]

Since \( m \) varies while \( \xi \) is, presumably, the same, from element to element it is possible from observations of, say, H and He lines to obtain a value both of \( T \) and \( \xi \). Such a procedure, however, frequently results in temperatures too low to excite He I—Jeffries and Orrall, P. ten Bruggencate and Elste. If lines of a third element (say Fe I) are added, it seems to be largely a matter of chance as to whether or not their widths are compatible with those of the other two elements. This is commonly regarded as requiring a separation of the atmosphere into regions, each of which is responsible for emission of a separate element—Jeffries and Orrall, Tandberg-Hanssen and Zirin.

Redman and Suemoto\(^9\) have tried similar techniques for analysis of profiles of the flash spectrum lines of H, He I, and metals. For H they inferred a temperature \( \sim 11,000^\circ \text{K} \) at \( \sim 1000 \text{ kilometer height} \), which seems reasonably in accordance with other results—Thomas and Athay. However, combining it with the He I lines according to Eq. (6) reduces this to \( \sim 5000^\circ \text{K} \), a value again too low to support He I emission—Jeffries.\(^11\) Combining the metal and hydrogen widths reduces \( T \) to \( \sim 10,000^\circ \text{K} \); but this is then incompatible with the He line widths. This type of situation appears too frequently in the solar atmosphere to be accidental.

The difficulty is, apparently, that macroscopic velocities and kinetic temperatures may, in gen-
eral, all be functions of depth while the line emissions can have depth variations different for each element. The atmospheric parameters are, therefore, of such diversity as to allow a fit with any observations. At the same time these analyses can only be expected to be satisfactory for non-self-absorbed lines and these, in their very nature, can reflect only mean conditions. This whole question is considered in more detail by Orrall; the general results are, however, not such as to inspire much confidence in any current procedures for inferring kinetic temperatures from the shapes of weak lines.

For strong lines the line shapes and strengths reflect also the influence of self-absorption. Their analyses can only be undertaken in terms of the transfer Eq. (3) or its essential equivalent, Eq. (4). This immediately raises what may be termed the fundamental problem of line formation, namely, the specification of $S_1$. To analyze the total emission of lines showing no self-absorption requires a computation of the number of emitting atoms in the line of sight. Thus in both cases a specification of level populations is needed; we consider the general problem first.

THE SPECIFICATION OF $S_1$

For the centers of strong lines, or for lines formed in the absence of continuum emission, $r$, may be set equal to zero. The specification of $S_1$ as a function of the local atmospheric parameters requires a specification separately of the emission and absorption coefficients. These coefficients, in turn, depend on four separate quantities, (1) the particular transition, through the oscillator strength, (2) the profile of the absorption coefficient, (3) the profile of the emission coefficient, and (4) the ratio of the number of atoms in the upper and lower states of the transition.

For a given transition and for a given location in an atmosphere, the specification of the oscillator strength and of the profile of the absorption coefficient are problems of physics which have received considerable attention, particularly in the last few years.

A good deal of information, both theoretical and experimental, is available on oscillator strengths. This is summarized in various places, see e.g., Unsold, Allen, and Pearce. The frequency variation of the absorption coefficient has been widely discussed in various reviews—see e.g., Margenau and Lewis, Breene, Unsöld, and in individual studies e.g., by Anderson, Baranger, and Kolb and Griem. While obviously of the most fundamental importance to a specification of $S_1$—and so to the study of line spectra—both of the above matters are rather beyond the scope of the present review, and as such are not considered further here.

We may, in spite of the many problems of principle and practice involved in specifying these quantities, regard them as comparatively well-known. In any case they are independent of the radiation field whose value is sought from Eq. (3), and, in this fundamental respect, differ from the dependence entering under (4).

The Frequency Dependence of the Emission Coefficient

An atom may be excited by absorption of radiation or by collisional impacts. For collisional excitation the fundamental assumption of a Maxwellian distribution of velocities assures us that, within a particular energy level, no special energies are preferentially populated over those under conditions of thermodynamic equilibrium. Thus for emission resulting from collisional excitation we expect the profiles of the emission and absorption coefficients to be identical, from the principle of detailed balance, so long as the energy distribution of the colliders is flat over the energies in the lines.

If, however, excitation is due to absorption of radiation, a subsequent reemission—known as a scattering—may reasonably be expected to show some frequency coherence with that of the photon absorbed. Two extreme cases are considered in the literature according to whether (1) the re-emitted frequency is the same as, or (2) is entirely uncorrelated with that of the absorbed photon. These are, respectively, the cases of coherent scattering and complete redistribution. Various agencies act to destroy coherence; the "natural widths" of the states allows a reemission to fall back to a different region of the lower state to that from which it was absorbed; the reemission may be in a different line altogether—the case of interlocking; perturbations due to the interaction of the radiator with other atoms, ions, or electrons may be significant during the lifetime of the atom in the excited state, while the thermal Doppler effect will act to destroy coherence existing in the frame of the atom. These influences have been discussed by various authors, Woolley.
and Stibbs,20 Spitzer,21 Zanstra,22 Henyey,23 Unsöld,4 Thomas,4 and Jeffreys and White.25 For the core of the line, there seems no doubt that the thermal Doppler effect alone will destroy any frequency coherence existing in the frame of the atom; for the line wing the matter is still unclear, but urgently in need of clarification. Evidently the matter is not trivial. As pointed out by Eddington,26 in complete redistribution the radiation intensities are locked together by interchange among the different frequencies. In that case the whole line profile would be essentially controlled by conditions at the line center—because the absorption coefficient is largest there. For coherent scattering, on the other hand, each frequency is transferred through the atmosphere independently. We could expect (and indeed we find) the line centers to have about the same strengths on either extreme assumption. However, the line profiles, and especially the wing strengths, show considerable sensitivity to the scattering mechanism.

The Population Ratios

The other factor entering \( S_L \), namely the ratio of the upper and lower state populations, has been treated in two general fashions.

Firstly, there are the methods introduced by Milne27 and Eddington26 in which the population ratio is determined from the equations of statistical equilibrium. These state that the rates of entry and exit from a particular state are equal, so that the population (at any atmospheric location) of each level remains constant in time. Formally, they may be written

\[
\sum \frac{P_{ij}n_i - \sum P_{ji}n_j}{n} = 0 \quad (\text{all } i \neq j) \quad (7)
\]

where \( P_{ij} \) is the rate per atom in the \( i \) state of the transition \( i \to j \), and \( n_i \) is the number per unit volume of atoms in state \( i \). In general, \( P_{ij} = A_{ij} + C_{ij} \) where \( A_{ij} \) is a radiative and \( C_{ij} \) a collisional rate. For absorption transitions (and stimulated emissions) \( A_{ij} \) involves the radiation field, \( C_{ij} \) is proportional to the density of colliders—usually electrons are the important source of collision excitation having much greater cross sections than heavy particles.

The second general approach has bypassed the solution of Eq. (7) by assuming the ratio of any two populations to be given by the Boltzmann law for the "temperature" applying at the particular atmospheric location. This is the so-called assumption of local thermodynamic equilibrium (LTE). Although originally introduced for analyses of stellar continua, it has been very widely applied, in the past 20 years, in attempts to interpret line spectra. Its validity is considered later.

The Two- and Three-Level Approximations

In his original study of a two-level atom, Milne27 supposed that reemission following absorption was coherent in frequency. In keeping with the more modern conclusion that scattering is better represented as complete redistribution, we may write Milne’s expression for \( S_L \) in the form given by Thomas,24

\[
S_L = \frac{\int I_\nu d\nu + \epsilon B(T_L)}{1 + \epsilon} \quad (8)
\]

where \( I_\nu \) is the mean intensity, \( \phi_\nu \) the normalized profile of the absorption coefficient, \( B(T_L) \) the Planck function, and \( \epsilon \approx C_{11}/A_{11} \) the relative proportion of collisional to radiative de-excitations. The two terms in this expression represent, respectively, the reemission due to previous absorption of photons, and the new or "true" emission of radiation which arises directly as the result of collisions. This latter term, therefore, represents the direct conversion of thermal into radiant energy.

Since it seems (Thomas)24 that a simple atomic structure of two levels is a fairly good approximation for a discussion of certain resonance lines, it is worthwhile to consider it in some detail.

Firstly, it is evident that \( \epsilon \) is a fundamental parameter whose value controls the line behavior, since \( 1/1 + \epsilon \) is the probability of destruction of the photon at an absorption. In a stellar atmosphere, the electron density \( n_e \) in the region of formation of strong lines may be \( \sim 10^{14} \) to \( 10^{15} \) cm\(^{-3} \), while the spontaneous transition probability is \( \sim 10^8 \) sec\(^{-1} \). The value of \( \epsilon \) is, therefore, small—as pointed out first by Pannekoek28—ranging somewhere in the neighborhood of \( 10^{-4} \) to \( 10^{-6} \), depending on the transition. In other words, \( S_L \) will depend strongly on the local radiation intensity. This intensity must, in general, decrease towards the top of the atmosphere simply because of the presence of a boundary, and so \( S_L \) must also decrease towards the top.

Now it is a well-known consequence of the transfer Eq. (3) that, if \( \tau_s > 1/\mu \) where \( \tau_s \) is the total optical thickness of the atmosphere,

\[
I_s(0, \mu) \simeq S_L(\tau_s = 1/\mu) \quad (9)
\]
\( I_\nu(0,\mu) \) is the emergent intensity in the direction \( \mu \). What general consequences then follow from the fact that \( S_t \) is controlled by the local value of \( I_\nu \), i.e., that \( \epsilon \) is small? Consider first an atmosphere of negligible continuous opacity—such as the solar chromosphere—which is completely homogeneous (no gradient in \( T_e \) or \( n_e \)) and free of internal macroscopic motion. Since \( S_t \) decreases toward the surface and since \( \kappa_r \) has its maximum value at the line center, \( S_t(\tau = 1/\mu) \) will increase away from the line center and continue to increase until frequencies are reached such that the atmosphere has optical thickness less than about 1/\( \mu \). From then on the emission drops to zero with the optical depth. From Eq. (9), therefore, the observed line shape will be reversed in the center. This phenomenon is observed in all strong chromospheric lines, though its interpretation seems usually to have been sought in instrumental terms, in velocity fields, or in a temperature structure.

In the corresponding case of disk observations, the presence of a boundary results in formation of absorption lines in a completely isothermal atmosphere. The central intensity of the line is readily shown to be determined almost completely by the value of \( \epsilon \) being, roughly, \( \sqrt{\epsilon} \) times of the continuum. It is to be noted that \( \epsilon \) is only very slightly dependent on \( T_e \); it is controlled mainly by \( n_e \) and, of course, the \( f \) value and the collision cross section.

Evidently the assumption of LTE applied to analysing these spectral lines would predict a temperature decreasing towards the surface and a value much too low near the top. This simple example is indicative of the difficulties to be expected in trying to infer temperatures in a stellar atmosphere. It is also, perhaps, suggestive of the reliability which one may associate with strong line analyses based on the LTE assumption.

Similar difficulties would apply to the inference of \( T_e \) from line shapes formed in accordance with this simple two-level atom model. The chief difficulty may be summarised in that both \( n_e \) and \( T_e \), and their gradients, influence the line intensity and profile, together of course with any velocity fields, and to separate their individual effects seems possible at present only on the basis of experience gained from solutions for individual atmospheres. For this reason Thomas and the author have, in a series of papers\(^{22}\) considered some of the consequences of the form in Eq. (8) for \( S_t \), on the shapes of spectral lines emitted from stellar atmospheres. In this case the transfer Eq. (4) is

\[
\frac{1}{\sigma^2} \frac{dI_\nu}{dr} = I_\nu \int I_\nu \, dr - \lambda_r B(T_e)
\]

with

\[
\lambda_r = 1 - \omega_r = (\epsilon + \tau_r)/(1 + \tau_r)
\]

and it is assumed that \( S_t = B(T_e) \). Replacing the integral by a quadrature and solving the resulting coupled equations with appropriate boundary conditions, one may find \( S_t \) and so \( I_\nu(0,\mu) \). Depth variations in \( B(T_e) \) can be readily treated, in particular we have considered the case of a positive temperature gradient (as in the solar chromosphere) for which,

\[
B(T_e) = S_t(1 + A e^{-\epsilon r})
\]

On this basis we have been able to account, at least qualitatively, for many line shapes observed in stellar spectra. For example, the self-reversed profiles of the Ca II lines, H and K, have been shown to follow as a logical consequence of the temperature increase in the solar chromosphere; the shape of the solar Lyman-\( \alpha \) line has been accounted for along the same lines by Morton and Widing\(^{23}\) and self-reversals observed in many solar emission lines from the chromosphere, prominences, and flares appear as almost inevitable consequences of this simple physical approach.

For resonance lines of neutral metals, and for most strong subordinate lines, Eq. (8) is modified since ionization to, and recombination from, the continuum then becomes important in maintaining the upper and lower state populations. In this case—as first pointed out by Stromgren\(^{24}\) for coherent scattering—Eq. (8) must be generalized, following Thomas,\(^{25}\) to the form,

\[
S_t \approx \frac{\int I_\nu \, dr + \eta B^* + \epsilon B(T_e)}{1 + \epsilon + \eta}
\]

where \( \eta \) and \( B^* \) are controlled by the rates of photoelectric ionization and recombination. The so-called photoelectrically controlled lines, for which \( \epsilon B(T_e) \leq \eta B^* \), are exemplified by solar H\( \alpha \). For them, the depth variation of \( S_t \), and so the line profile, seem hardly at all affected by temperature gradients; they should not, then, show the disk central reversal like "collisionally controlled" lines, e.g., H and K, for which \( \eta B^* \ll \epsilon B(T_e) \). Indeed, this is found to be so—the oc-
casional appearance of self-reversal of Hα in disk flares is not at variance with this since the physical conditions in flares are far removed from those of the quiet chromosphere.

Thus, for the strong lines for which $S_1$ is described by Eq. (12), this simple theory seems to give reasonable agreement with observations. It is somewhat premature, however, to regard it as adequate for much more than the roughest estimates of temperature in a stellar atmosphere.

The influence of density gradients have not been studied in detail; some preliminary results are given by Miyamoto. Their influence would be to make $\kappa$ a function of depth and so to modify the depth variation of $S_1(\tau)$. This would be reflected in a changed line profile. When $\kappa$ varies with depth, however, the equations no longer have constant coefficients and this mathematical difficulty has tended to defer its proper study.

The Multilevel Atom

To consider simultaneously the transfer of radiation in many lines is hardly feasible at present, since an n level atom emits, in general, $n^2/2$ separate radiations. The main progress so far has been for cases in which the atmospheric opacity is very high or very low for all lines, so that either the radiative processes balance in detail or absorptions are specified by the strength of the incident radiation. This procedure has been widely used to determine the level populations and the consequent emission in optically thin lines. Thus it is the basis of much work on the excitation of planetary nebulae—Menzel et al., and of much of that on H, He I, and He II excitation in stellar atmospheres—Giovanelli, Thomas, Jeffries, Zirker, Zirin, Krat and Sobolev, and Athay and Johnson.

Some preliminary attempts to account for interlocking effects have been made by Pottash and Thomas, and by Jeffries. The latter work has demonstrated that $S_1$ for two strong lines with a common lower level such as H and K, Na D1 and D2, and Lα and Lβ will have a common depth dependence for $S_1$ below a certain optical depth, whose value depends on the atom and the excitation conditions. This has been borne out in detail for the line core by Waddell’s recent observations of the solar Na D lines. It has important consequences for analyses such as those of Goldberg, Mohler, and Muller on solar H and K since, on making this assumption, it is possible to obtain from the two profiles a run of Doppler width with depth; this procedure has been suggested also by Athay and Thomas, and de Jager. On less certain grounds theoretically is the use by Unno of this procedure for pairs of multiplet lines of metals with complex atomic level structures.

Until a satisfactory solution to this general problem is obtained, analyses of line spectra will be hindered. Yet one is appalled at the difficulty of a direct attack even if the relevant atomic collision and radiative parameters were available. It seems, to this author at least, that further progress may be possible on two general lines. Firstly, if a direct approach is attempted one must somehow assess the influence of simplifying the level structure. This may be possible from general solutions of the equilibrium equations—Rosslund, White, and Jeffries. The latter paper shows that a general solution can be written in the form

$$\frac{n_j}{n_k} = \frac{\sum P_{k\rightarrow j \rightarrow k}}{\sum P_{k\rightarrow j \rightarrow k}}$$

where, e.g., $q_{j\rightarrow k}$ is the probability of a chain of transitions leaving 1 which arrive in $j$ without any stage going through $k$. It should be no great problem to determine the perturbation on the $q$ due to the addition of one or more states. Even then, however, a rather large number of simultaneous transfer equations will remain and these will, in general, be nonlinear. This is not insuperable, but neither is the necessary machine computation likely to throw much light on the physics of line formation.

The second possible procedure is to adopt what we may call a “morphological” approach, following Zwicky. The radiation emitted by an atom in all its lines and continua is a property of the atom as a whole, not just of the particular levels giving rise to this emission. It is, therefore, essentially wrong to consider the transfer in one line without simultaneously accounting for all the others. Thus, the depth variation of $S_1(Lα)$ is an essential function of that in all other lines and continua emitted by hydrogen; at high enough depths they can all be described in terms of a common parameter: This is suggestive that an attempt to describe the whole radiation field may be profitable; such a “thermodynamic” approach has been suggested by Wildt, but its application appears difficult.
The Hypothesis of Local Thermodynamic Equilibrium

The evident difficulties associated with a complete study along the above lines have prompted many investigators in the past 20 years to bypass the whole problem and assume that

\[ S[fT(\tau)] = B[fT(\tau)] \]

the LTE hypothesis. It was certainly a procedure worth trying had its physical validity proved amenable to consistent testing. However, the almost infinite possibilities—using any theory—for choosing a model consistent with observation requires us to consider its justification in some detail, since a great many temperature determinations in astrophysics rely on its truth.

Firstly, it is obvious that lines formed under conditions where collision excitation is predominant in all transitions will be formed essentially in LTE. Roughly, this requires formation in regions where \( n_e \geq 10^{6} \text{cm}^{-3} \), some possible exceptions are considered later.

The main argument for the applicability of LTE in stellar atmosphere is that given by Unsold, which runs as follows. If the radiation in a line is enclosed so that it cannot escape, we can define a thermodynamic temperature. If any scattering is largely noncoherent, \( S_f \) for all frequencies is a function of the optical depth at the line center, hence, as Unsold states, for high enough optical depth in the line center the intermediate field will be bounded and, in consequence, LTE may be assumed. However, the crucial question is, how great must this optical depth be? Unsold implies that it is unity at the line center so that, since the observed radiation in all but the line center arises at lower depths, the whole line may be taken to be formed in LTE.

This conclusion is, however, premature. The condition of boundedness only has meaning when applied to the radiation in all its emitted frequencies (see the Foregoing). A photon which, on absorption, has a finite probability of reemission in another spectral region which escapes from the atmosphere, is not bounded. Thus the radiation field needs to be bounded in all frequencies emitted by the atom before one can expect it to couple so completely with the thermal energy that LTE will be \textit{a priori} applicable. This condition is somewhat relaxed if the continuum radiation field is roughly Planckian since then it is only necessary that those radiations be bounded which contribute more to the local intensity than does the continuum. Such lines, however, would not be observed in stellar spectra.

The fundamental point, then, is that the radiation fields interlock strongly, and so cannot be considered separately. Most attempts to prove the applicability of LTE to spectral lines, e.g., the above argument of Unsold and that of Mitchell, seem to have failed to recognize this.

However, many atomic levels will be populated almost as in LTE if they lie close to the ionized levels, e.g., the high hydrogen levels. The lines originating from such levels will, in general, be weak and so less able to provide much information on stellar atmospheric structure. At present, no simple criterion can be suggested as to whether or not a line is formed in LTE. In any case, a line which is self-absorbed, and so able to reflect atmospheric gradients, seems unlikely to be formed in LTE; it is not even certain that this assumption is valid for the wings, though it is used almost universally.

\section*{TOTAL EMISSION IN OPTICALLY THIN LINES}

It has already been pointed out that the widths of nonself-absorbed lines should, in principle, give some measure of the thermal and macroscopic velocities. The emission is given by an integral of the form

\[ I_v = \frac{A h \nu}{4 \pi} \int \chi_v(z) n_u(z) dz \]

where \( A \) is the spontaneous transition rate, \( \chi_v(z) \) the normalized profile of the emission coefficient, and \( n_u \) the upper state population. The integration is over the total thickness of the atmosphere in the line of sight. The assumption that \( n_u(z) \) is a common function for all elements is normally employed to relate the widths of lines of various elements. When this procedure fails to result in a unique \( T_e \) and \( \xi \), as it frequently does, we must seek the explanation in terms of differential depth dependences of \( n_u \). Evidently, any set of observations of line \textit{widths} can be made compatible by the simple (if unsatisfying) procedure of requiring the emission from each element to arise in a separate atmospheric region characterized by different values of the Doppler width.

To consider the \textit{total} emission, the form of the
absorption coefficient is immaterial, since in that case

\[ \int I_t \, dz = (A_{\text{He}}/4\pi) \int n_e \, dz \quad (15) \]

Unless we are prepared to make some geometric assumption on the depth distribution of \( n_e \), we can hope only to determine mean values from an estimate of the total thickness. Even so, this is sometimes very useful in placing limits on the relevant electron temperatures.

Thus Seaton has considered the dependence of emission in certain forbidden lines as a function of electron temperature and density. Comparing the observed and computed relative emissions in two lines, he has been able to deduce mean values for \( n_e \) and \( T_e \) for planetary nebulae. This question has been treated extensively by Seaton and is not considered further here.

This method has also been applied to the solar chromospheric He I emission whose spectrum is quite sensitive to temperature due to the high excitation potentials (\( \sim 20 \) eV) of the first excited state. The very appearance of He I lines seems to require \( T_e \gtrsim 12,000 \text{K} \) if the excitation is due to collisions—Jefferies, Athay and Johnson, and Krat and Sobolev. This fact is frequently useful in determining whether or not temperatures derived from line widths are reliable. The He I emission of the solar chromosphere as observed at eclipse has been analyzed from solutions of the equations of the statistical equilibrium by Athay and Johnson, and by Jeffries; however, the results are very different—the former two papers deduce \( T_e \sim 50,000 \text{K} \), the latter \( \sim 15,000 \text{K} \).

The total emission in thin lines seems, at present, capable of giving more reliable information on electron temperatures than any other procedure using spectral lines, provided the equilibrium equations can be solved: This requires knowledge of the transition rates, radiative and collisional, which in turn requires an analysis of the radiation field. Thus even this procedure comes back to the necessity for a specification of \( S_i \), or at least the ability to set criteria for optical thickness and thinness.

Finally, we may note that optically thin lines are incapable of giving details of the gradients of the atmospheric parameters unless other assumptions on distribution of emission are made.

**SUMMARY AND CONCLUSIONS**

The interdependent influences of electron temperature, density, velocities, and opacity on the observed profile of a spectral line make their interpretation exceedingly difficult. As has been stressed, the fundamental problem in the theory of formation of spectral lines lies in the specification of \( S_i \) as a function both of frequency and depth.

The frequency dependence near the line core seems to be well represented by complete redistribution. In the wings, however, the situation is less clear; Zanstra has suggested that, for a resonance line, the degree of coherence in scattering is given by \( \gamma_n/(\gamma_m + \gamma_c) \) where \( \gamma_c \) is the collisional and \( \gamma_n \) the radiative damping constant. An essentially similar effect is found in the influence of velocity fields on line profiles, an influence which confuses temperature determinations quite as much as opacity effects. A preliminary study of this is given by Pecker and Thomas.

The determination of the depth variation of \( S_i \) involves the enormous complication of interlocking. In some cases it seems possible to simplify the atomic model to a two or three-level atom. This has been, in any case, quite a fruitful assumption in giving a qualitative description of spectral line shapes.

**References**